6a

Rice’s Theorem

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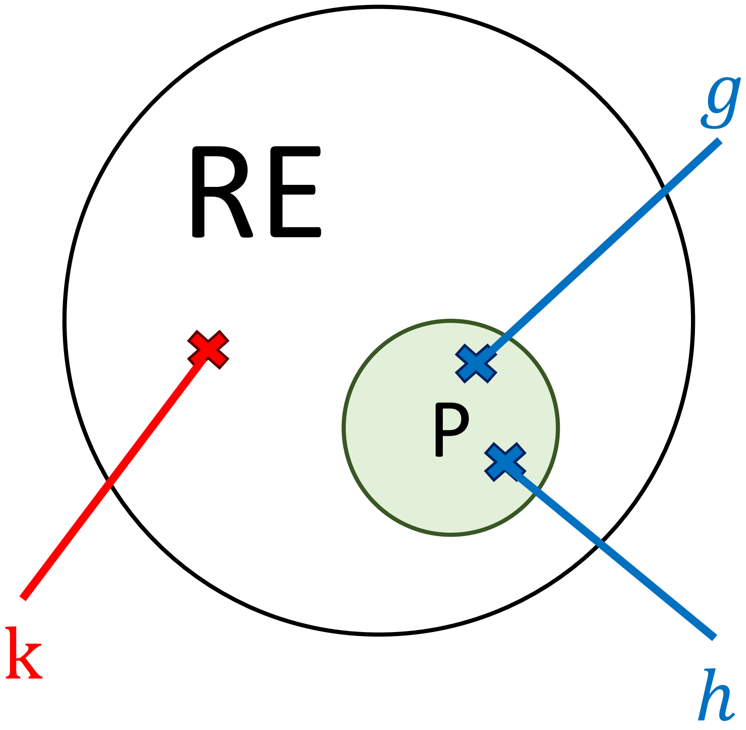
Image generated with ChatGPT 4o

Informal statement

“Any non-trivial property of acceptable problems is undecidable”

"… property of acceptable problems…"



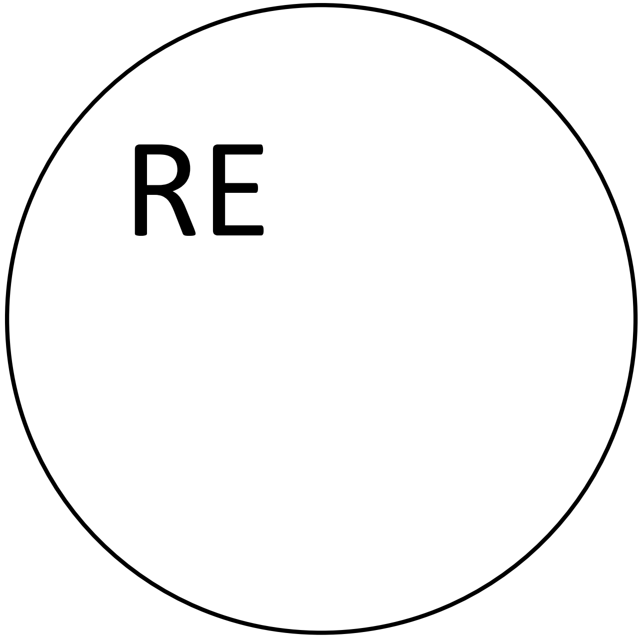
Property P: having a finite number of TRUE inputs

$$g(w) = \begin{cases} TRUE & |w| \leq 8 \\ FALSE & otherwise \end{cases}$$

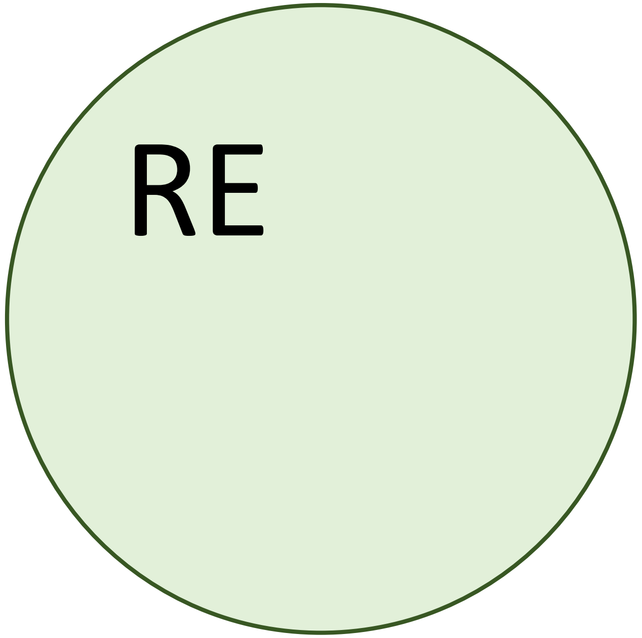
$$h(w) = \begin{cases} TRUE & w = 10101 \\ FALSE & otherwise \end{cases}$$

$$\mathrm{k}(w) = \begin{cases} TRUE & w\ ends\ in\ 0 \\ FALSE & otherwise \end{cases}$$

“… trivial property…”

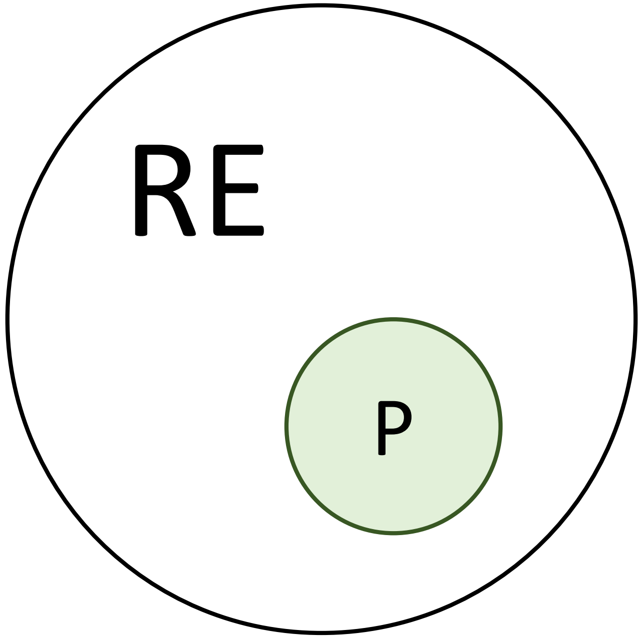


$P = \emptyset$



$P = RE$

"… property […] is undecidable…"



$$hasP(f) = \begin{cases} TRUE & f \in P \\ FALSE & otherwise \end{cases}$$

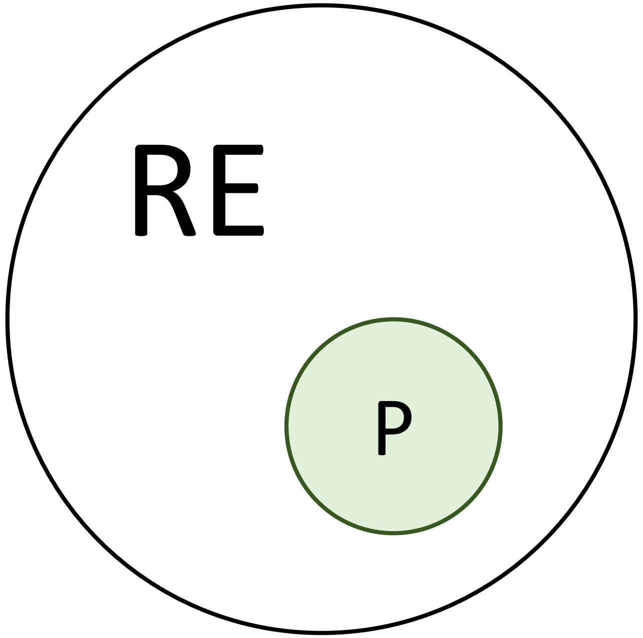
Property P: having a finite number of TRUE inputs

$$g(w) = \begin{cases} TRUE & |w| \leq 8 \\ FALSE & otherwise \end{cases}$$ hasP$(g) = TRUE$

$$h(w) = \begin{cases} TRUE & w = 10101 \\ FALSE & otherwise \end{cases}$$ hasP$(h) = TRUE$

$$\mathrm{k}(w) = \begin{cases} TRUE & w\ ends\ in\ 0 \\ FALSE & otherwise \end{cases}$$ hasP$(k) = FALSE$

“… property […] is undecidable…”



$hasP: \Sigma^* \rightarrow \{FALSE, TRUE\}$

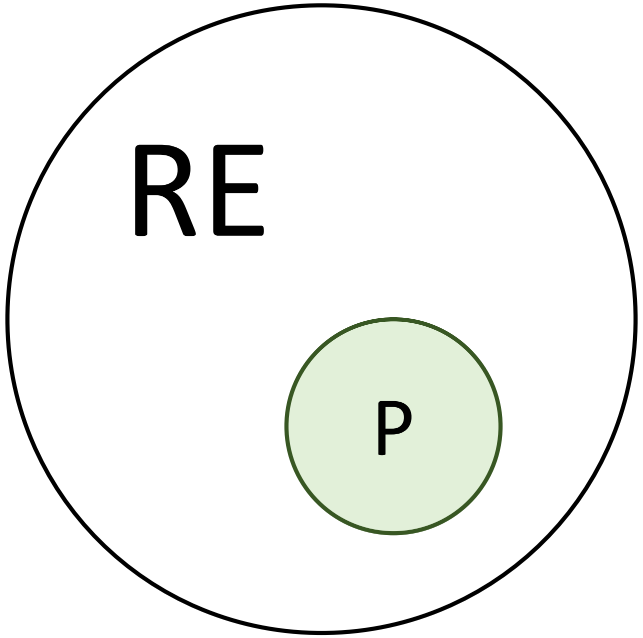
$$hasP\big(enc(A_f)\big) = \begin{cases} TRUE & f \in P \\ FALSE & otherwise \end{cases}$$

A_f *is a Turing Machine that accepts* f

Informal statement (2)

"Given a Turing Machine A_f, we can't decide whether the problem it accepts belongs to some subset of RE."

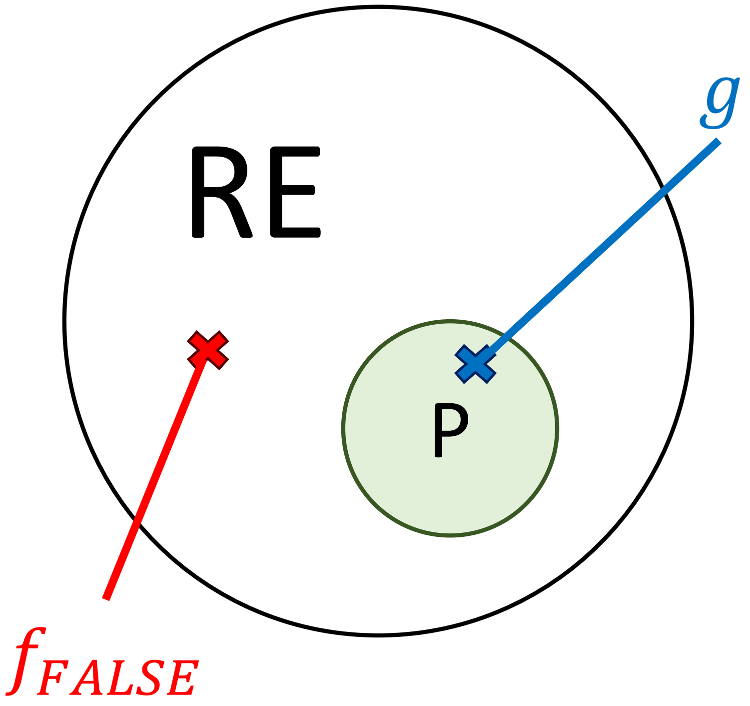
Formal statement



$$\text{hasP}\big(enc(A_f)\big) = \begin{cases} TRUE & f \in P \\ FALSE & otherwise \end{cases}$$

$$(P \neq \emptyset \land P \neq RE) \Leftrightarrow \text{hasP} \notin R$$

Proof (HALT $\leq_m$ hasP)



$(f_{FALSE}(w) = FALSE, \forall w \in \Sigma^*)$

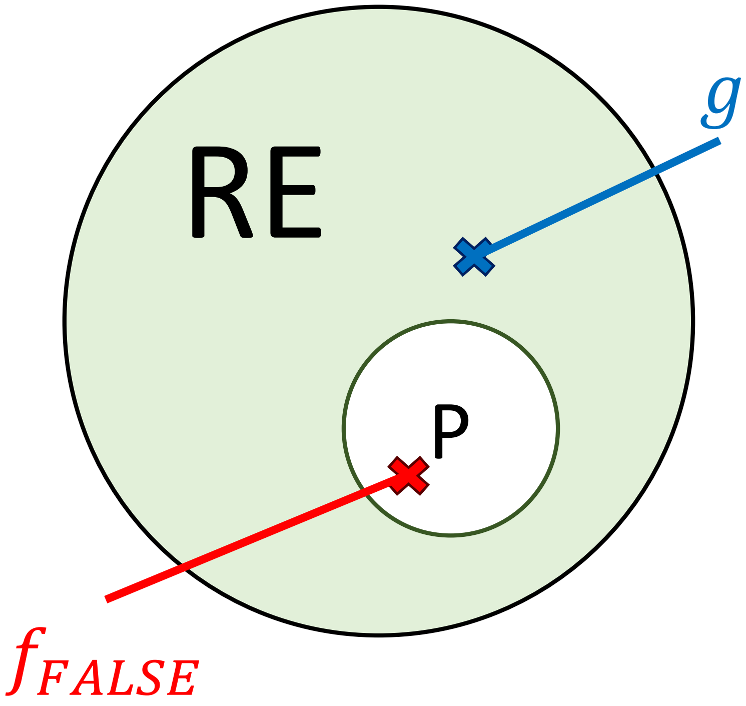
$P \neq \emptyset \Rightarrow \exists g \in P$

$\exists M_g \; accepts \; g$

$M'[v]$:

1. simulate $M[w]$
2. simulate $M_g[v]$

Proof $(\mathrm{HALT} \leq_m \overline{\mathrm{hasP}})$



$(f_{FALSE}(w) = FALSE, \forall w \in \Sigma^*)$

$P \neq RE \Rightarrow \exists g \in \bar{P}$

$\exists M_g \; accepts \; g$

$M'[v]$:

1. simulate $\mathrm{M}[w]$
2. simulate $\mathrm{M_g}[v]$