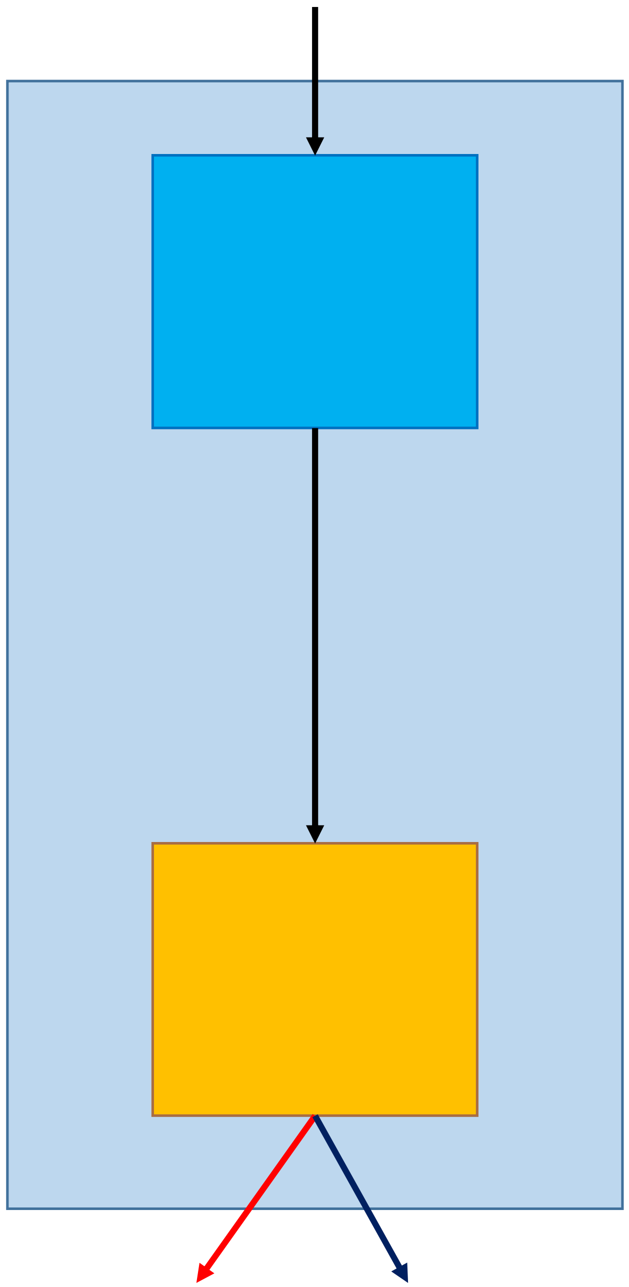
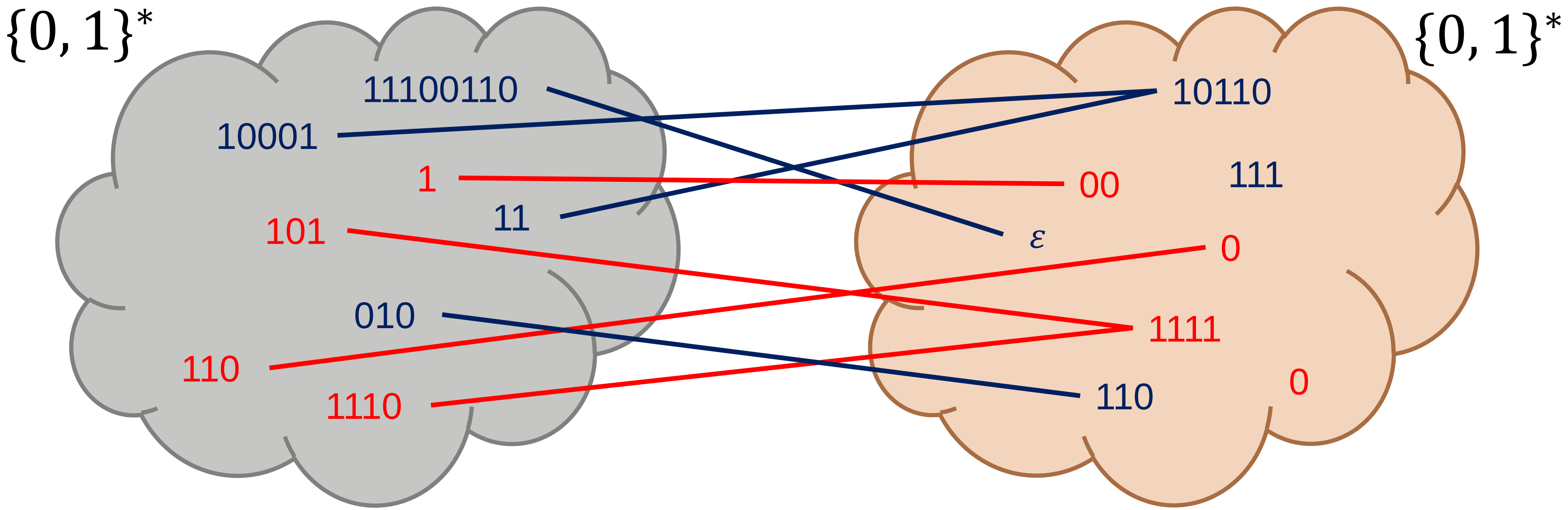


Reductions

Analiza Algoritmilor

Mapping reductions

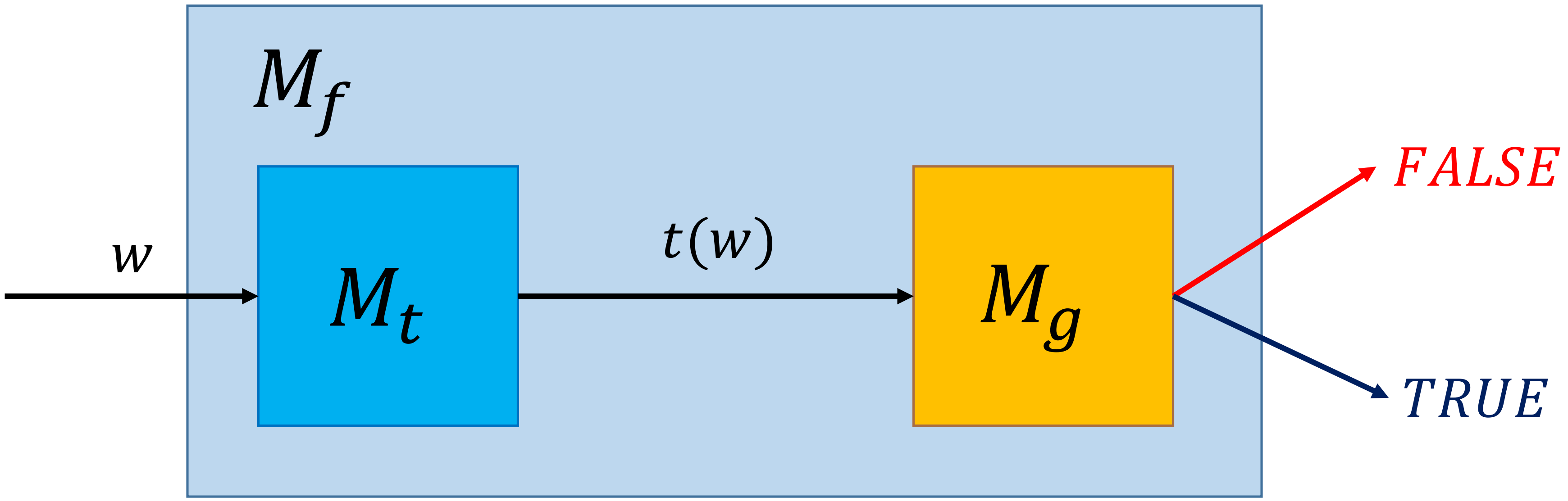


$f(\blacksquare) = TRUE$ $g(\blacksquare) = TRUE$

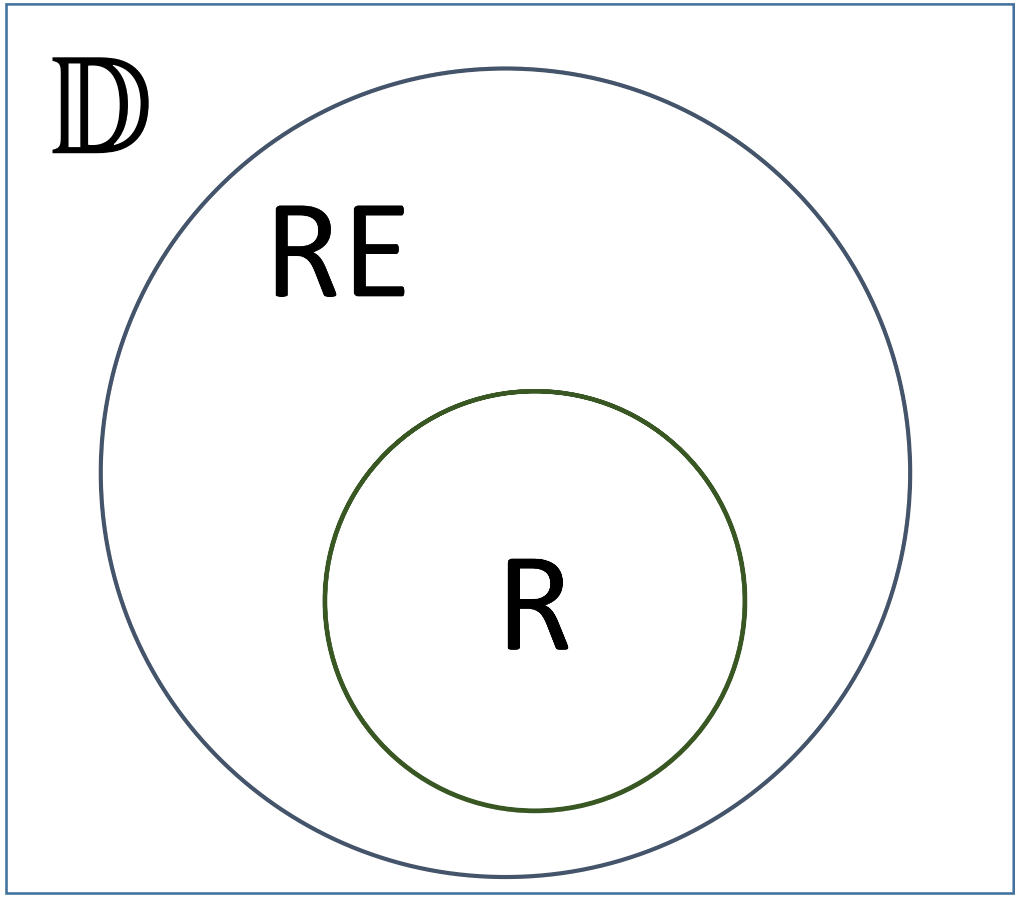
$f(\blacksquare) = FALSE$ $g(\blacksquare) = FALSE$

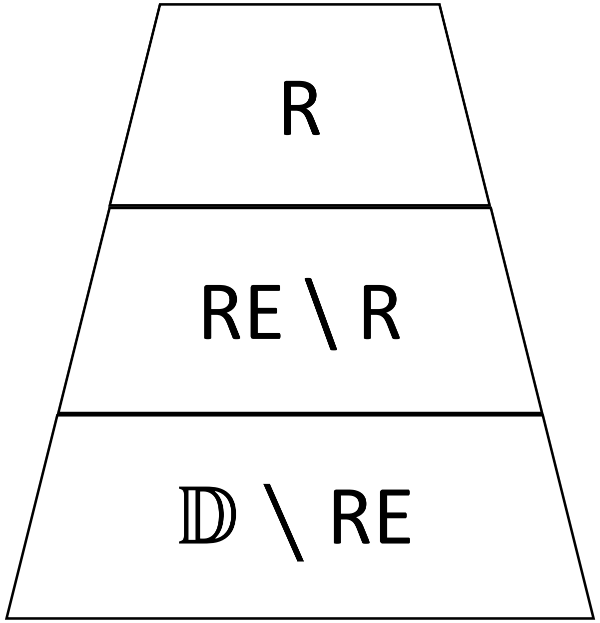
Mapping reductions (2)

$$f \le_m g \overset{\text{def}}{=} \exists t : \Sigma^* \to \Sigma^*, s.t. \begin{cases} t \text{ is computable} \\ \forall w \in \Sigma^*, f(w) = g(t(w)) \end{cases}$$



"Hardness"

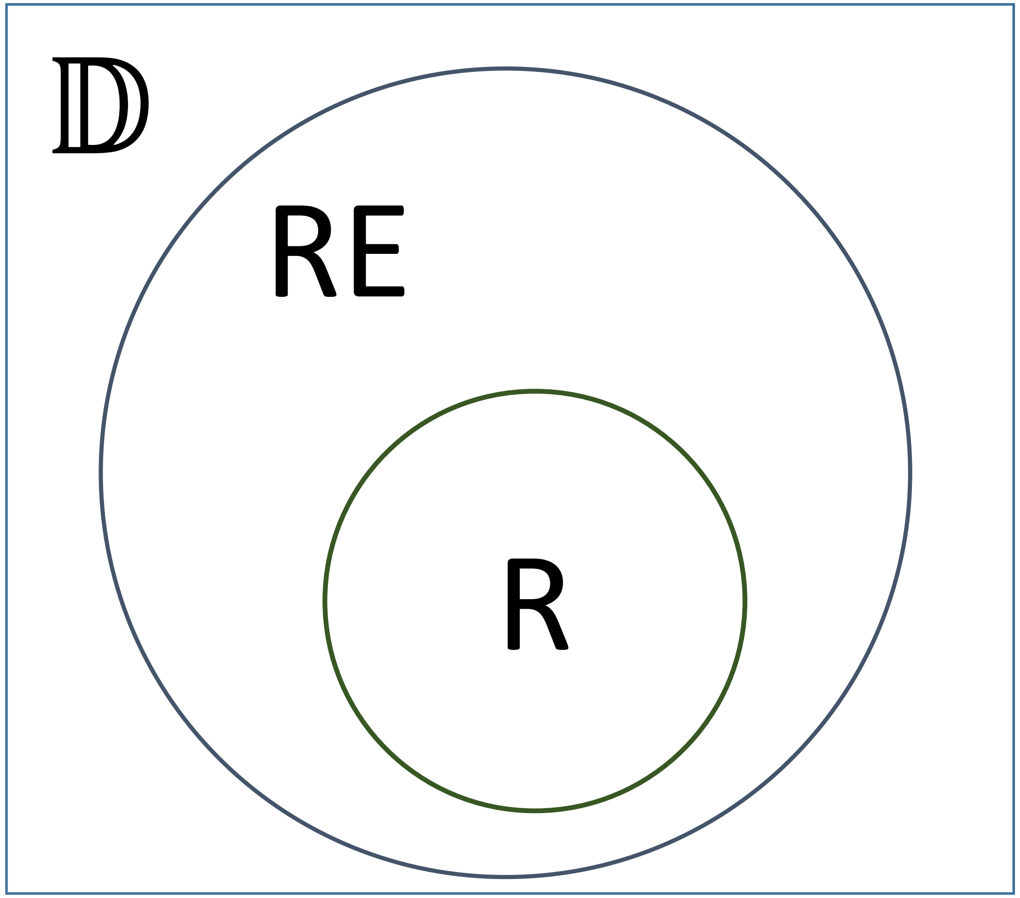


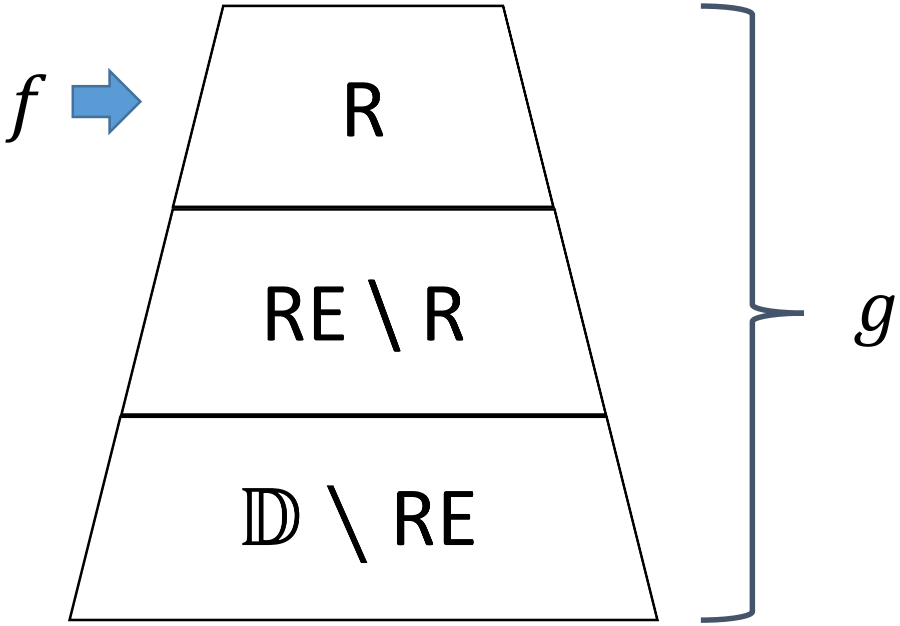


$$f \leq_m g$$

f is either "easier" or "as hard as" g

“Hardness”

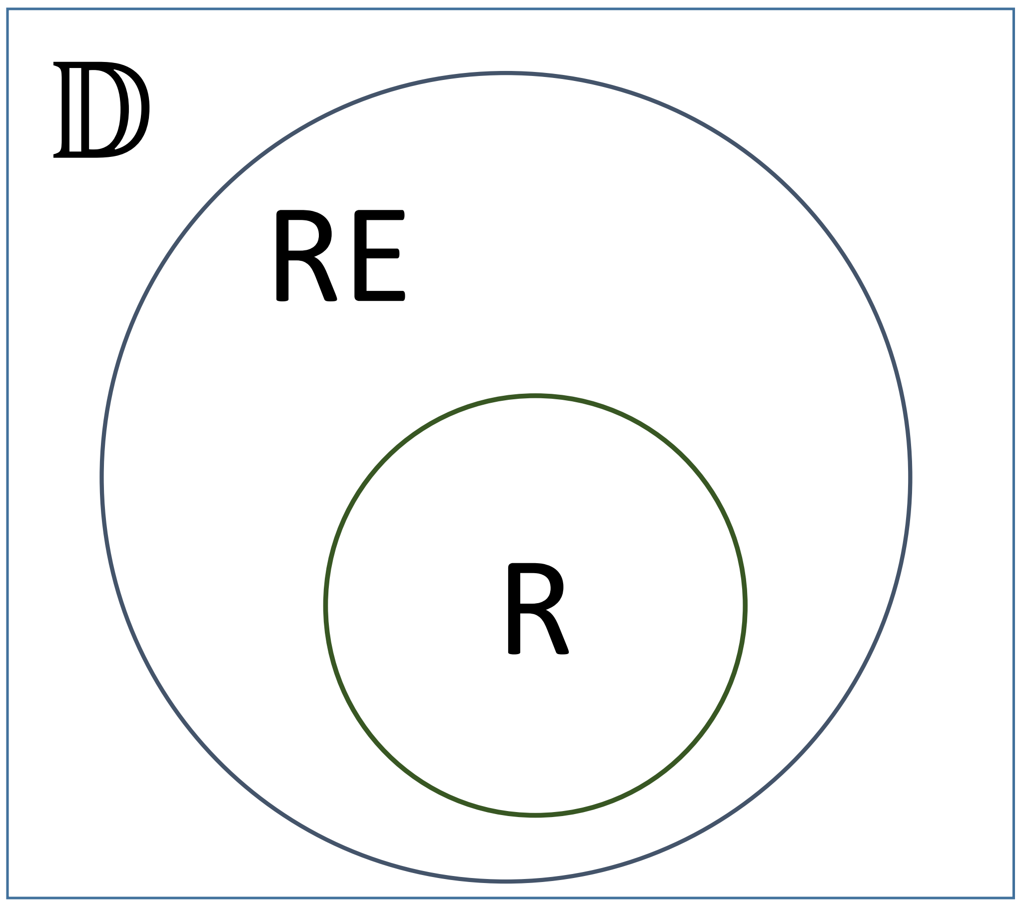


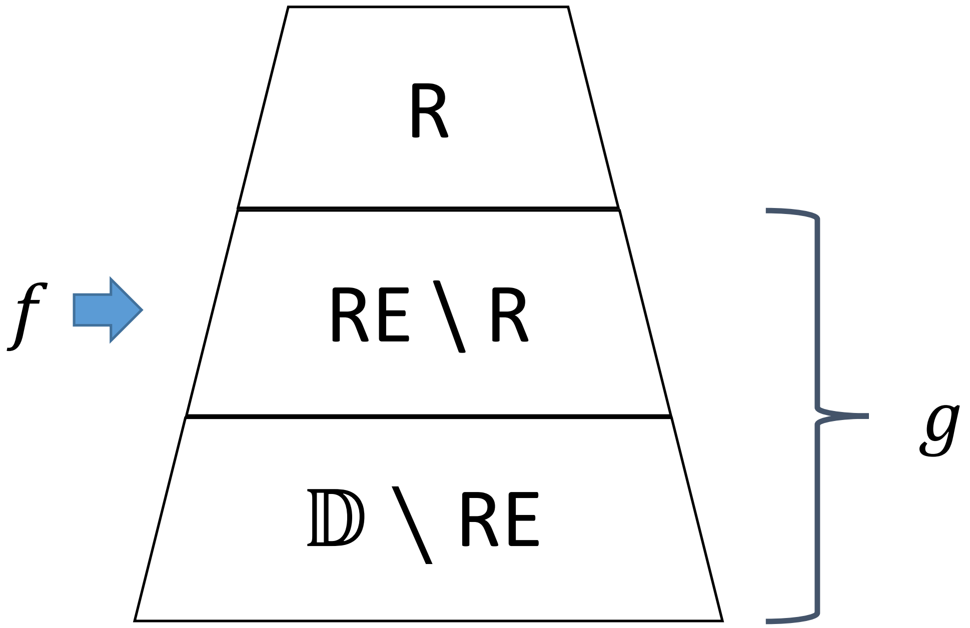


$$f \leq_m g$$

f is either “easier” or “as hard as” g

“Hardness”

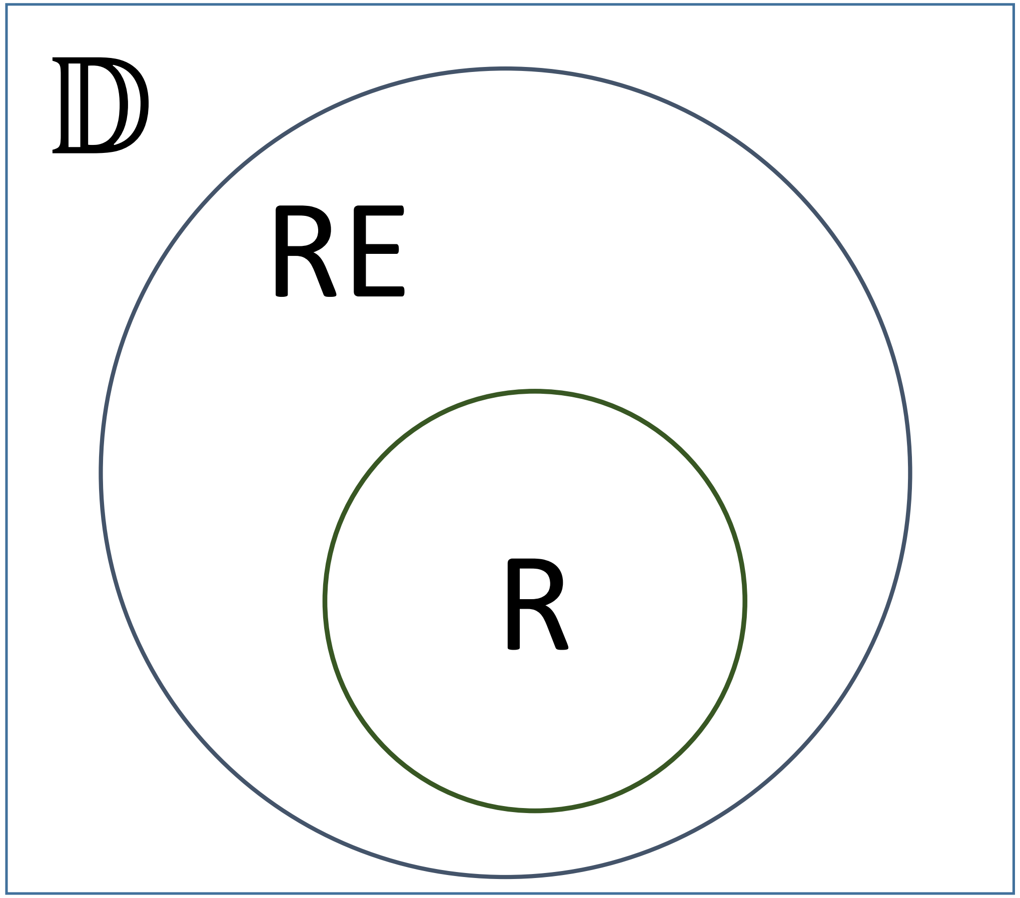


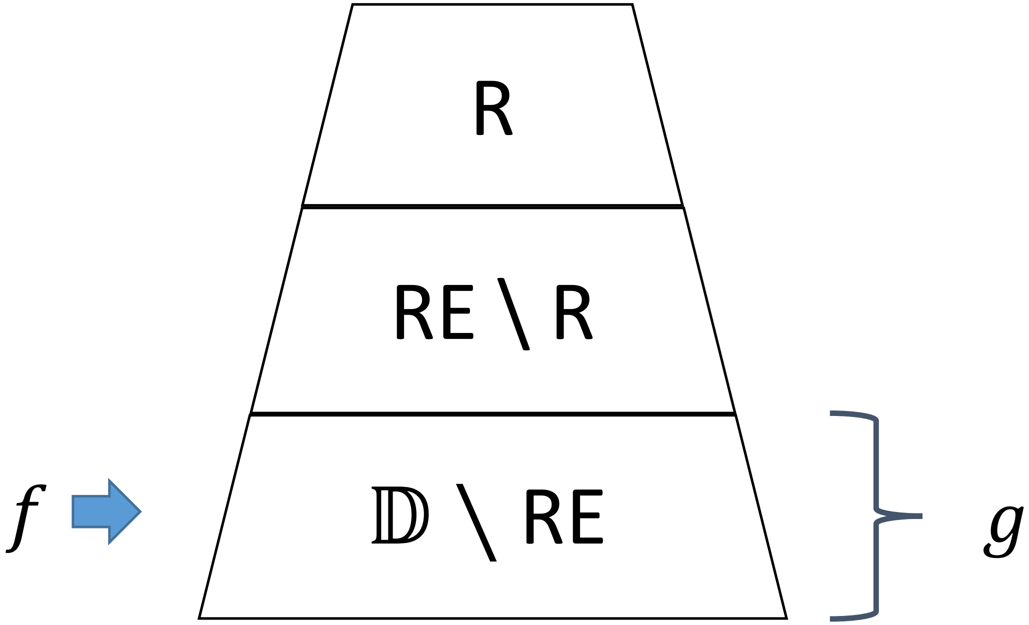


$f \leq_m g$

f is either “easier” or “as hard as” g

“Hardness”

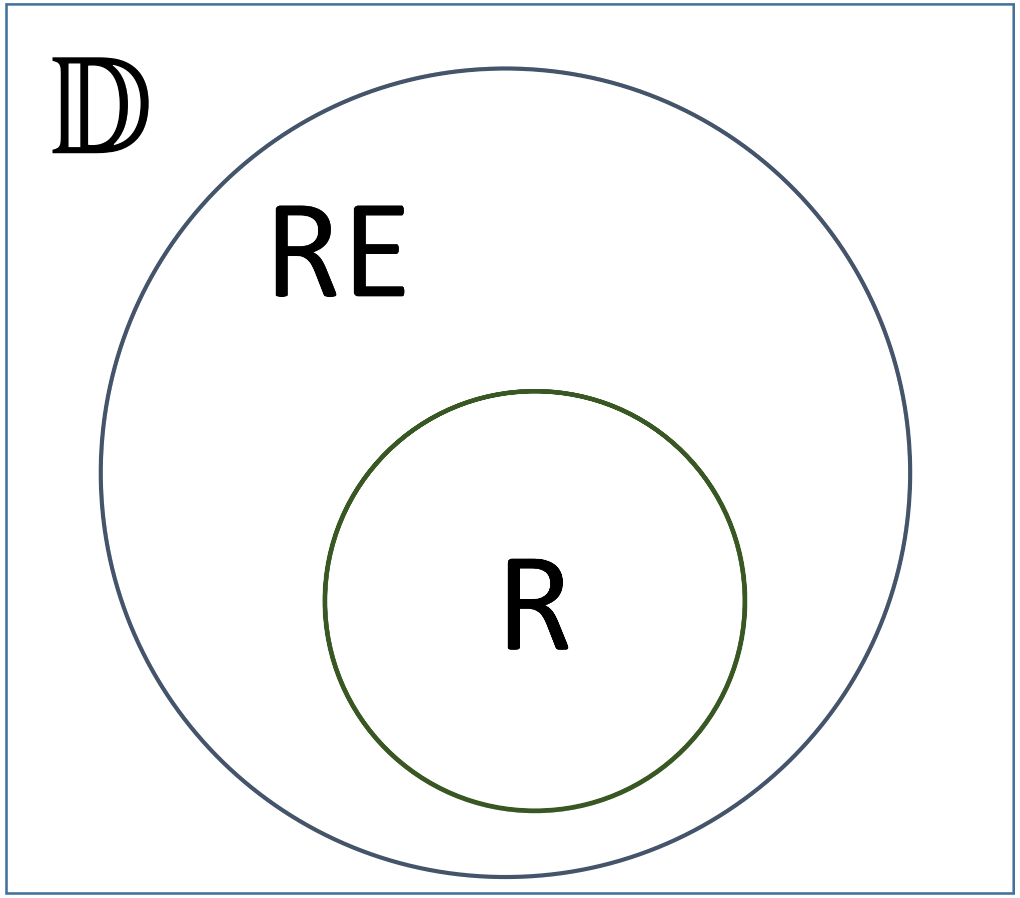


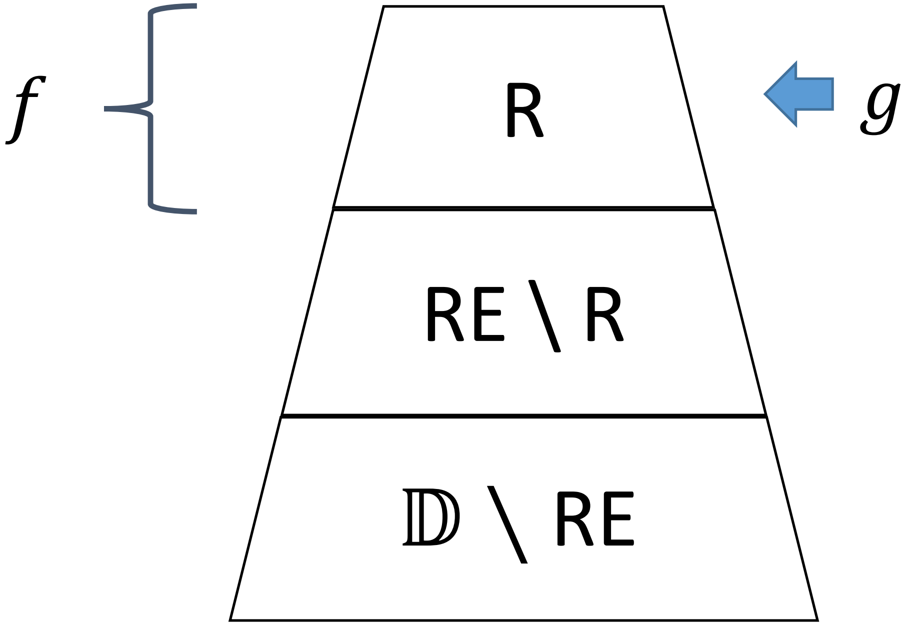


$$f \leq_m g$$

f is either “easier” or “as hard as” g

"Hardness"

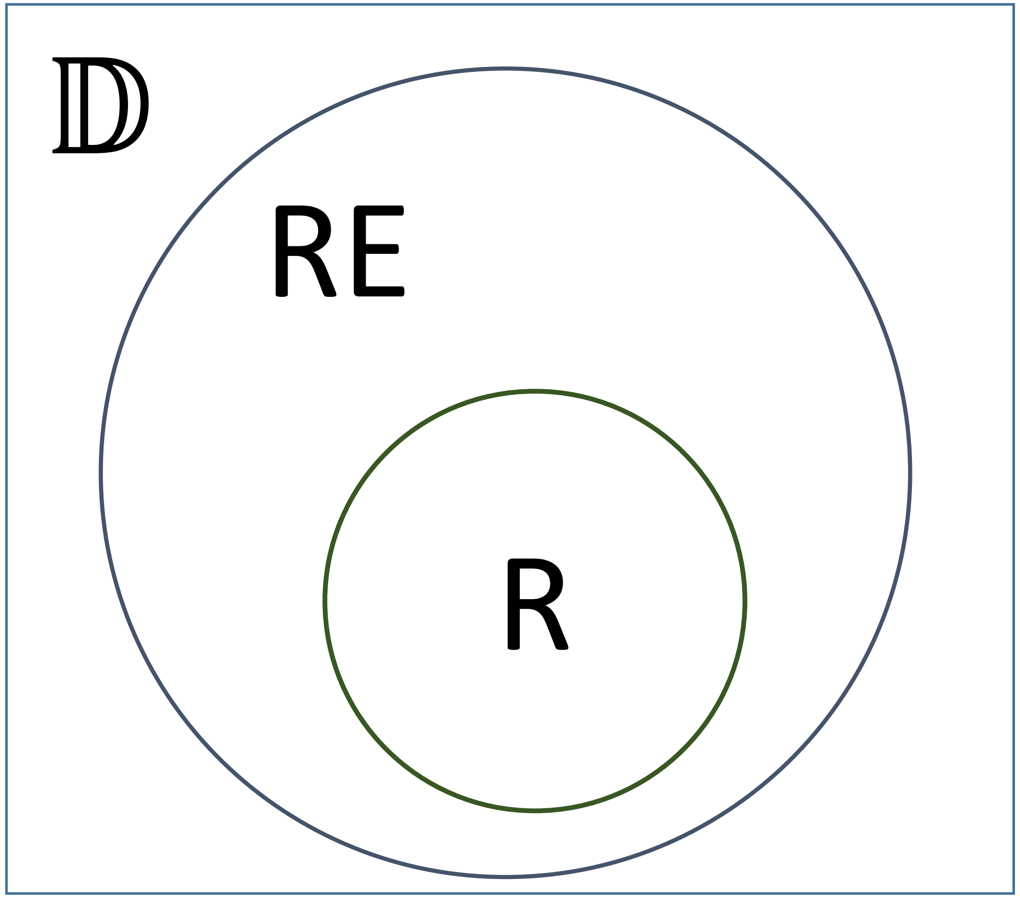


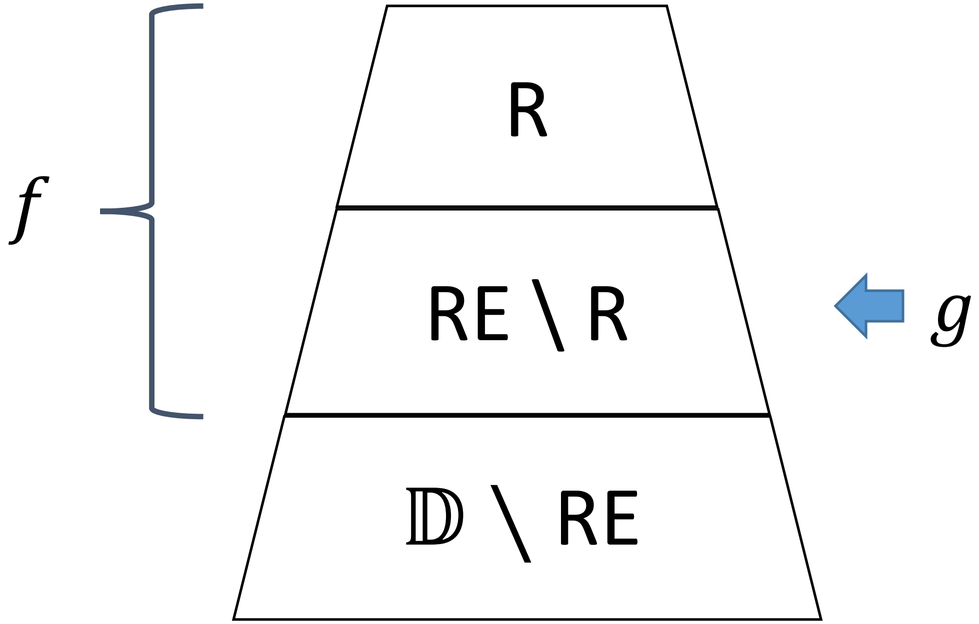


$f \leq_m g$

f is either "easier" or "as hard as" g

“Hardness”

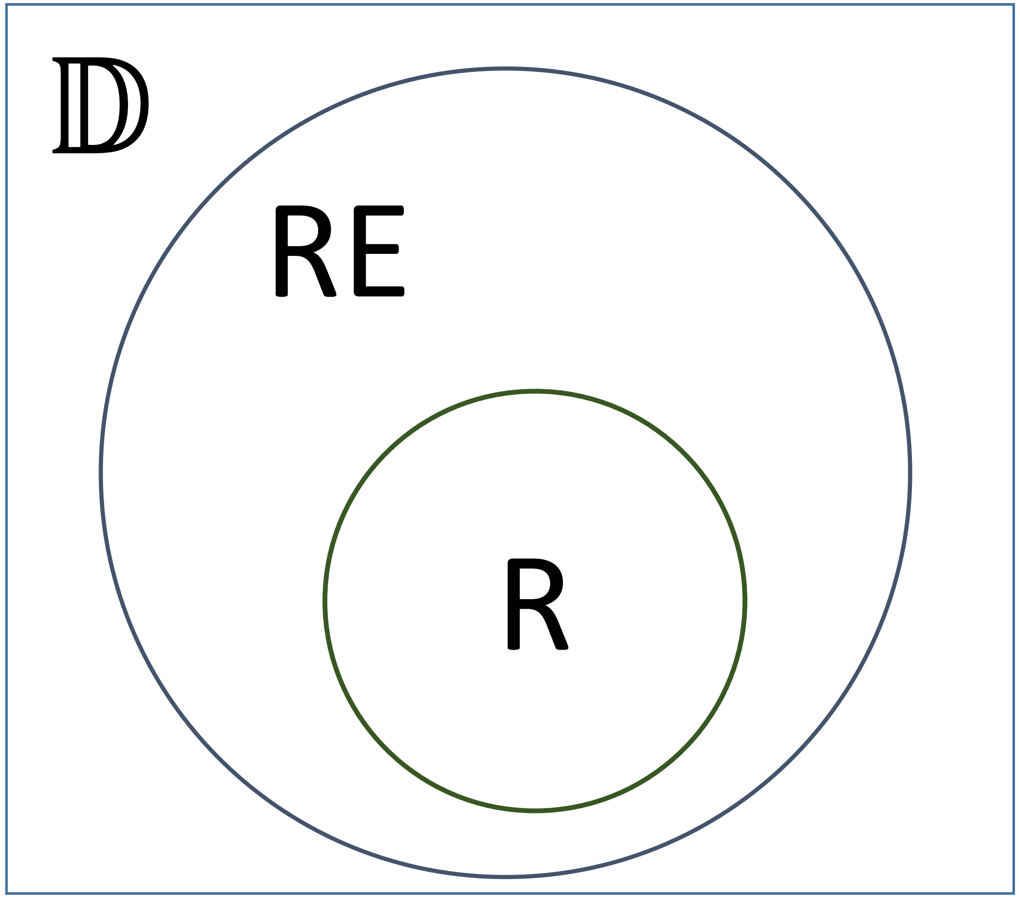


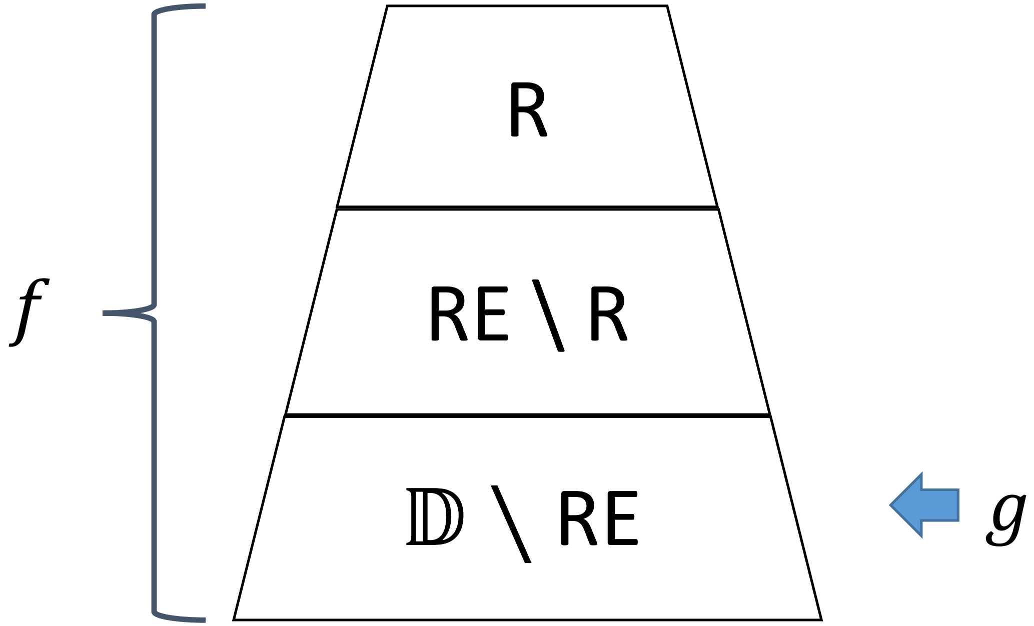


$f \le_m g$

f is either “easier” or “as hard as” g

"Hardness"





$f \leq_m g$

f is either "easier" or "as hard as" g

$$\text{ALL}\big(enc(M)\big) = \begin{cases} TRUE & \forall w \in \{0,1\}^*, M[w]\ halts \\ FALSE & otherwise \end{cases}$$

$$\text{ANY}\big(enc(M)\big) = \begin{cases} TRUE & \exists w \in \{0,1\}^*, M[w]\ halts \\ FALSE & otherwise \end{cases}$$

Complement of a problem

$f(\varepsilon) = TRUE$

$f(0) = FALSE$

$f(1) = TRUE$

$f(00) = TRUE$

$f(01) = FALSE$

$f(10) = TRUE$

$f(11) = FALSE$

$f(000) = FALSE$

$f(001) = TRUE$

$f(010) = FALSE$

$\ldots$

$\bar{f}(\varepsilon) = FALSE$

$\bar{f}(0) = TRUE$

$\bar{f}(1) = FALSE$

$\bar{f}(00) = FALSE$

$\bar{f}(01) = TRUE$

$\bar{f}(10) = FALSE$

$\bar{f}(11) = TRUE$

$\bar{f}(000) = TRUE$

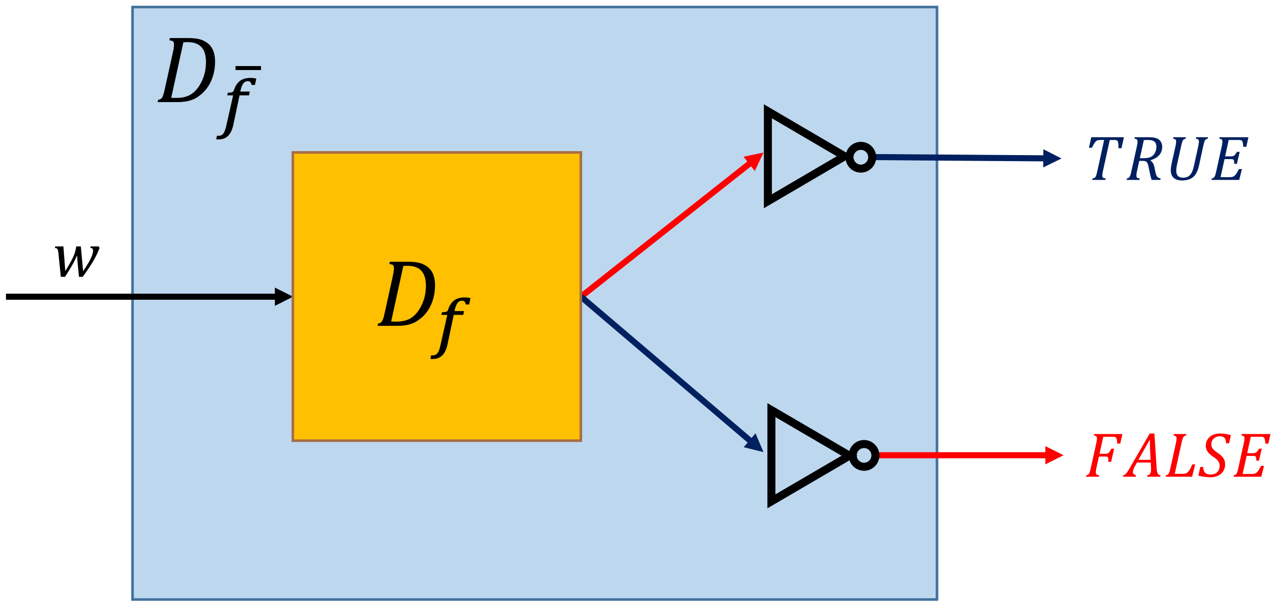
$\bar{f}(001) = FALSE$

$\bar{f}(010) = TRUE$

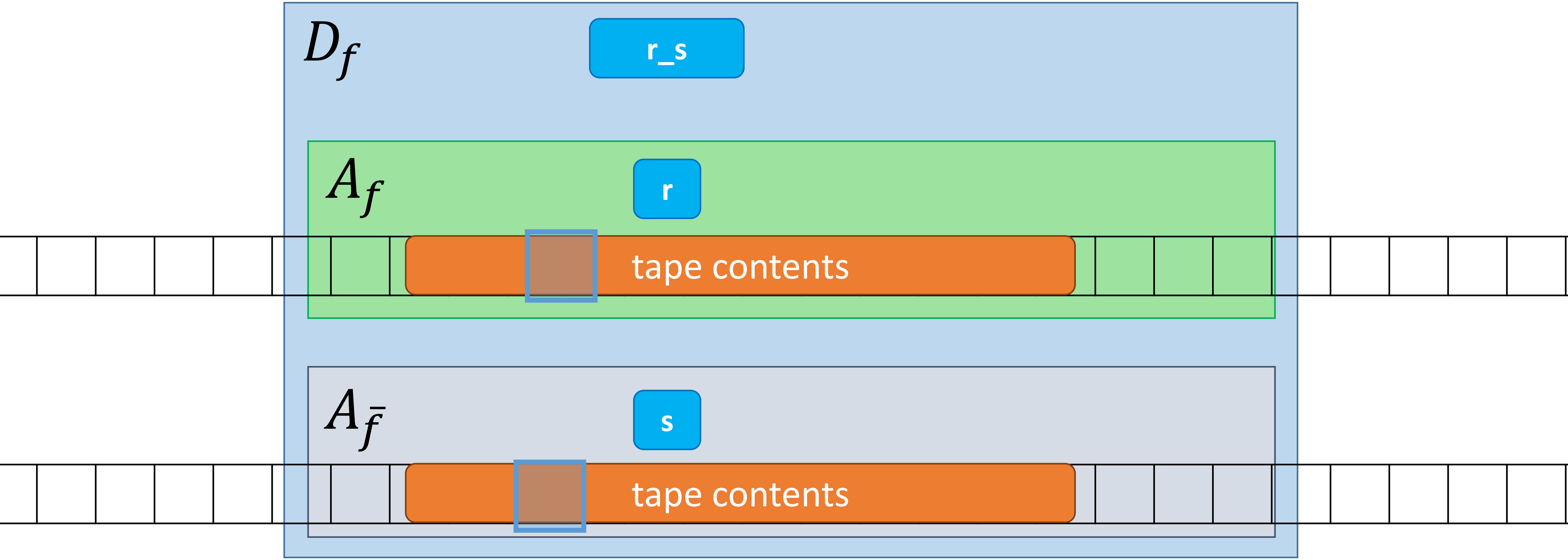
$\ldots$

$$\overline{HALT}\big(enc(M,w)\big) = \begin{cases} TRUE & M[w]\ \ doesn't\ halt \\ \\ FALSE & otherwise \end{cases}$$

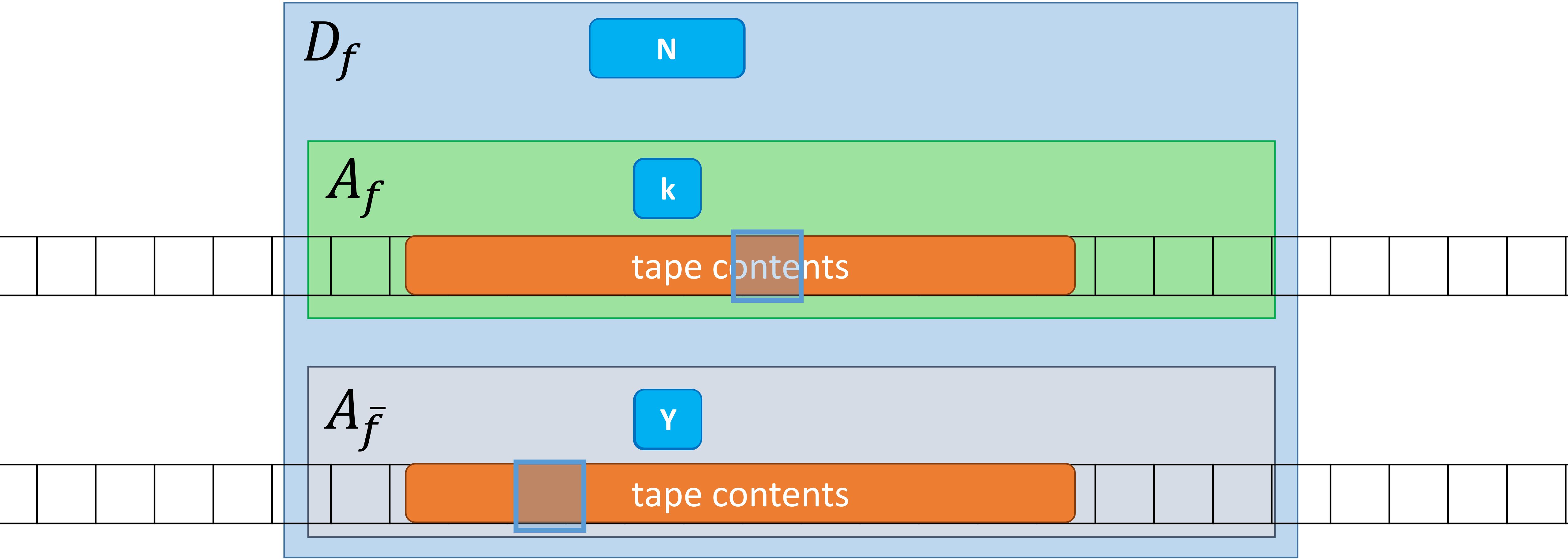
$$f \in R \Rightarrow \bar{f} \in R$$



$$(f \in RE) \wedge (\bar{f} \in RE) \Rightarrow f \in R$$

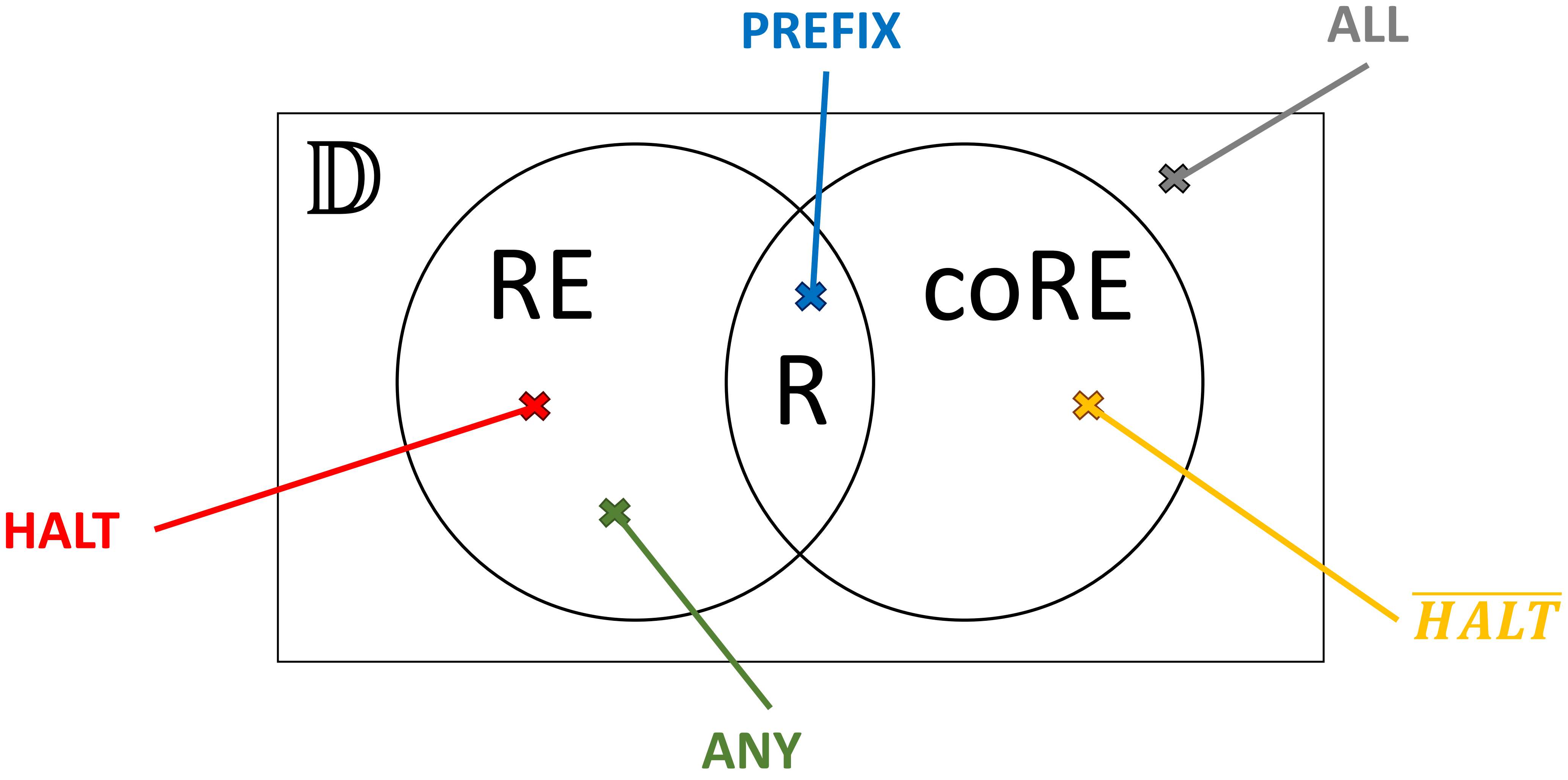


$$(f \in RE) \wedge (\bar{f} \in RE) \Rightarrow f \in R$$



$$\overline{ALL}\big(enc(M)\big) = \begin{cases} TRUE & \exists w \in \{0,1\}^*, M[w] \;\; doesn't\ halt \\ \\ FALSE & otherwise \end{cases}$$

Placement of problems



Programming challenge

Defeat a “Halting problem decider”

- You are provided with a program that attempts to solve the halting problem
- Provide an example of a (TM, input) pair that tricks the “decider”
- Provide a justification that indeed your pair does not halt
- **Soft deadline:** Wednesday, 30 OCT 2024, 23:55