

Classic induction

Goal: prove $\forall n \in \mathbb{N}, P(n)$

Base case: prove $P(0)$

Induction step: prove $P(n) \Rightarrow P(n + 1)$

Structural induction recipe

Prove **unary** properties: $P(x)$

1. For each **null constructor**, $C: T$, prove **base case**: $P(C)$
2. For each **external constructor**, $E: A_1 \times A_2 \times \cdots \times A_n \rightarrow T$, prove **base case**:
 $P(E(a_1, \dots, a_n))$
3. For each **internal constructor**, $I: \underbrace{T \times T \times \cdots \times T}_n \times A_1 \times A_2 \times \cdots \times A_m \rightarrow T$:
 - a) assume $P(t_1), P(t_2), \dots, P(t_n)$
 - b) prove $P(I(t_1, t_2, \dots, t_n, a_1, a_2, \dots, a_m))$