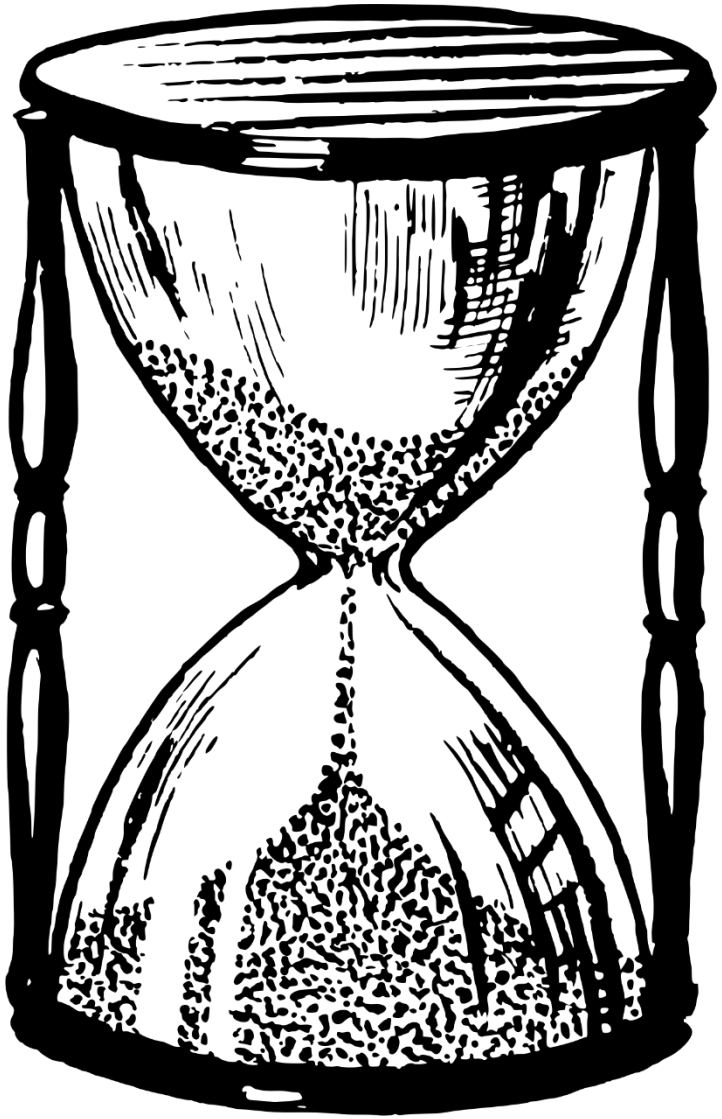
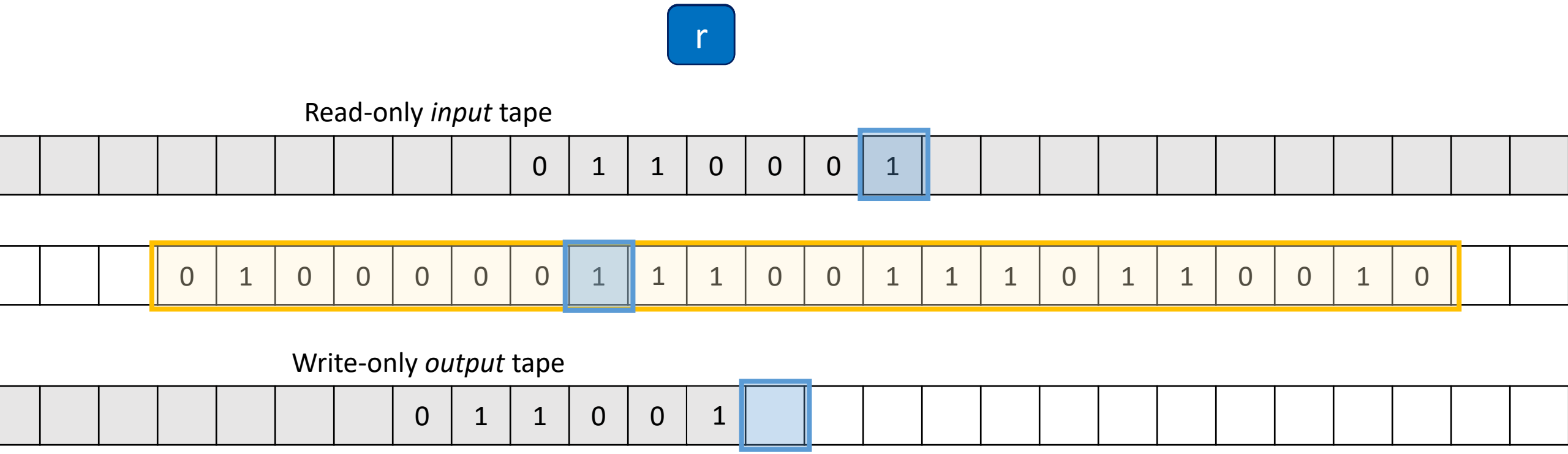


Computational Resources

Analiza Algoritmilor

Image source: https://openclipart.org/detail/262372/hourglass

Space and time



Transitions: **10**

Time is more important than space

In order to occupy one more cell,

we need to make one more transition

We *can reuse* cells by writing another symbol later

We *cannot reuse* transitions

Time as a function

$$t_M : \Sigma^* \to \mathbb{N}$$

$t_{isPalindrome}(\varepsilon) = 3$

$t_{isPalindrome}(0) = 6$

$t_{isPalindrome}(1) = 6$

$t_{isPalindrome}(00) = 9$

$t_{isPalindrome}(01) = 7$

$t_{isPalindrome}(10) = 7$

...

$t_{isPalindrome}(0000) = 15$

$t_{isPalindrome}(0001) = 11$

...

$t_{isPalindrome}(1010) = 11$

$t_{isPalindrome}(1011) = 12$

...

Worst-case complexity

Out of all $|\Sigma|^n$ inputs of size n focus on the one for which it takes *the most transitions*.

$$T_M : \mathbb{N} \to \mathbb{N}$$

$T_{isPalindrome}(4) = 15$ $T_{isPalindrome}(n) = 3n + 3$

"$isPalindrome$ runs in time $T_{isPalindrome}(n)$"

Moodle Quiz

Time function

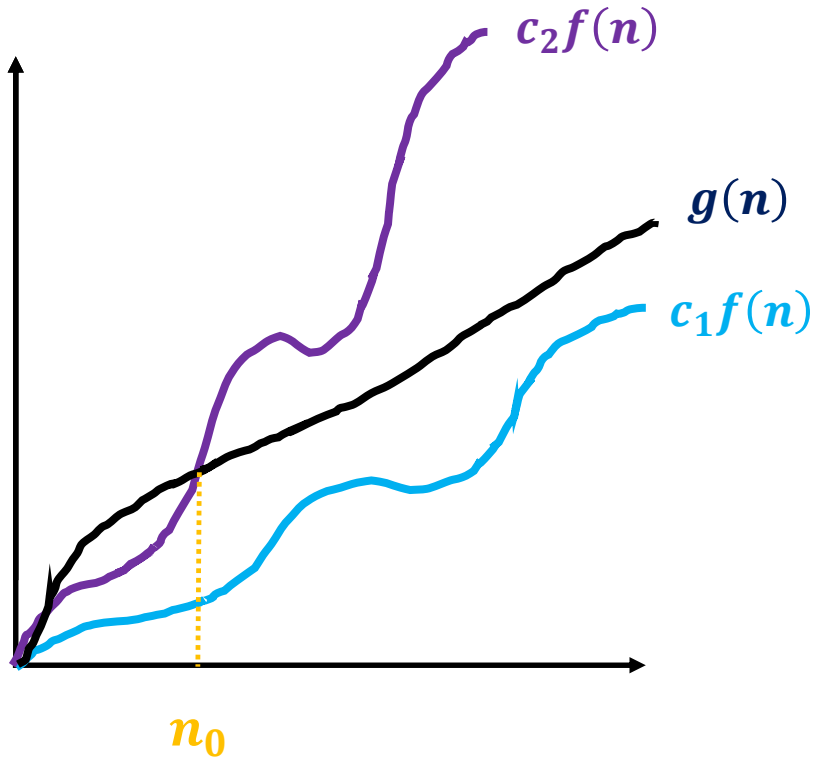
Complexity of a problem

Idea 1: the time complexity of the fastest machine that solves it

There is no fastest machine!

Idea 2: address linear speedup by abstracting away factors and constants

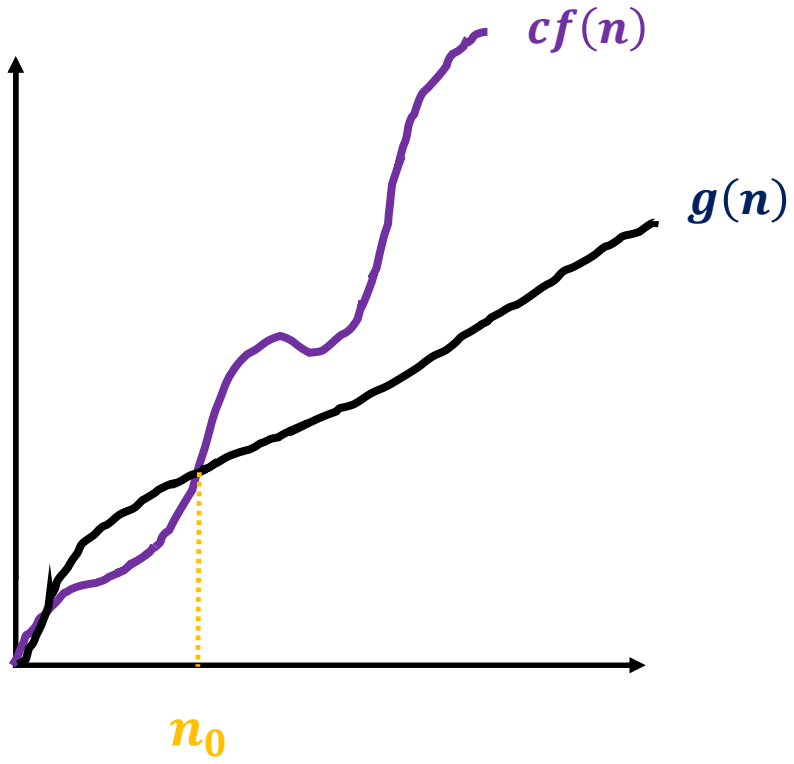
Asymptotic notations: $g(n) \in \Theta(f(n))$



$$\exists c_1, c_2 \in \mathbb{R}_+, \exists n_0 \in \mathbb{N}, \forall n \geq n_0$$

$$c_1 f(n) \leq g(n) \leq c_2 f(n)$$

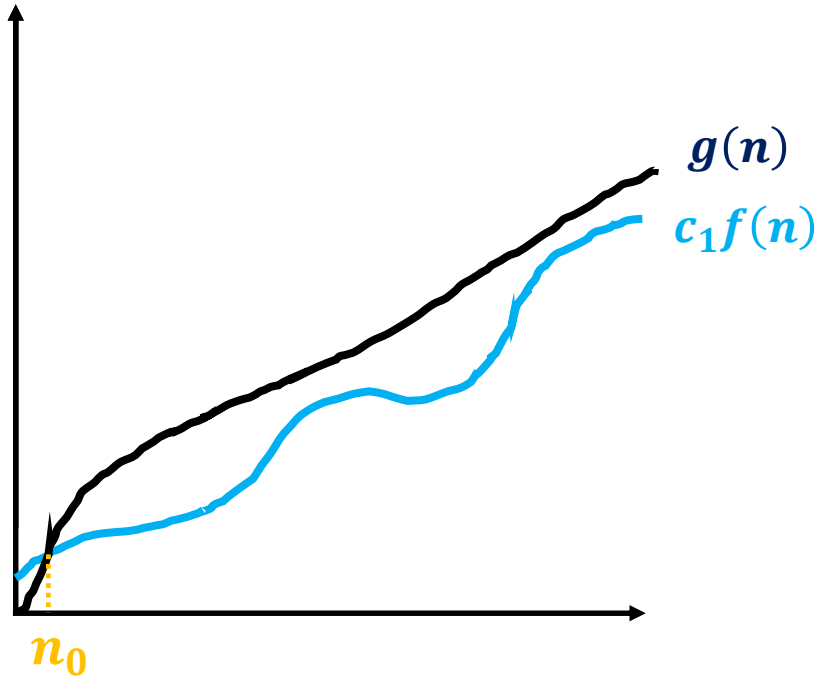
Asymptotic notations: $g(n) \in O(f(n))$



$$\exists c \in \mathbb{R}_+, \exists n_0 \in \mathbb{N}, \forall n \geq n_0$$

$$0 \leq g(n) \leq cf(n)$$

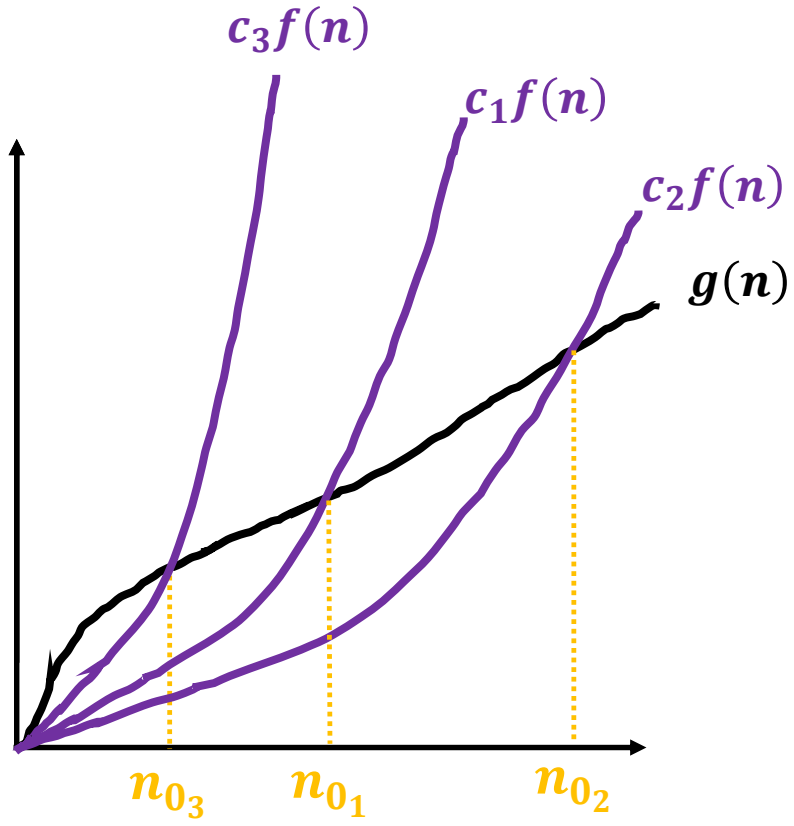
Asymptotic notations: $g(n) \in \Omega(f(n))$



$$\exists c \in \mathbb{R}_+, \exists n_0 \in \mathbb{N}, \forall n \geq n_0$$

$$cf(n) \leq g(n)$$

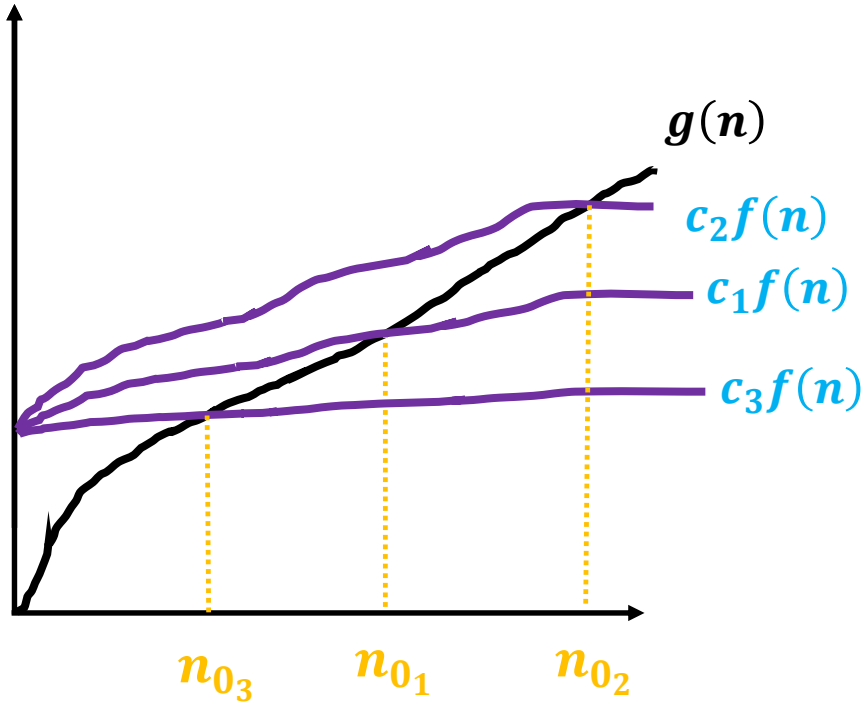
Asymptotic notations: $g(n) \in o(f(n))$



$$\forall c \in \mathbb{R}_+, \exists n_0 \in \mathbb{N}, \forall n \geq n_0$$

$$0 \leq g(n) \leq cf(n)$$

Asymptotic notations: $g(n) \in \omega(f(n))$



$$\forall c \in \mathbb{R}_+, \exists n_0 \in \mathbb{N}, \forall n \geq n_0$$

$$cf(n) \leq g(n)$$