

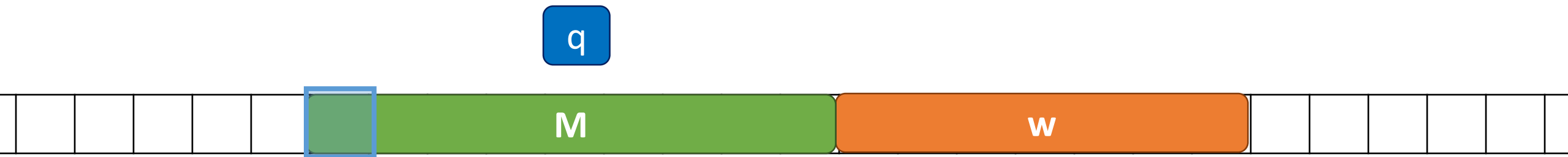
3b

Universal Turing Machines

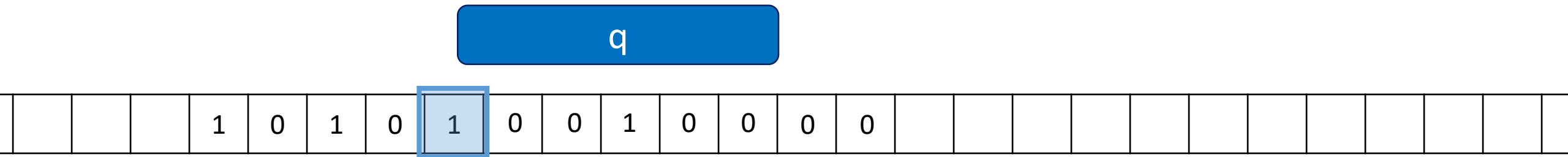
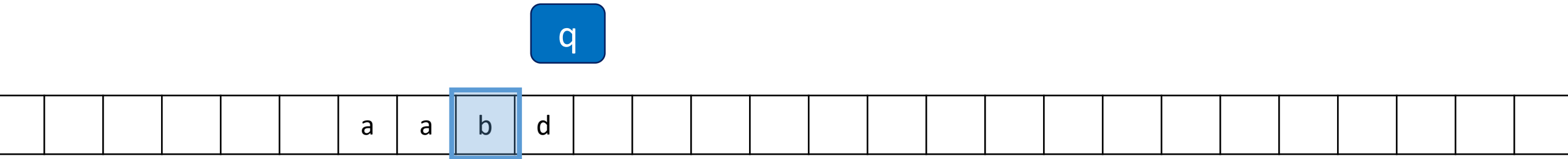
Analiza Algoritmilor

Image source: https://en.wikipedia.org/wiki/Wolfram%27s_2-state_3-symbol_Turing_machine

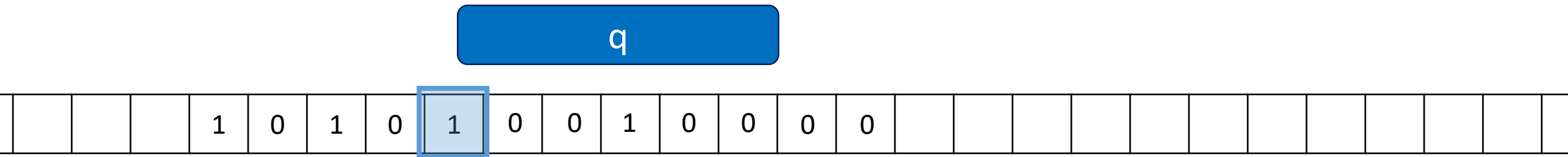
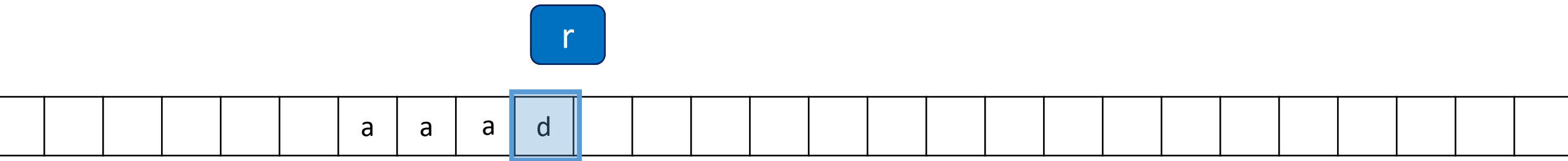
Vizualization



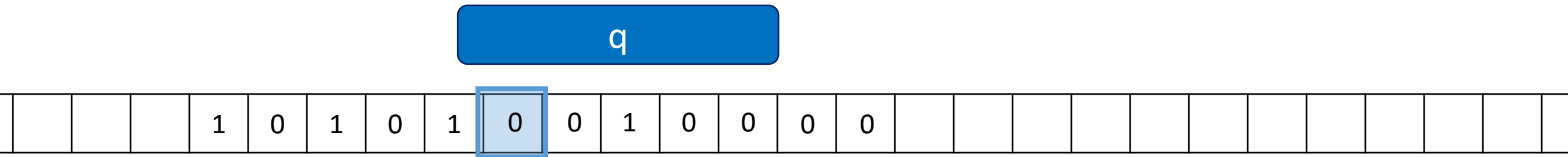
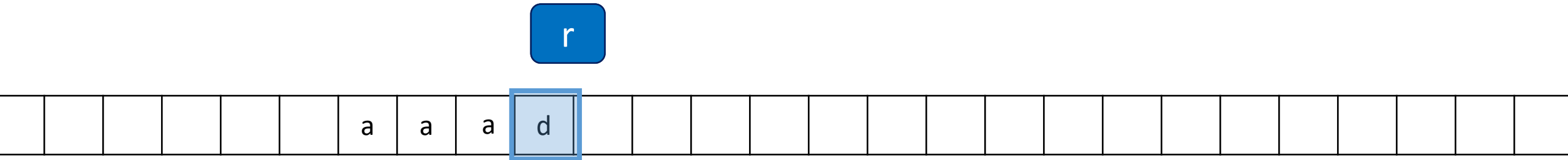
Dealing with different alphabets



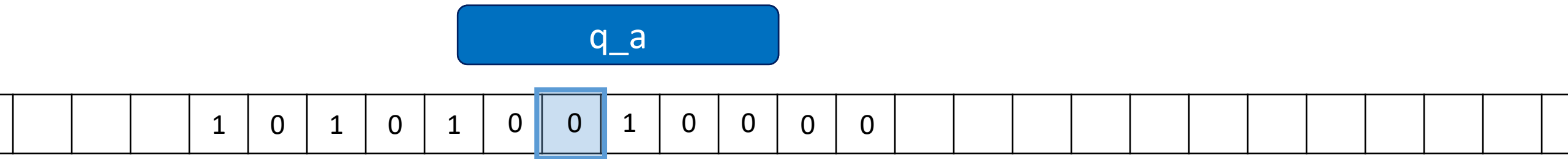
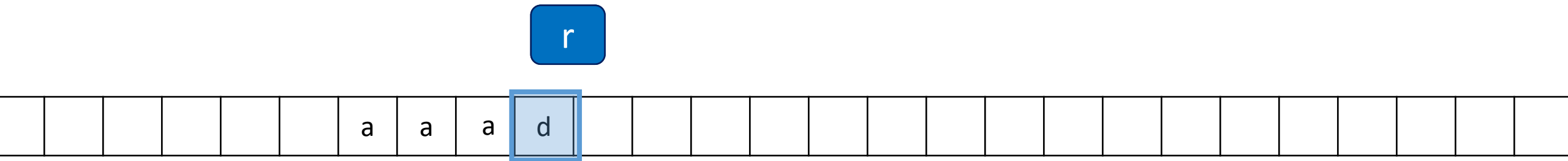
Dealing with different alphabets



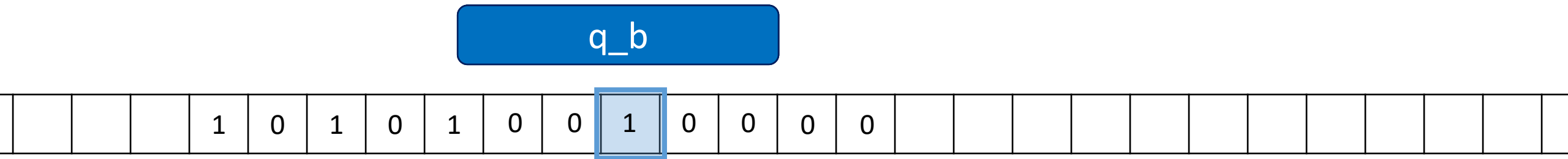
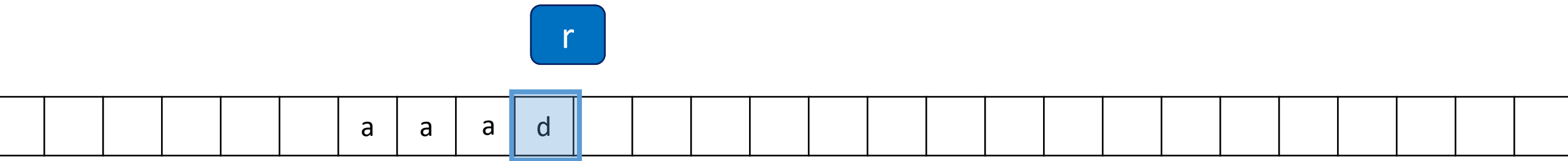
Dealing with different alphabets



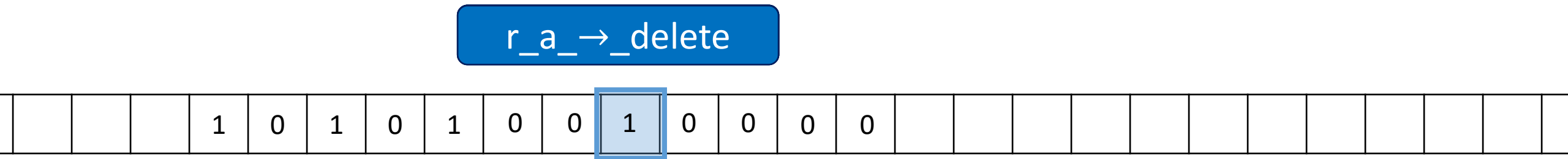
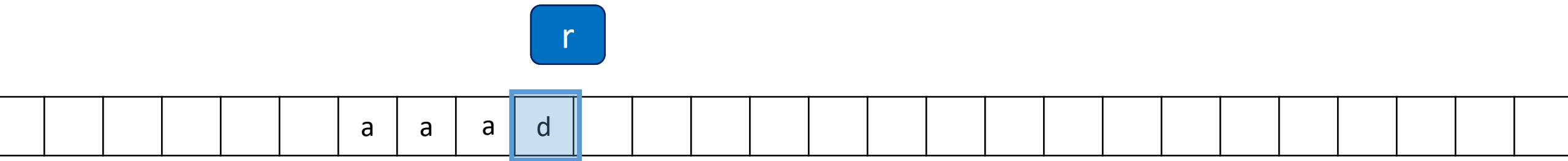
Dealing with different alphabets



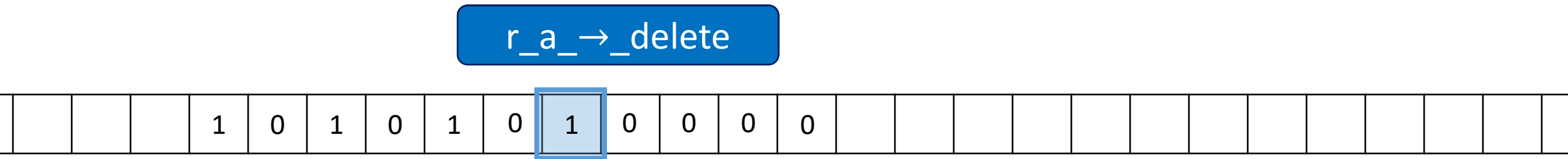
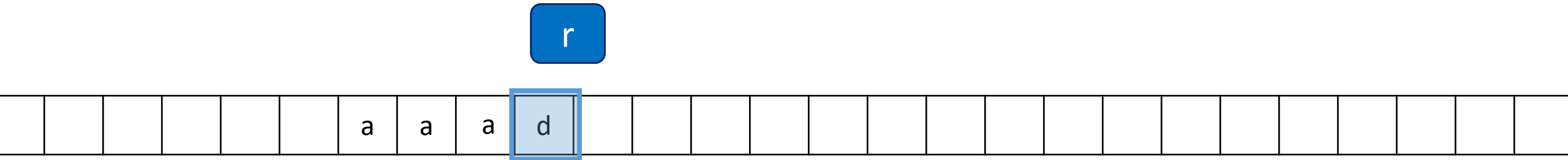
Dealing with different alphabets



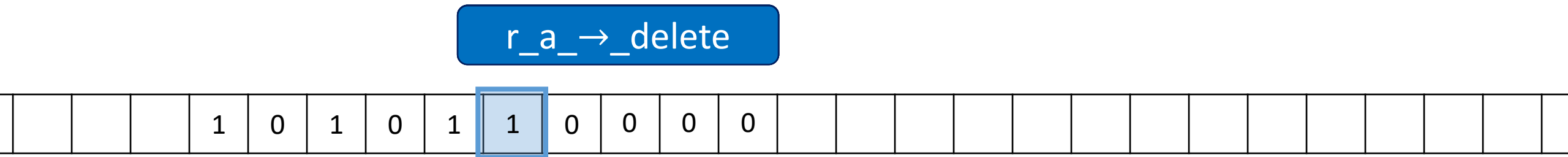
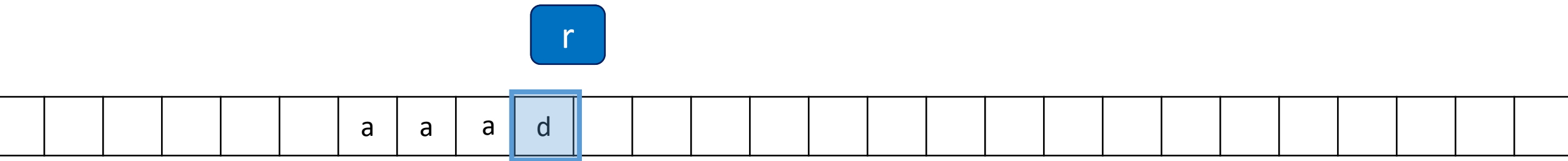
Dealing with different alphabets



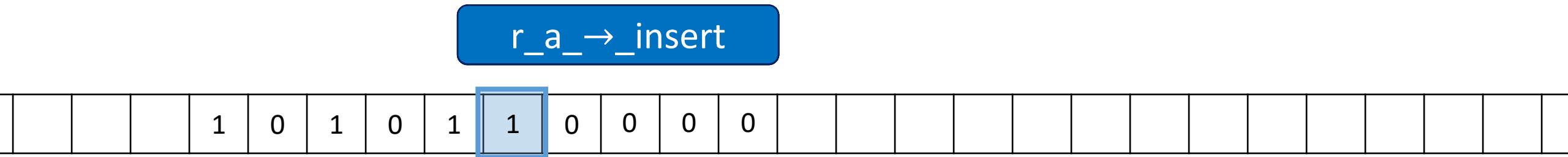
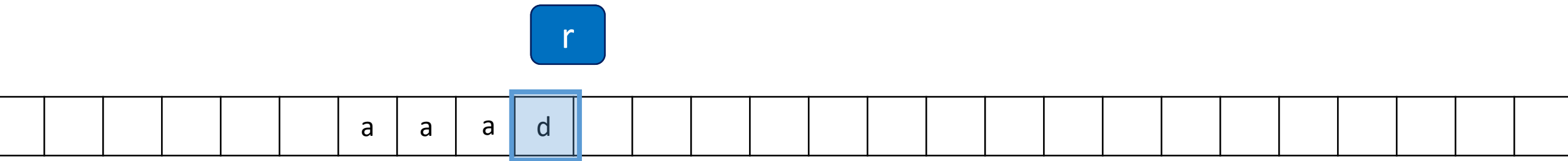
Dealing with different alphabets



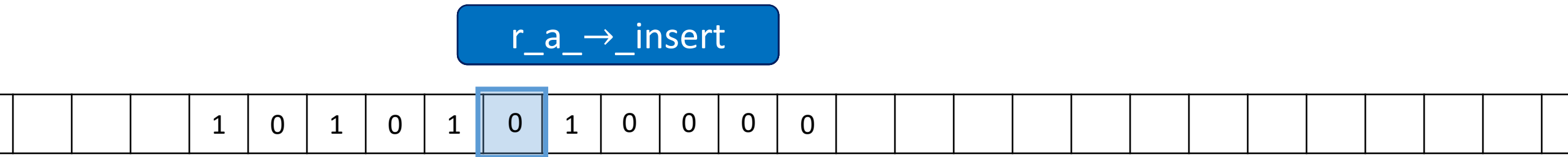
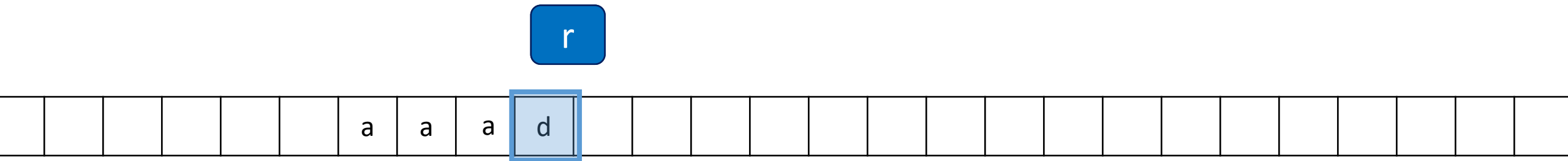
Dealing with different alphabets



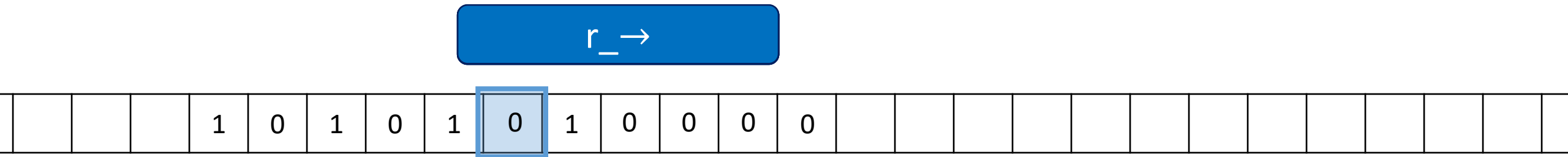
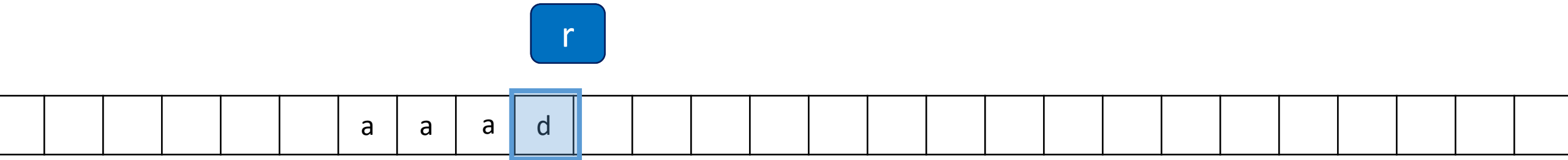
Dealing with different alphabets



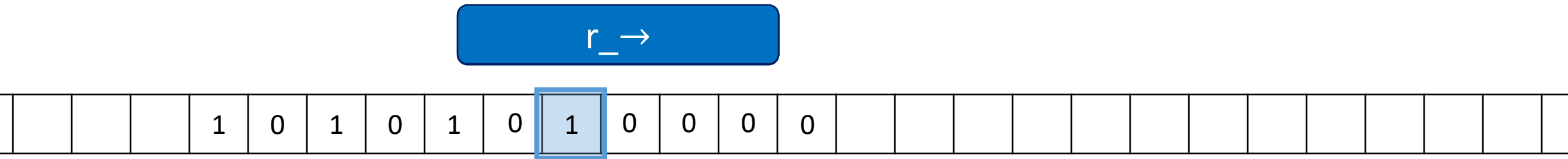
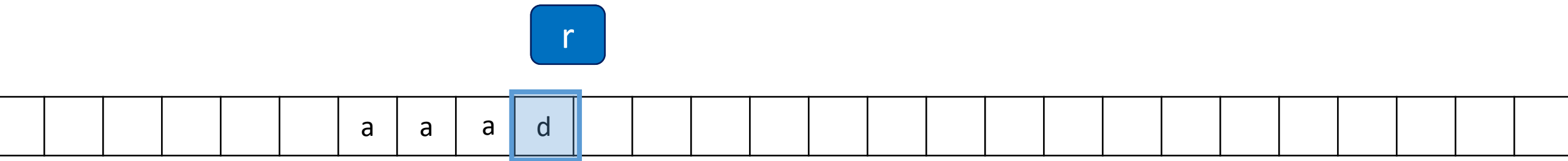
Dealing with different alphabets



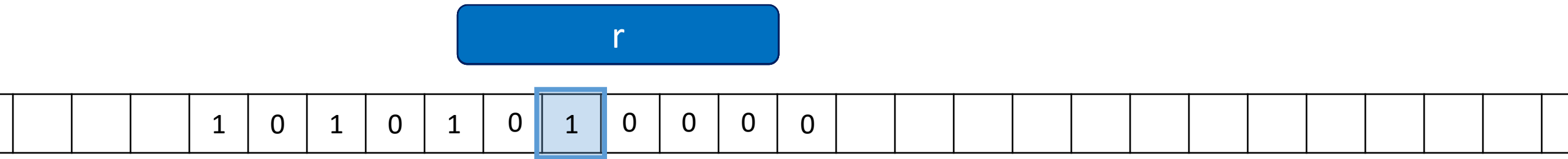
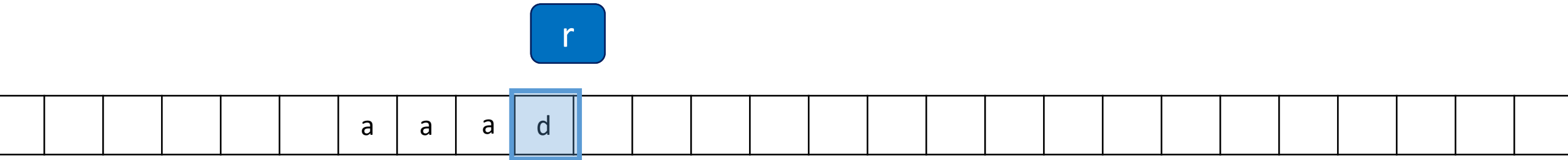
Dealing with different alphabets



Dealing with different alphabets



Dealing with different alphabets



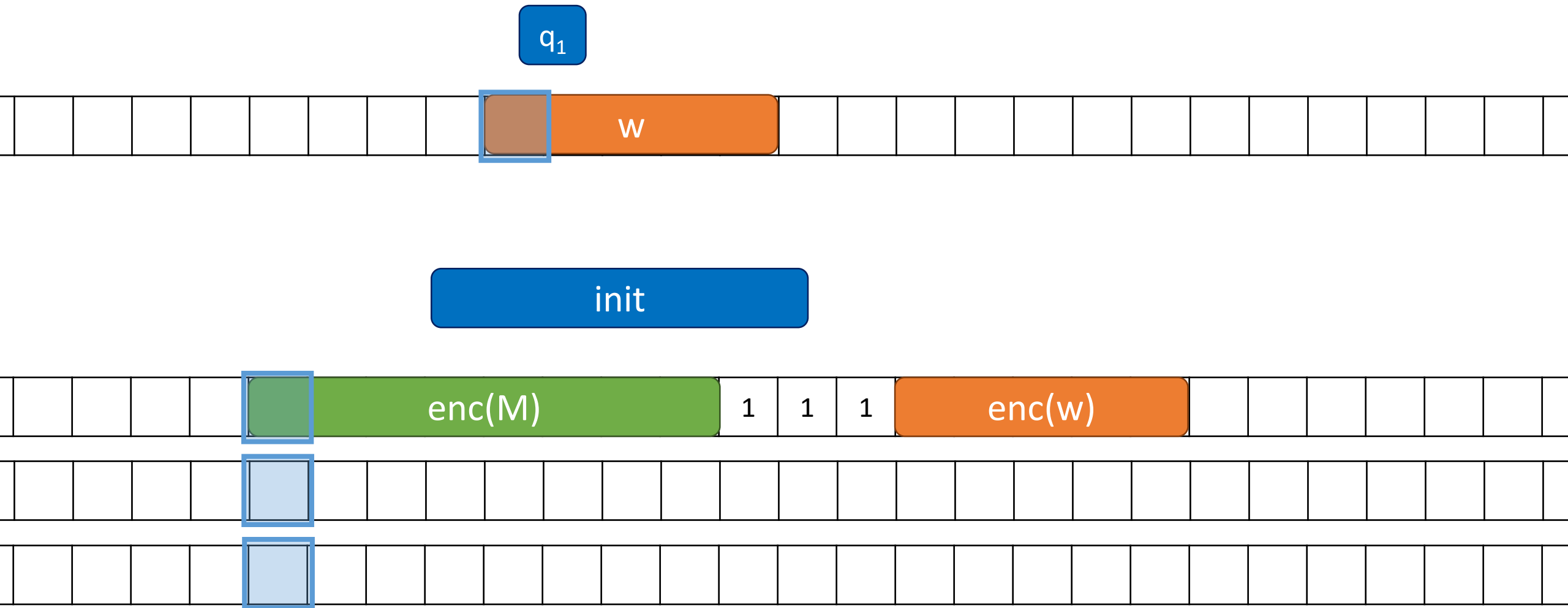
Encoding example – isEven

	0	1	$\square$
q_1	$q_1, 0, \rightarrow$	$q_1, 1, \rightarrow$	$q_2, \square, \leftarrow$
q_2	$Y, 0, -$	$N, 1, -$	$N, \square, -$

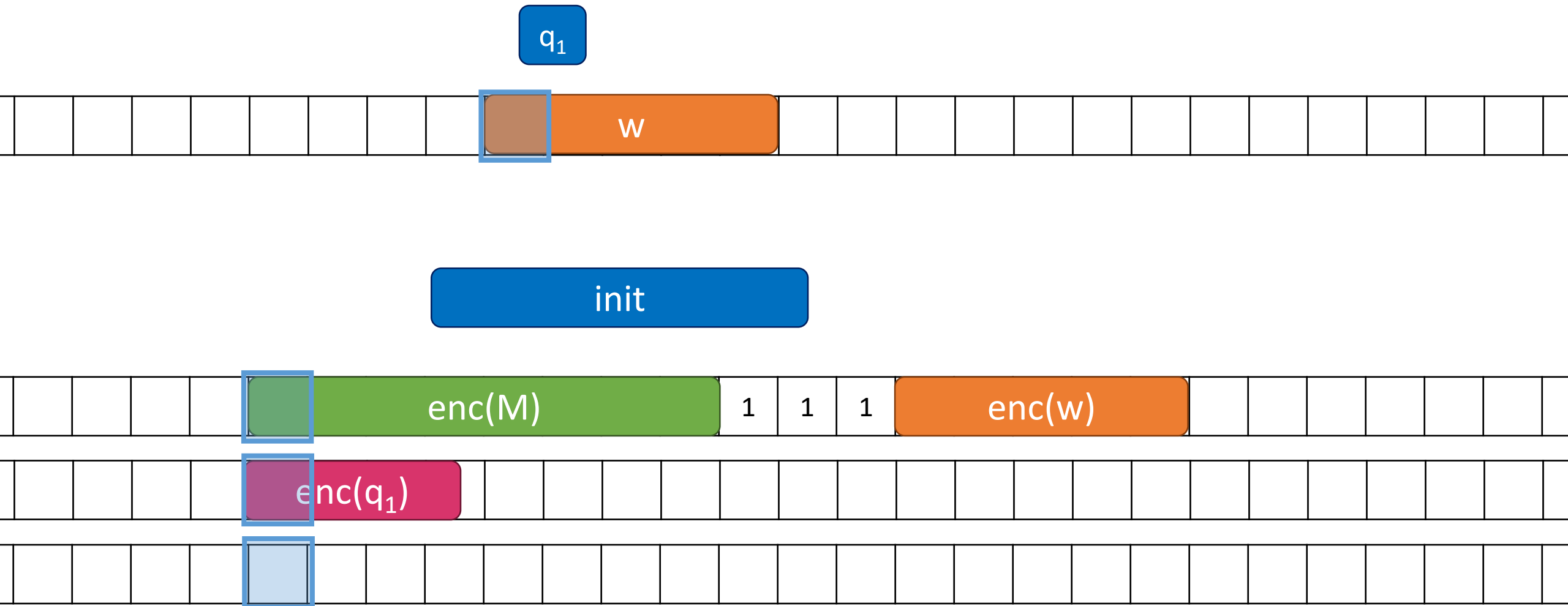
Encoding example – isEven

	g_1	g_2	g_3
q_1	q_1, g_1, d_2	q_1, g_2, d_2	q_5, g_3, d_1
q_5	q_2, g_1, d_3	q_3, g_2, d_3	q_3, g_3, d_3

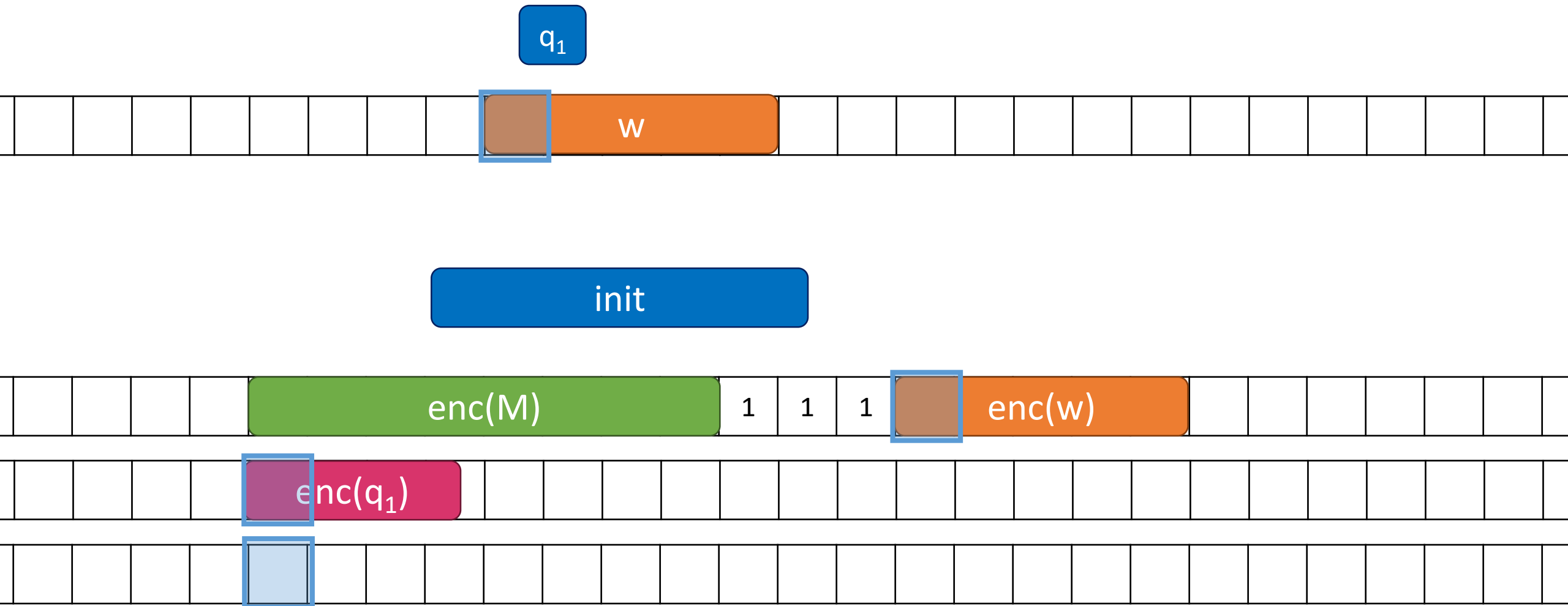
The UTM at work



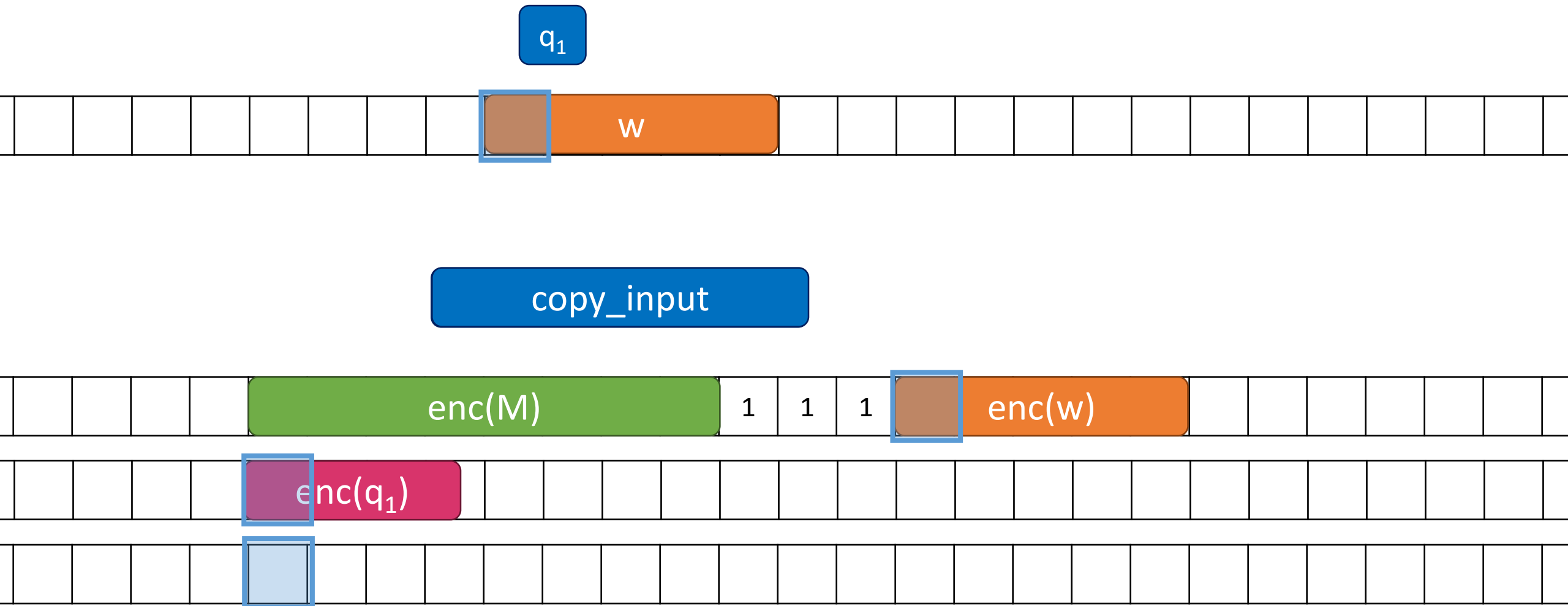
The UTM at work



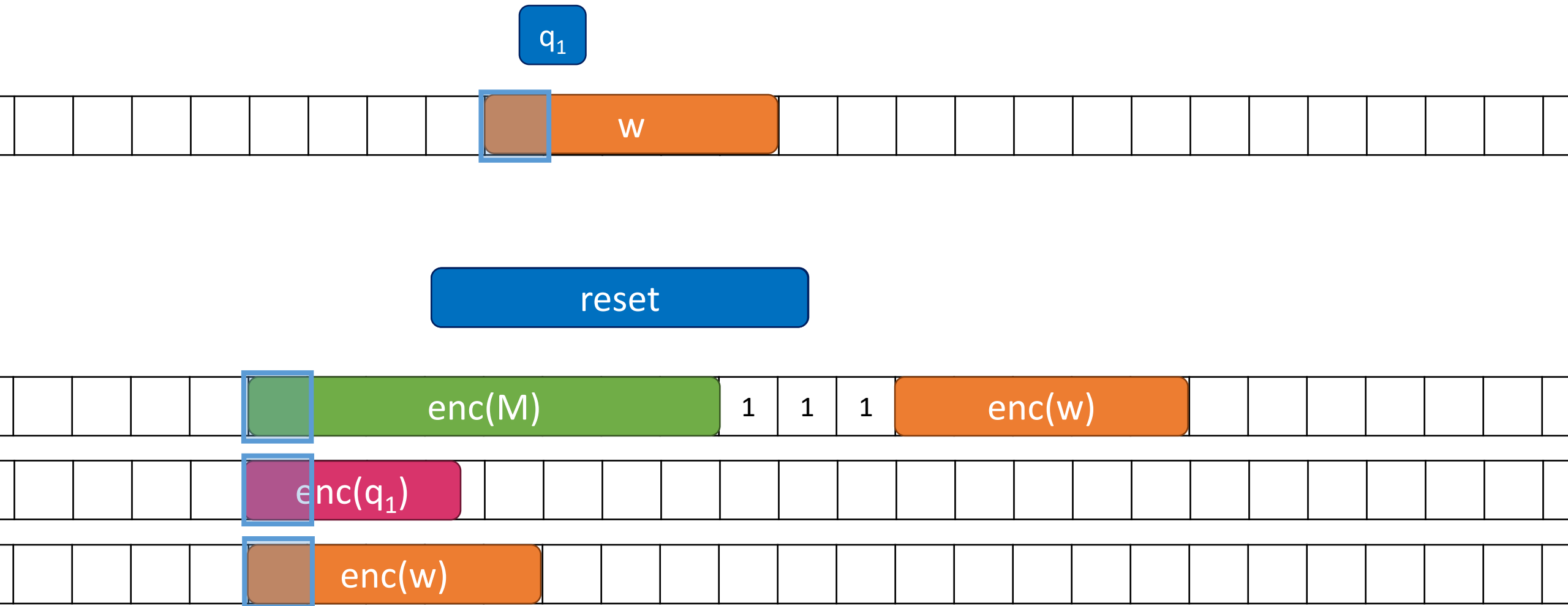
The UTM at work



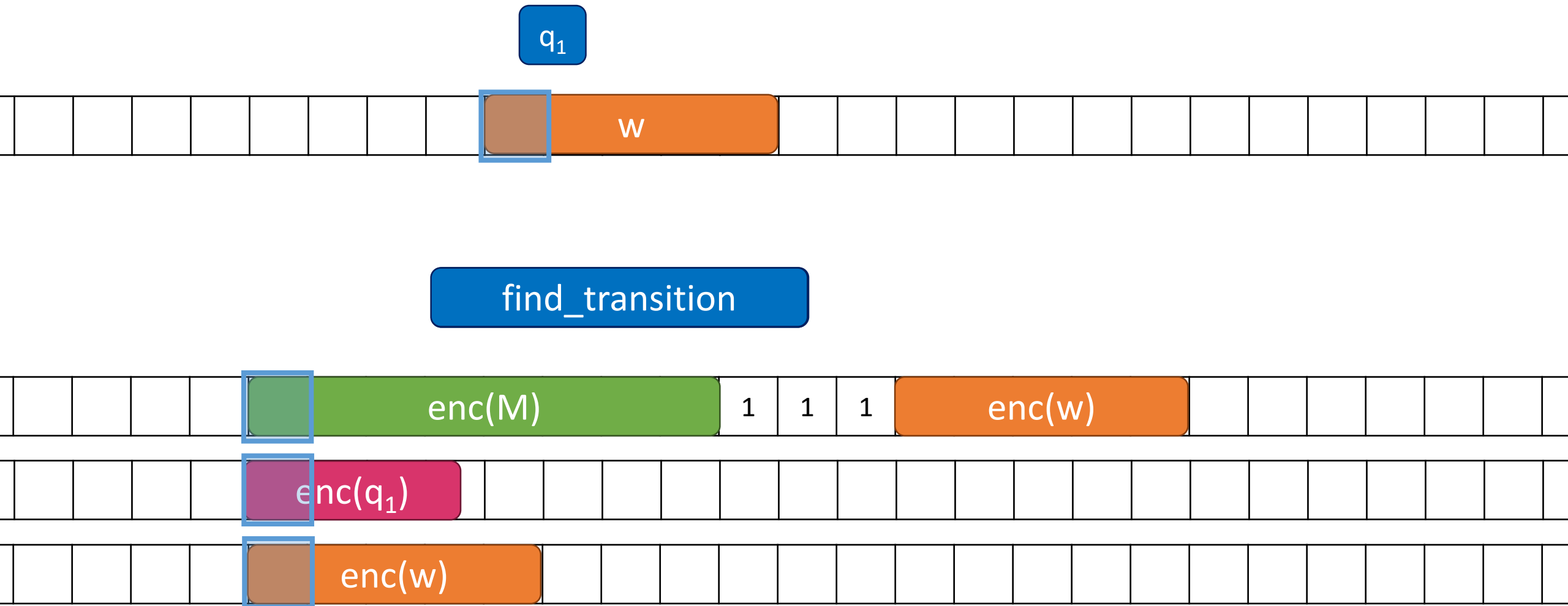
The UTM at work



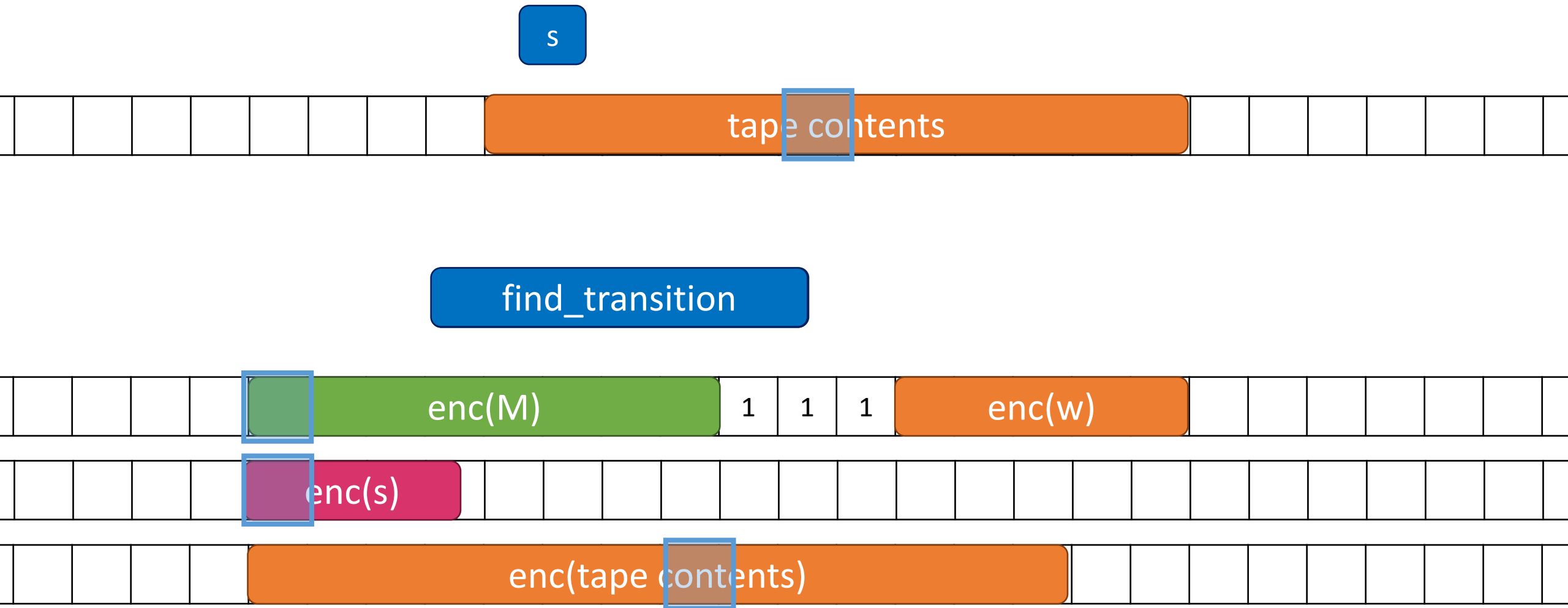
The UTM at work



The UTM at work



Midwork



Simulating one transition

S

z	0	1	0	1
---	---	---	---	---

check_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible]

0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Simulating one transition

S

z	0	1	0	1
---	---	---	---	---

check_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

0	0
---	---

0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Simulating one transition

S

z	0	1	0	1
---	---	---	---	---

check_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible]

0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1								
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--	--	--	--	--	--

Simulating one transition

S

[illegible]

check_state

1	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

z	0	1	0	1
---	---	---	---	---

check_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

0	0	0	0	0
---	---	---	---	---

0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Simulating one transition

S

[illegible]

go_to_symbol

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

[illegible]

go_to_symbol

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

[illegible]

check_symbol

1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible]

	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1								
--	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--	--	--	--	--	--

Simulating one transition

S

Z	0	1	0	1
---	---	---	---	---

check_symbol

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

0	0	0	0	0
---	---	---	---	---

0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Simulating one transition

S

[illegible]

delete_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

[illegible]

delete_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible]

	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1								
--	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--	--	--	--	--	--

Simulating one transition

S

[illegible]

delete_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible]

	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1								
--	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--	--	--	--	--	--

Simulating one transition

S

z	0	1	0	1
---	---	---	---	---

delete_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

0	0
---	---

0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Simulating one transition

S

[illegible]

delete_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible]

	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1								
--	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--	--	--	--	--	--

Simulating one transition

S

[illegible]

copy_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible]

	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1								
--	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--	--	--	--	--	--

Simulating one transition

S

[illegible]

copy_state

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible]

	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1								
--	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--	--	--	--	--	--

Simulating one transition

S

[illegible]

delete_symbol

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible]

	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1								
--	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--	--	--	--	--	--

Simulating one transition

S

[illegible]

delete_symbol

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

z	0	1	0	1
---	---	---	---	---

insert_symbol

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

0		0
---	--	---

0	0	0	1	0	1	0	0	1	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---

Simulating one transition

S

[illegible]

insert_symbol

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

[illegible]

move_head

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

[illegible]

move_head_0

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

[illegible]

move_head_00

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

[illegible]

move_head_000

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

[illegible]

move_head_000

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Simulating one transition

S

[illegible]

reset

1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

[illegible][illegible]

Takeaways

- A multitape TM can be simulated by a single tape one
- Any TM can be simulated by one with tape alphabet $\{0, 1, \square\}$
- TMs can be encoded as strings and used as input for other TMs
- Any TM's run on any input can be simulated by a UTM