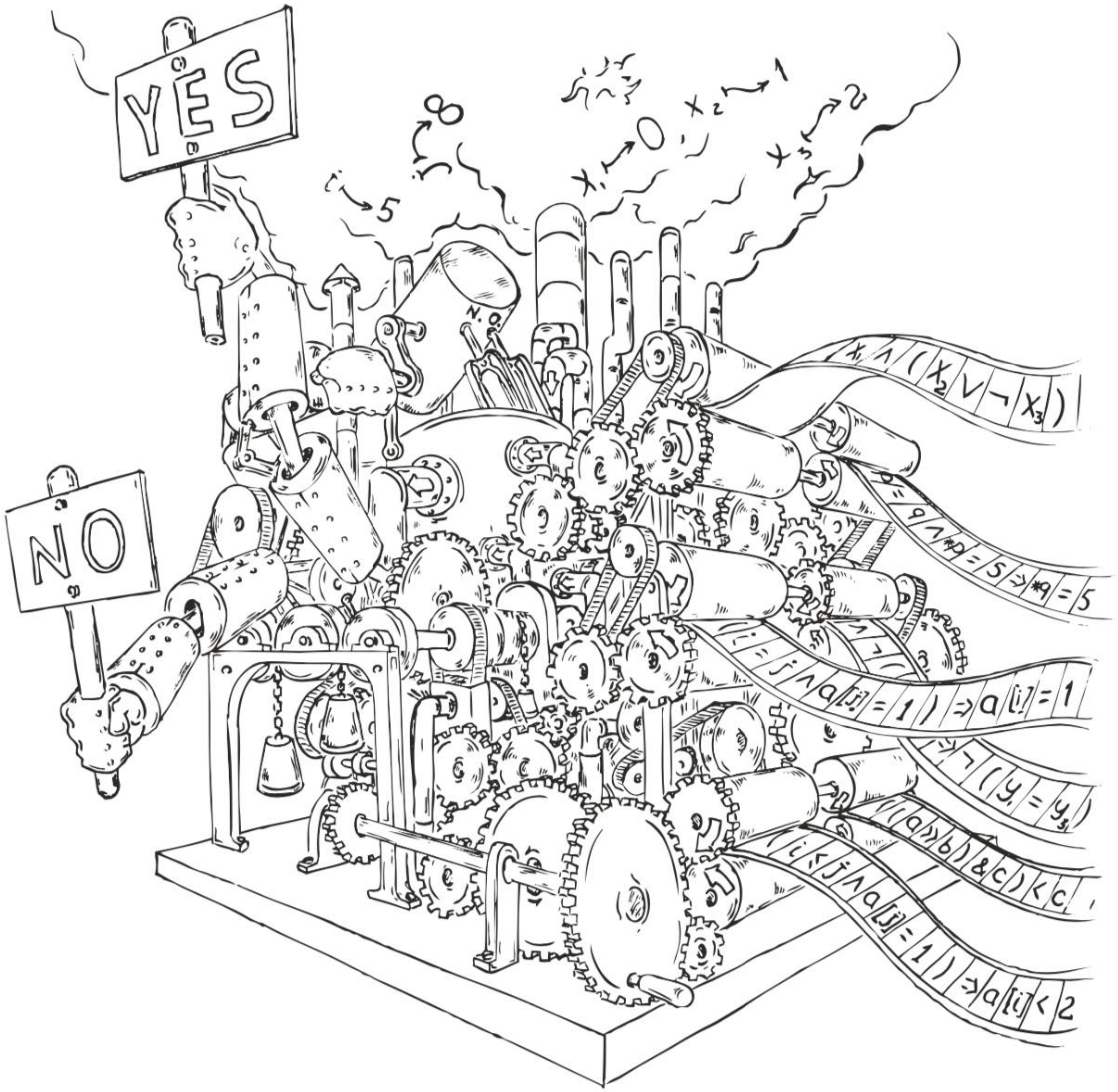
11

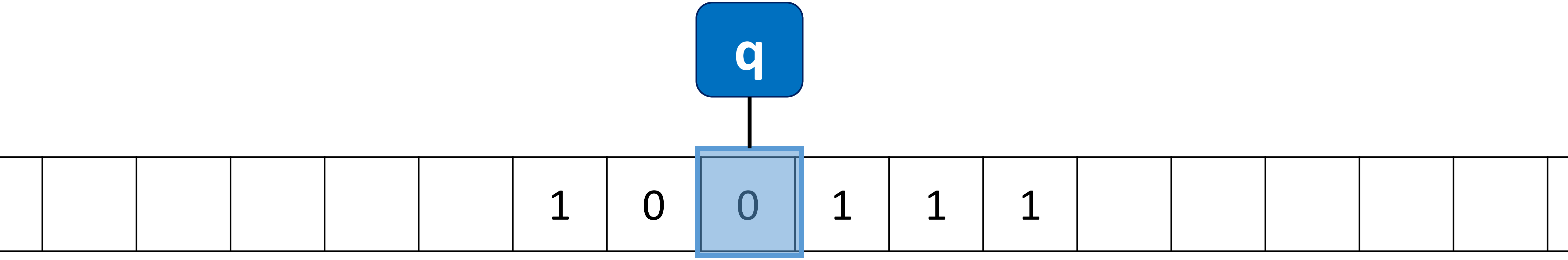


$\boldsymbol{P}$ vs. $\boldsymbol{NP}$

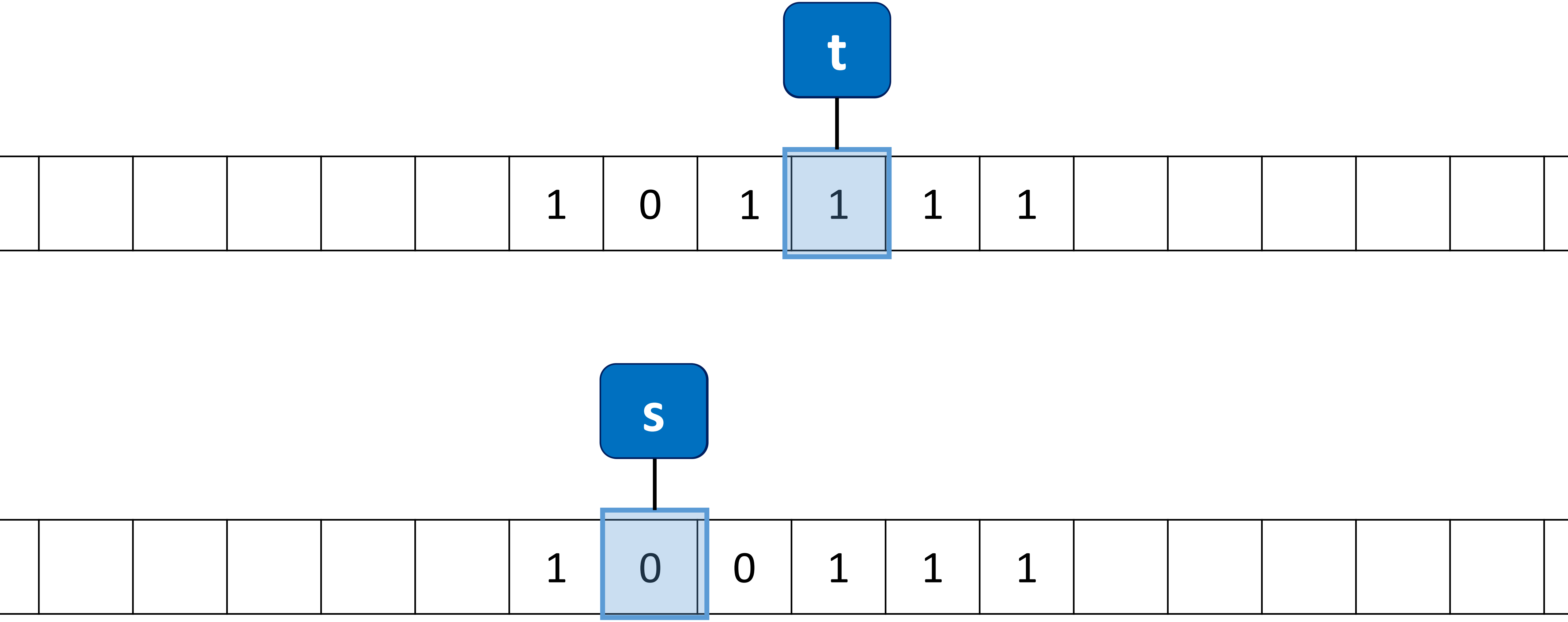
Analiza Algoritmilor

Drawing by Ilya Yodovsky Jr., from *Decision Procedures: An Algorithmic Point of View* by Daniel Kroening and Ofer Strichman
Source: https://decomposition.al/CSE290Q-2019-09/

Nondeterminism

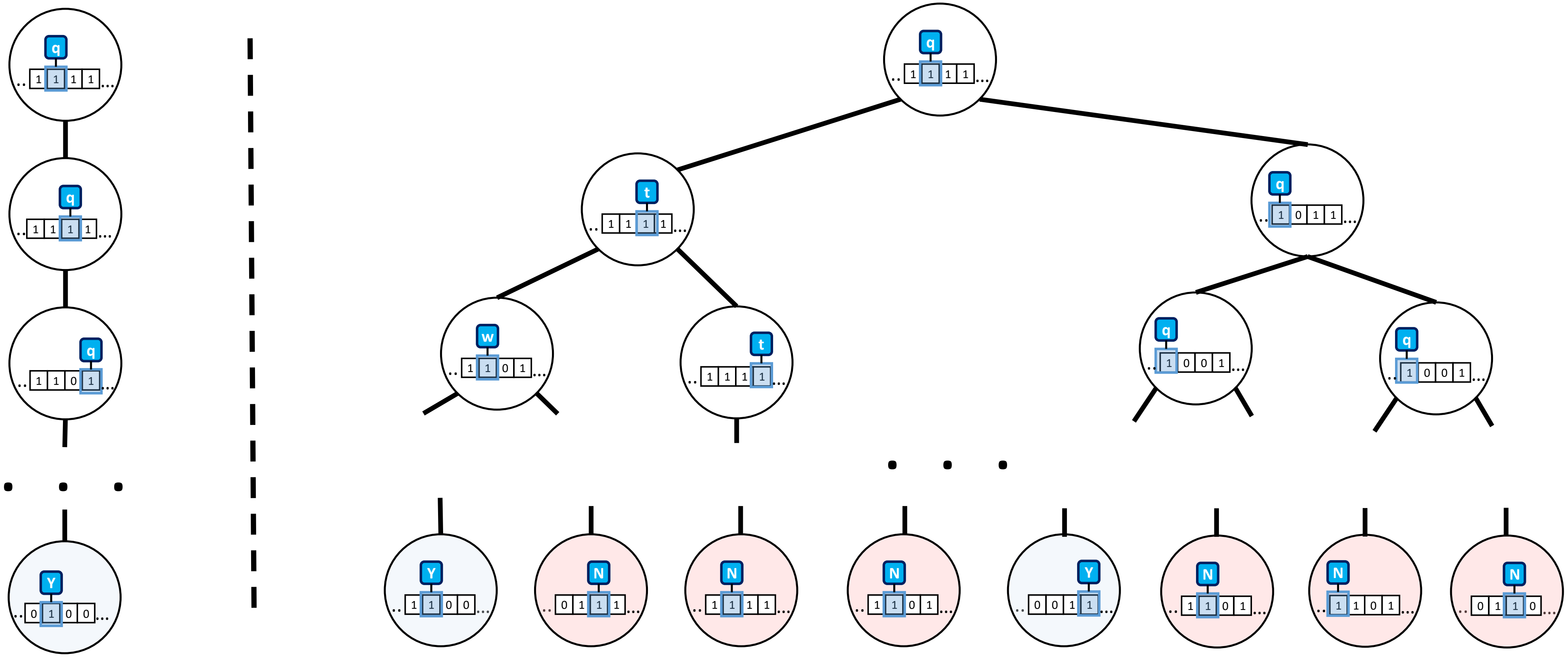


Nondeterminism



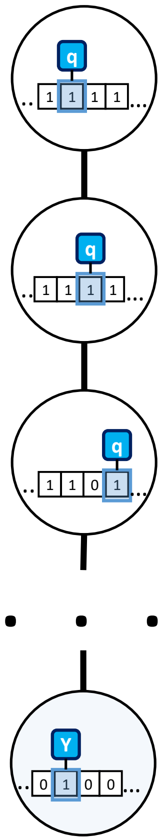
Computational histories

Deterministic Nondeterministic

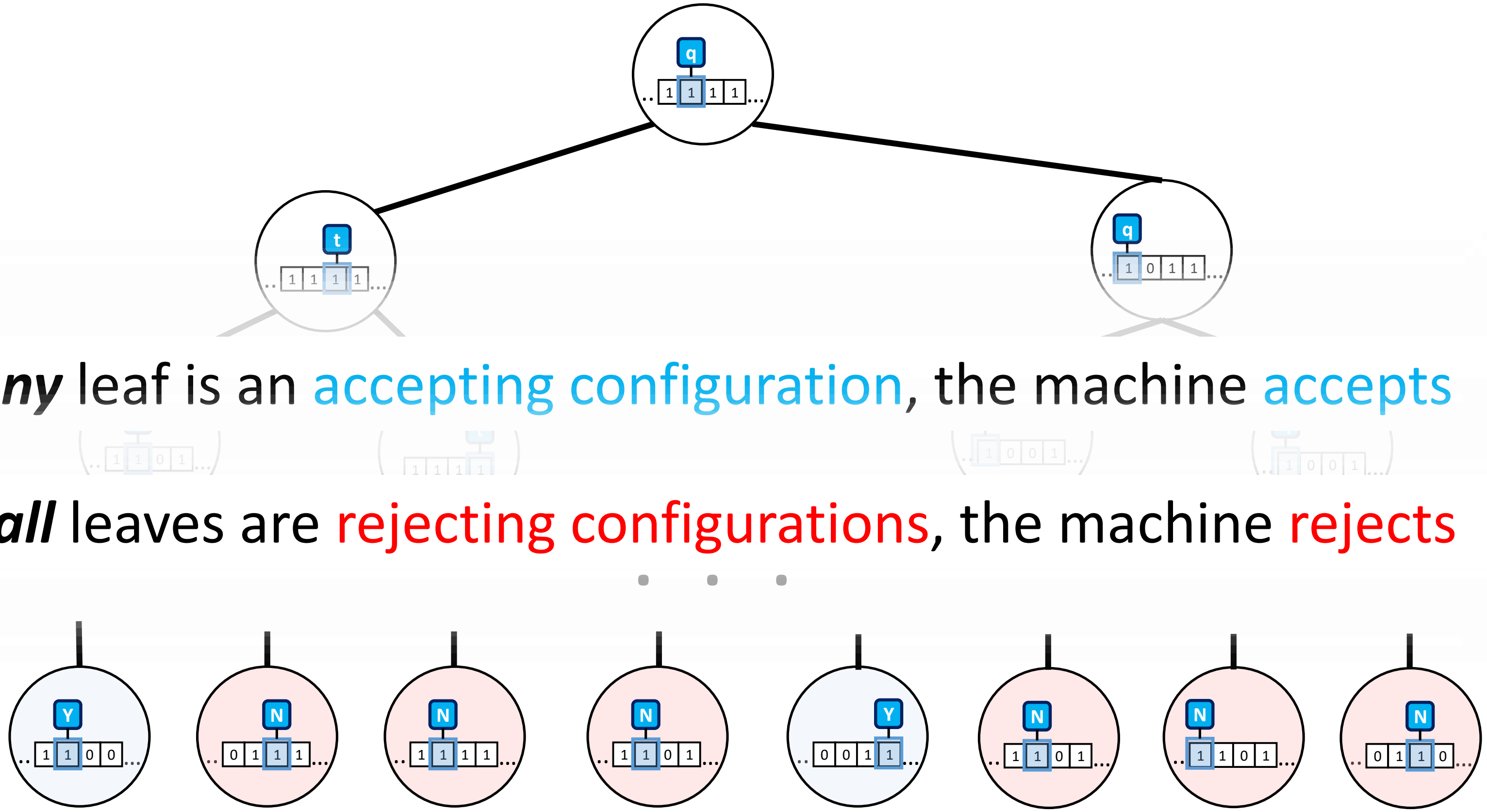


Computational histories

Deterministic



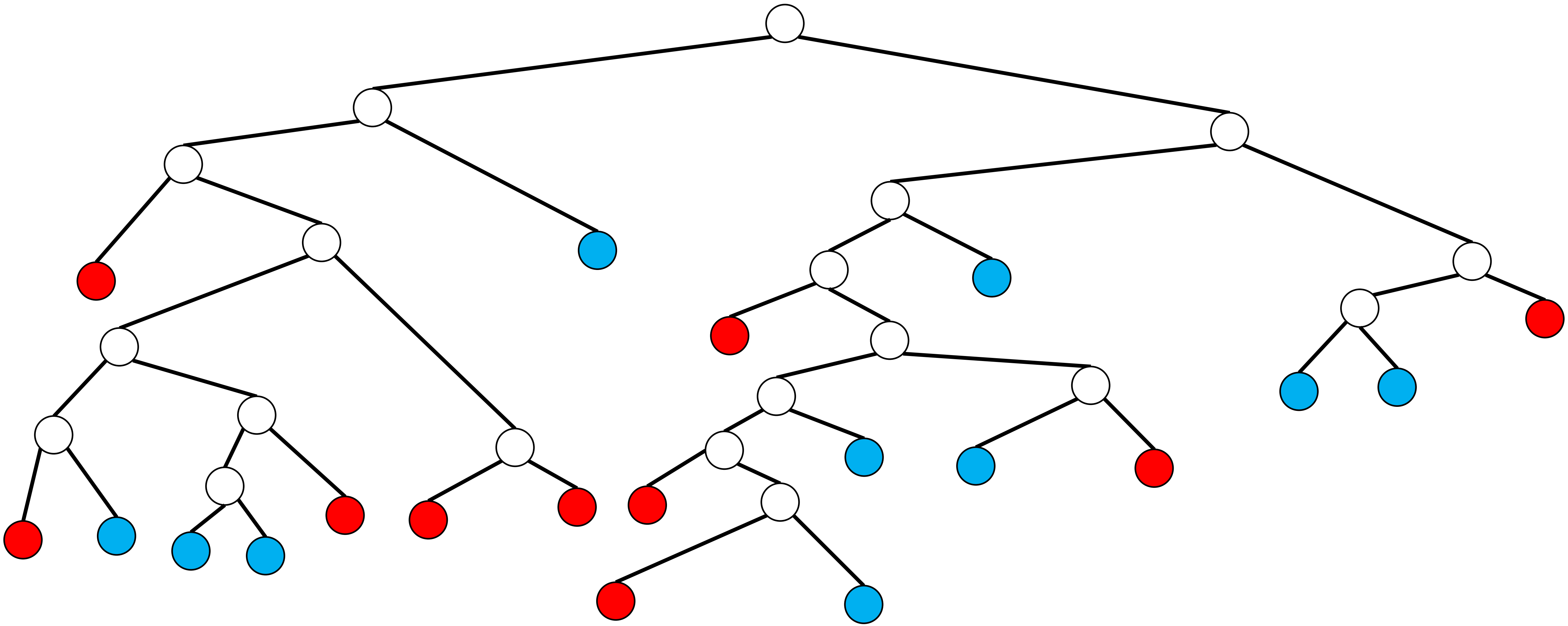
Nondeterministic



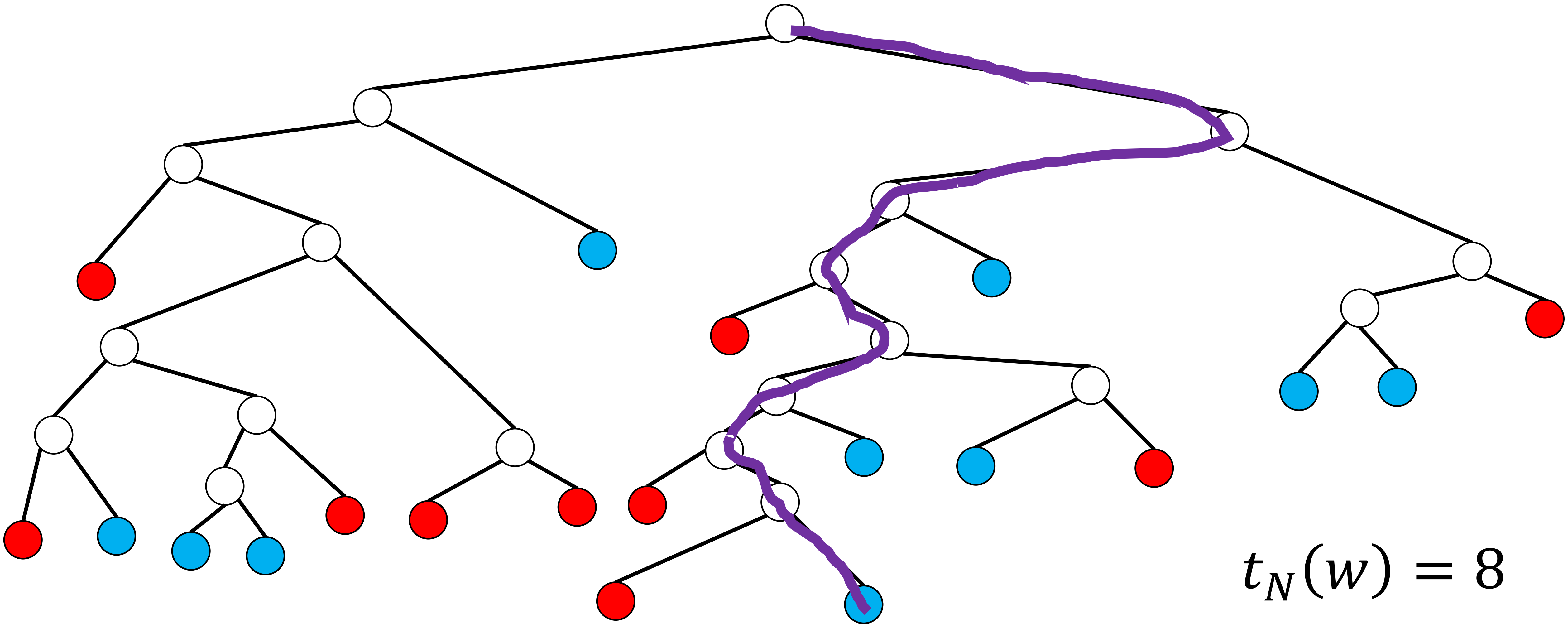
If ***any*** leaf is an accepting configuration, the machine accepts

If ***all*** leaves are rejecting configurations, the machine rejects

Running time for an NTM

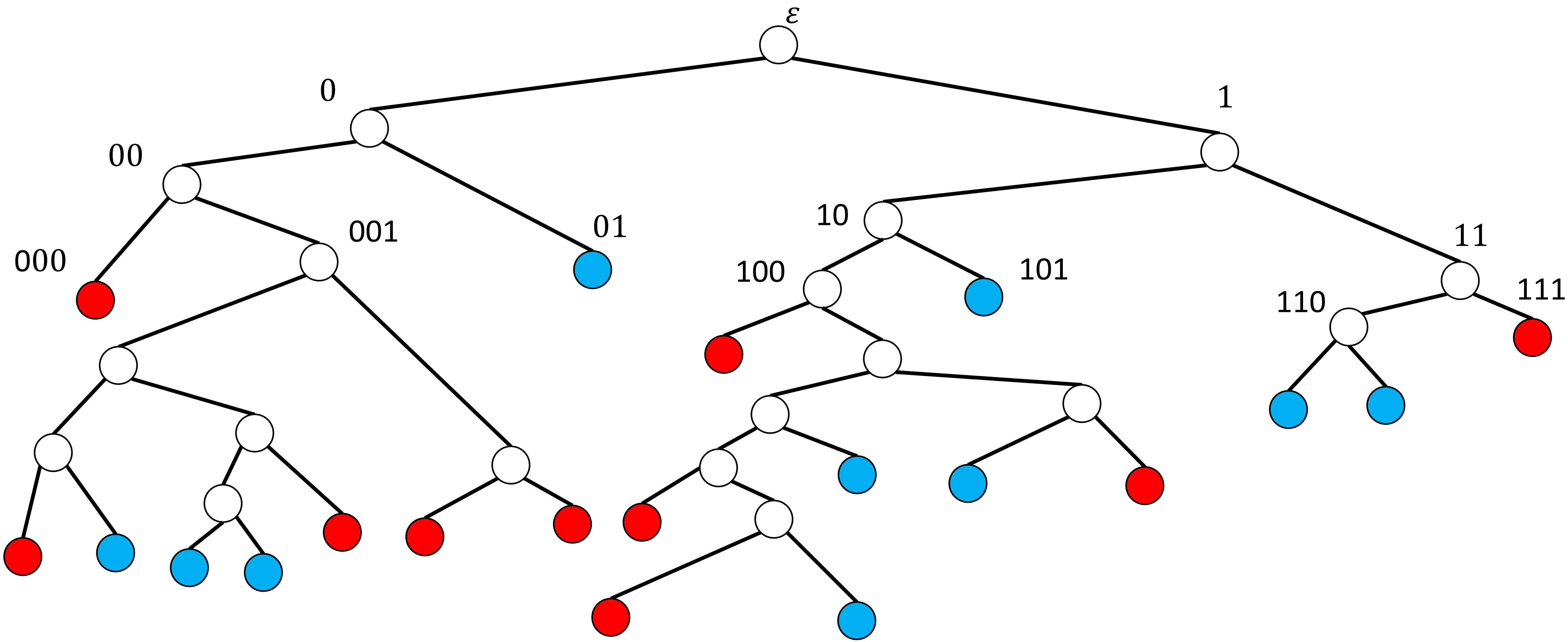


Running time for an NTM

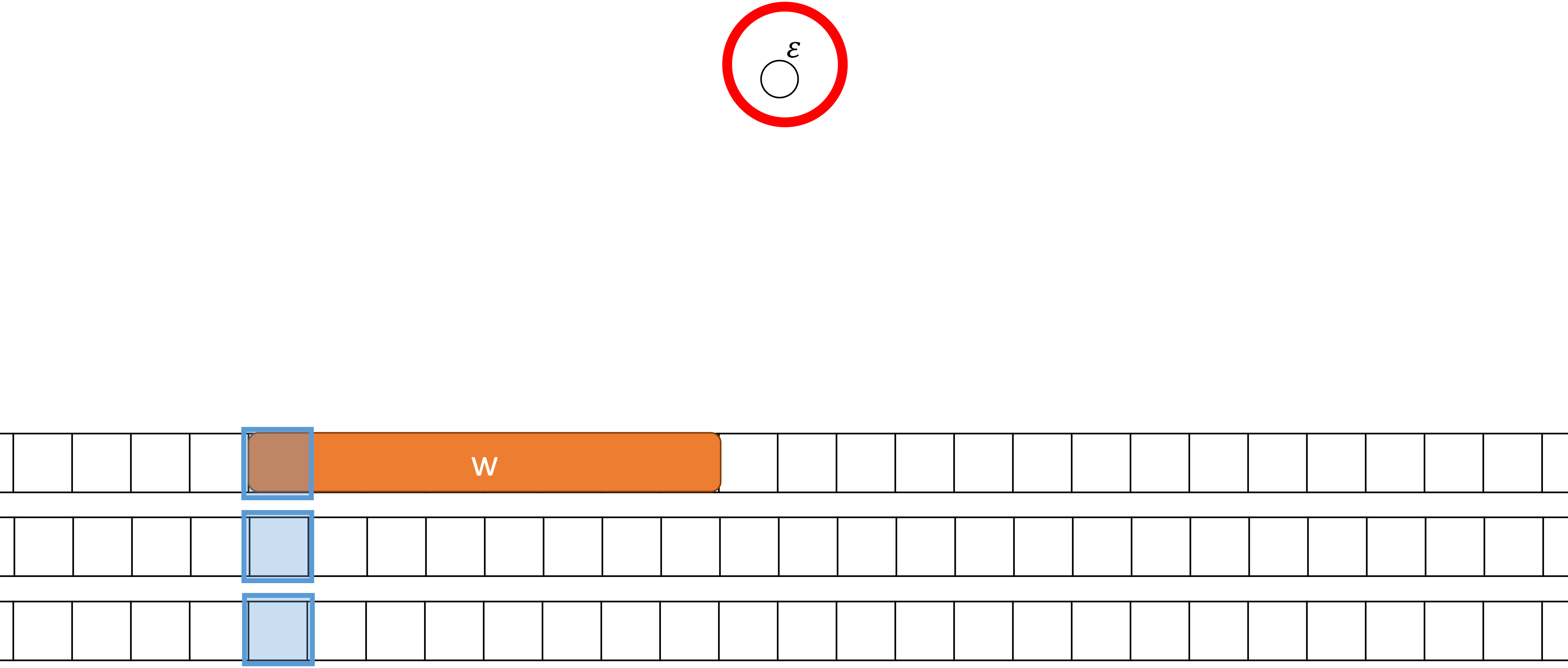


$$t_N(w) = 8$$

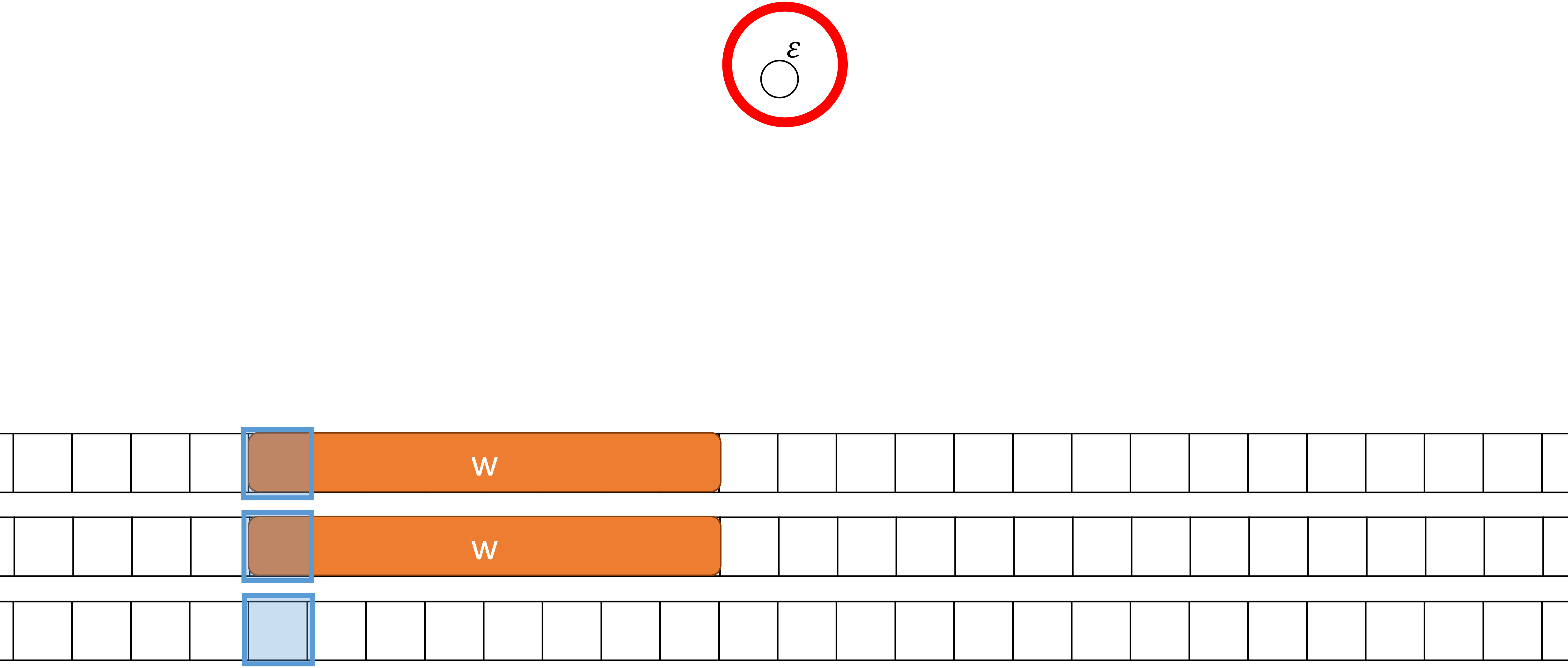
Address of a node



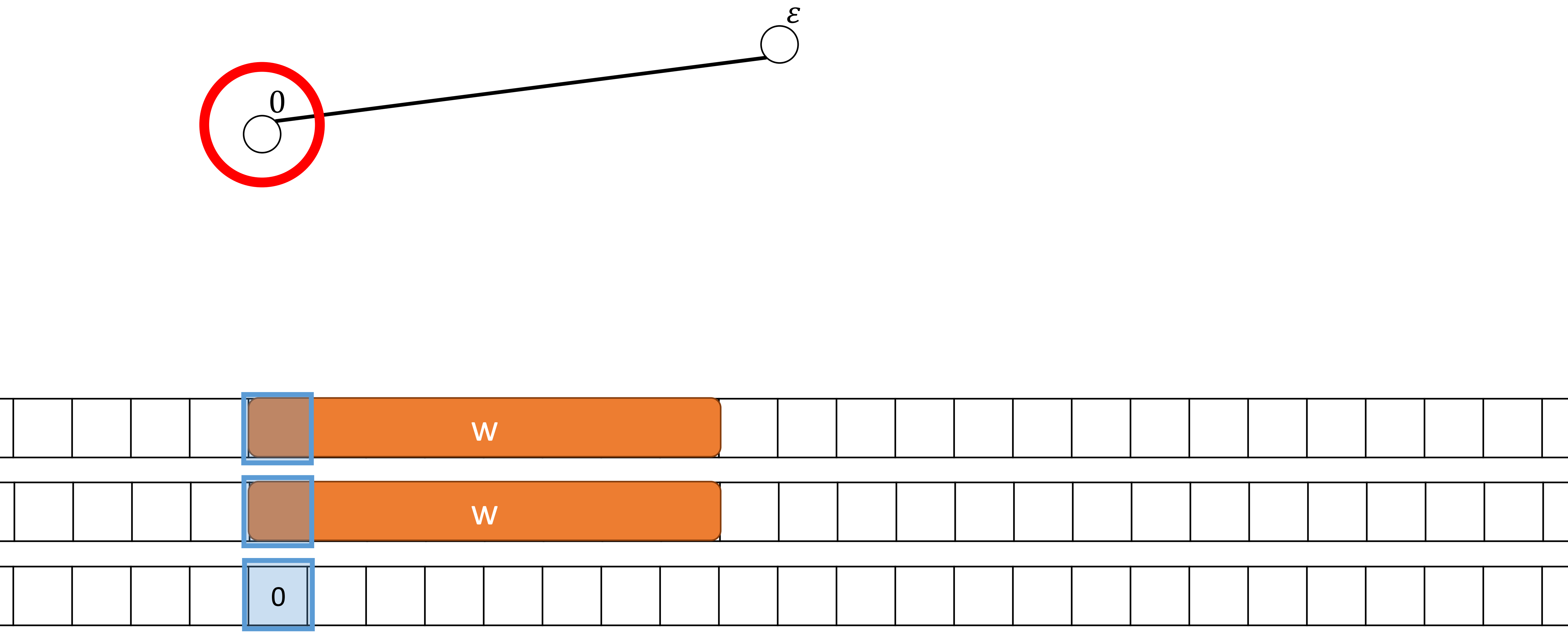
Deterministic simulation



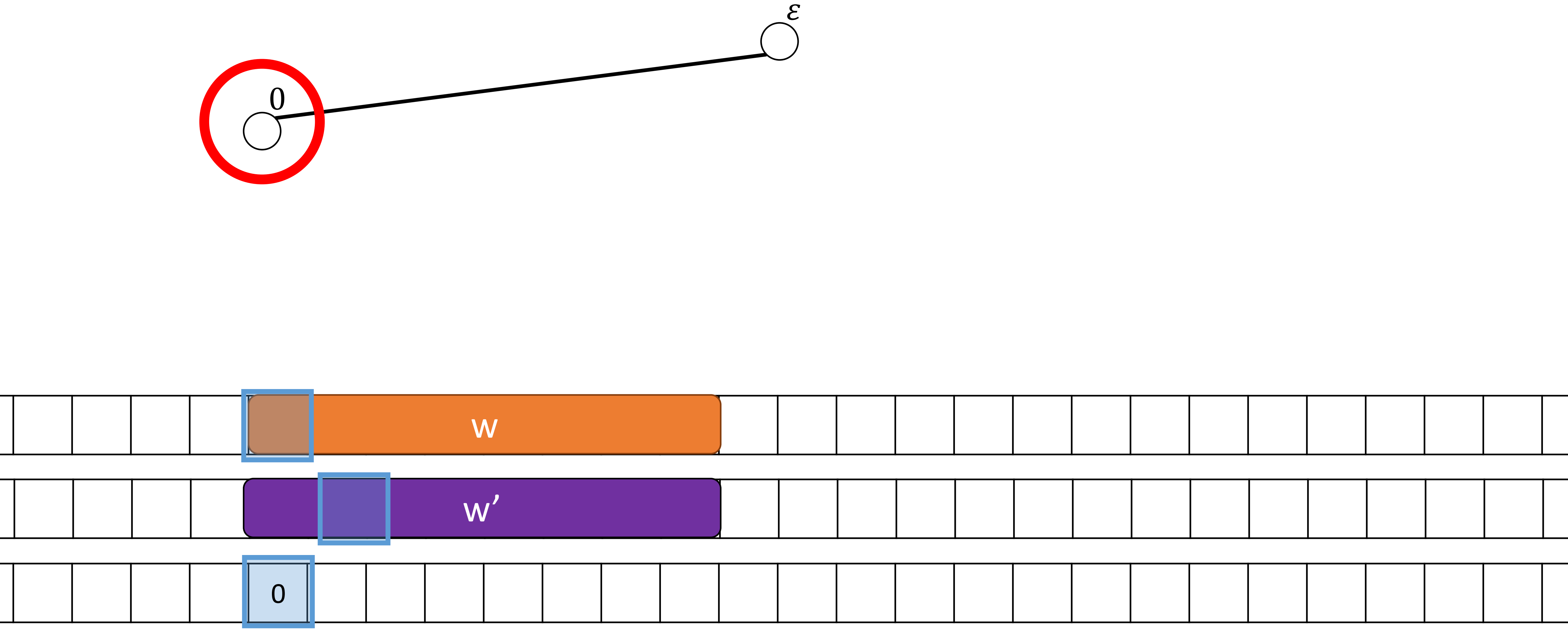
Deterministic simulation



Deterministic simulation



Deterministic simulation



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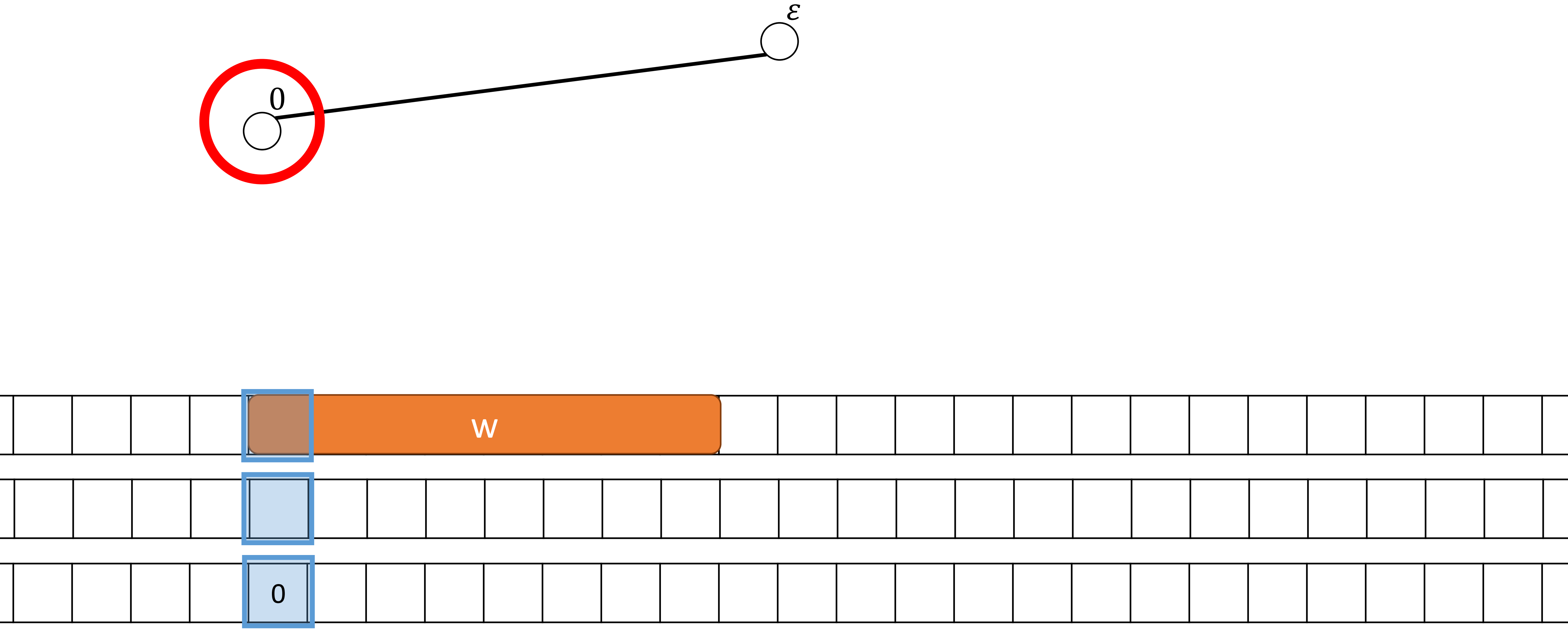

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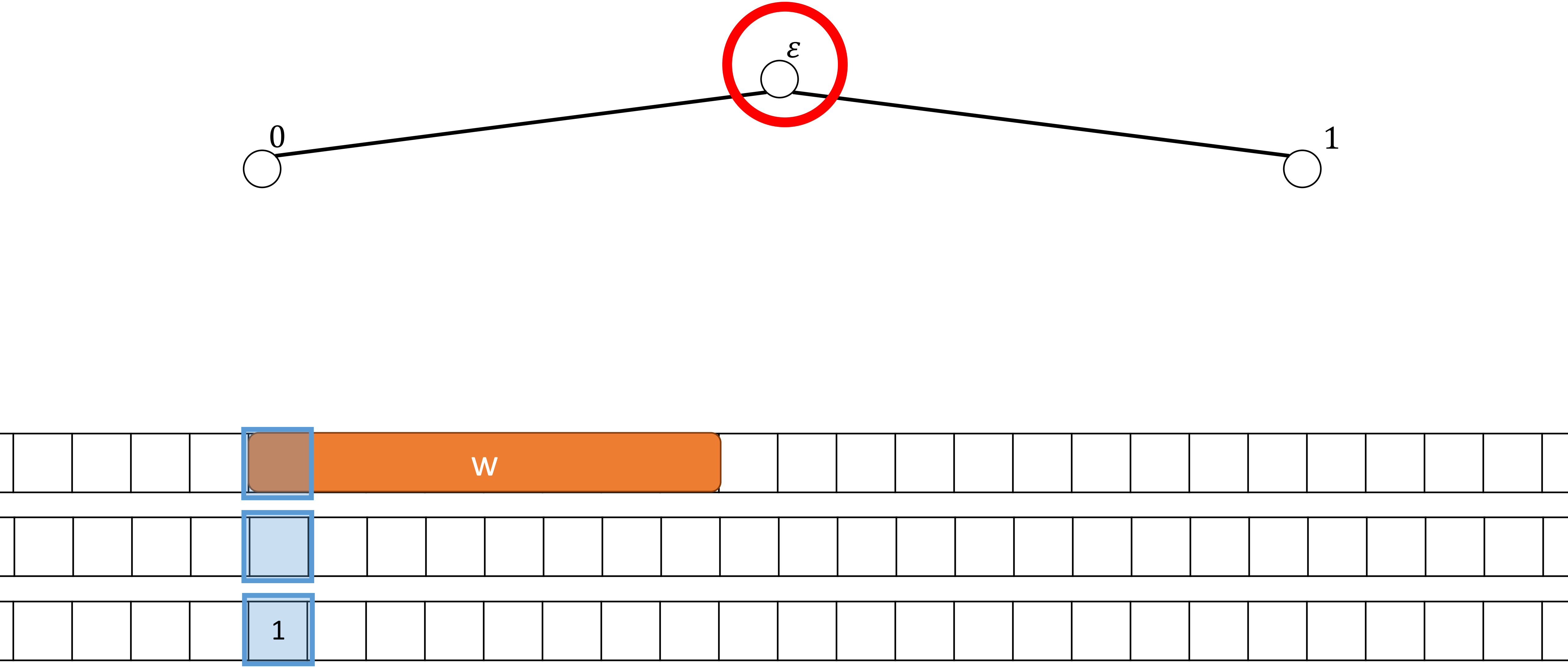
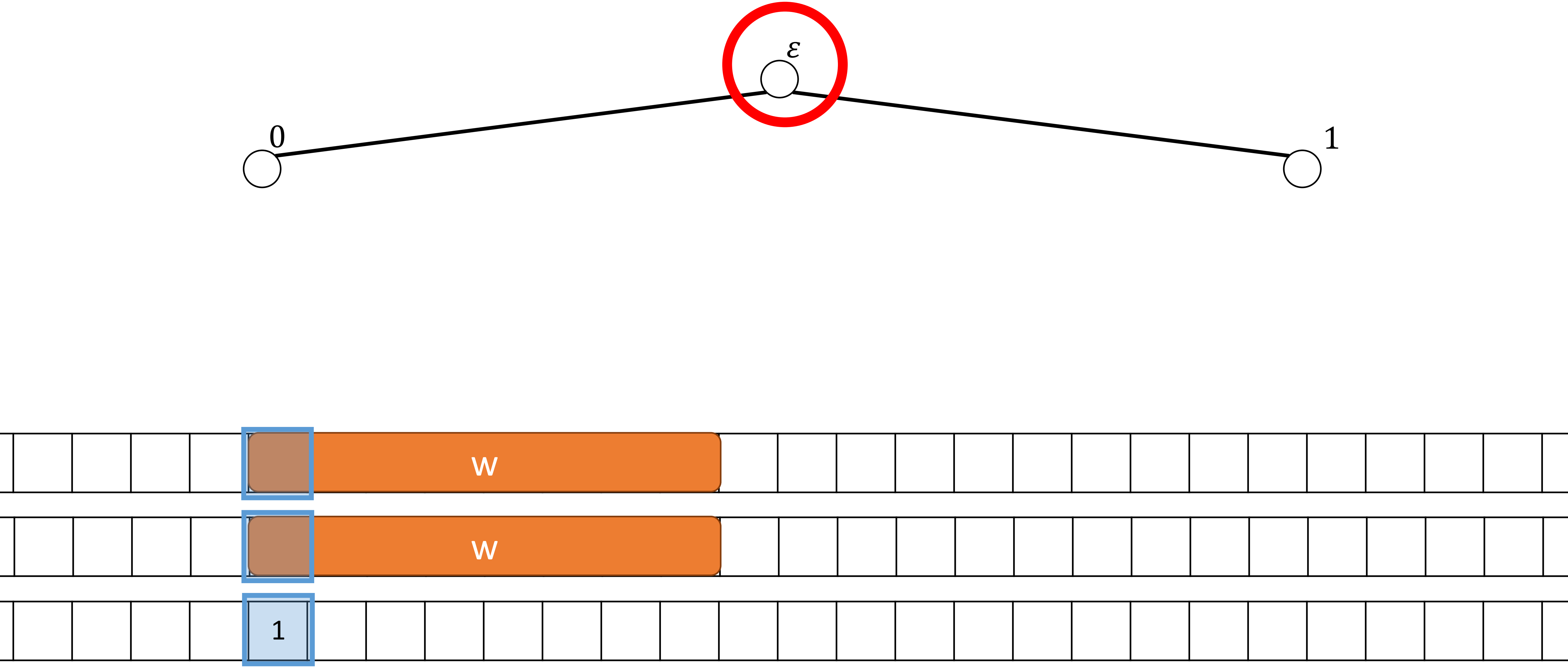
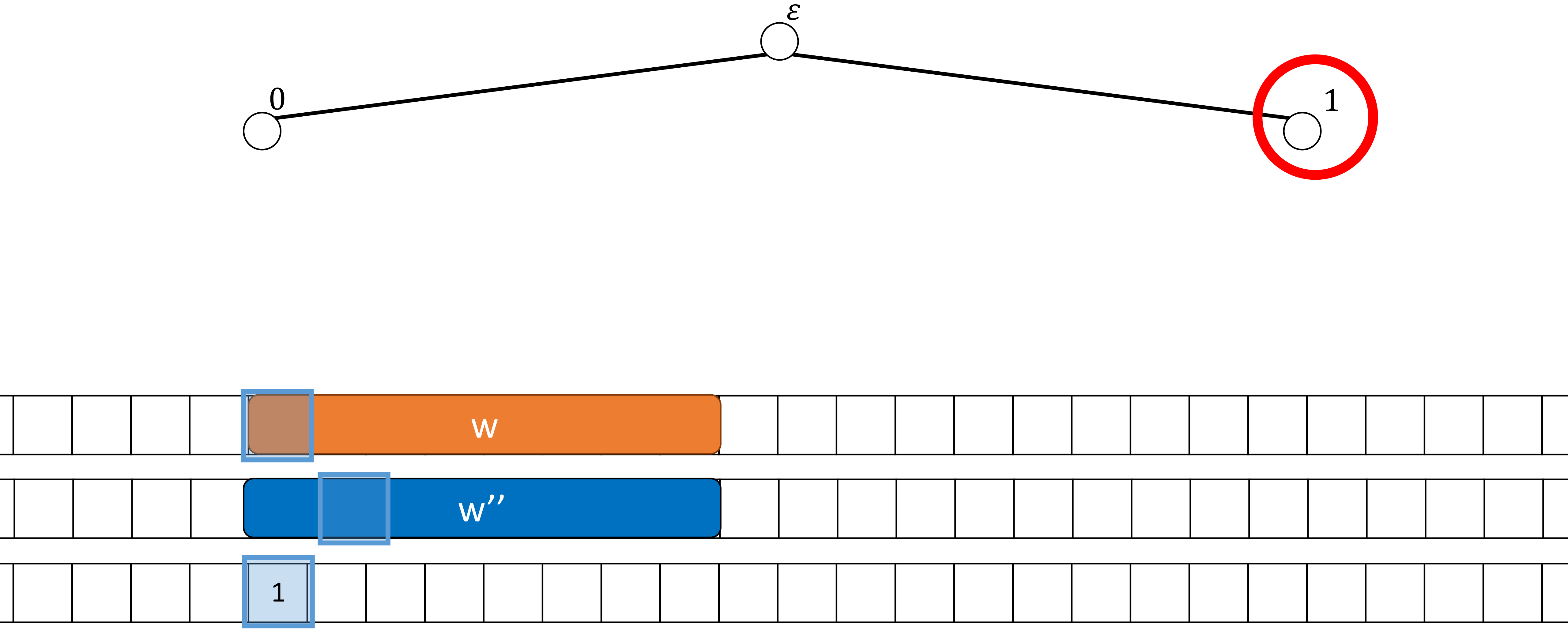


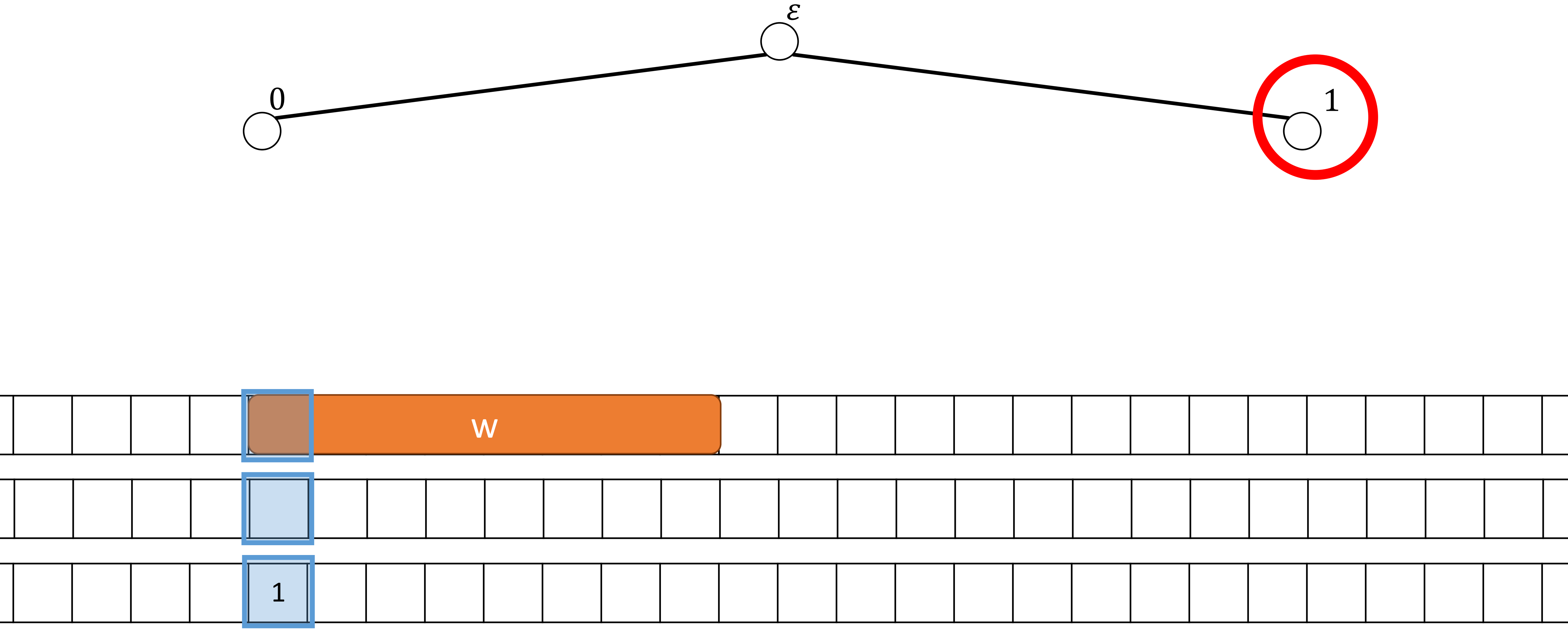
Deterministic simulation



Deterministic simulation



Deterministic simulation



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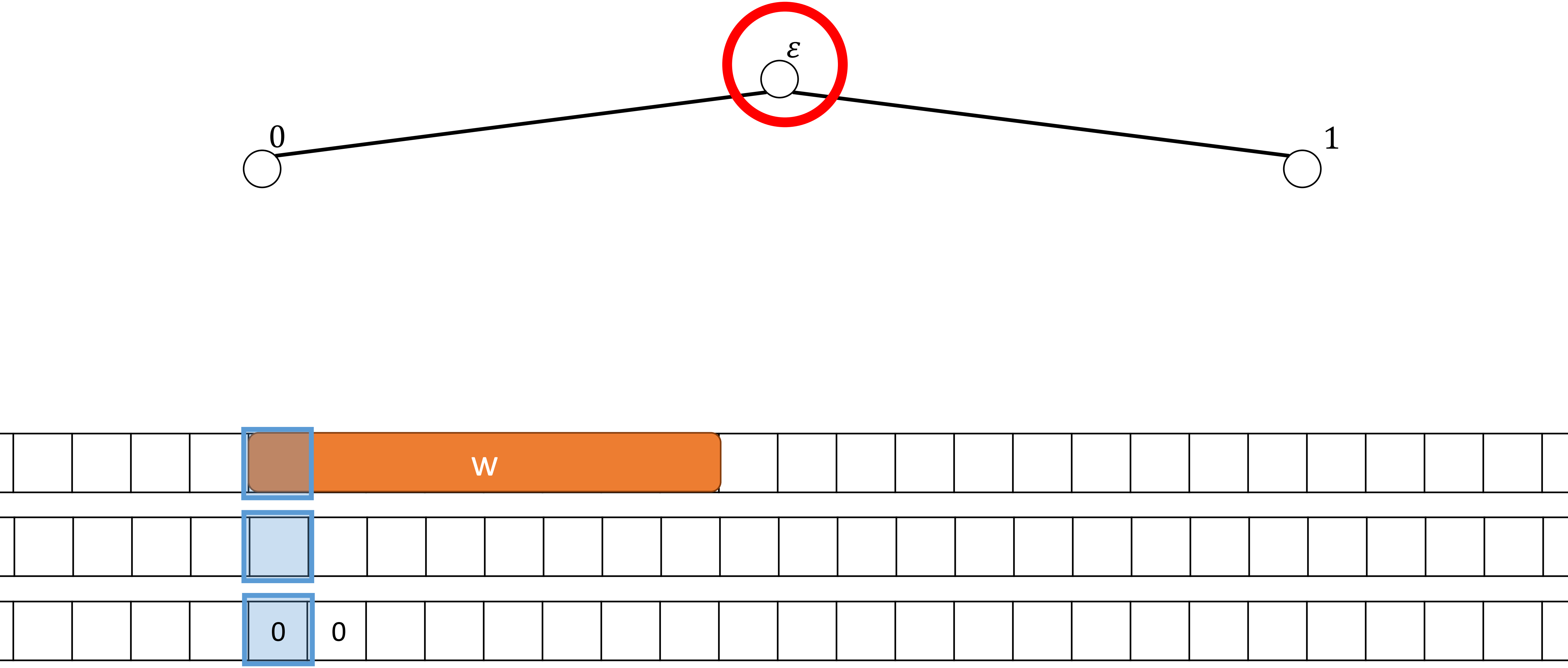
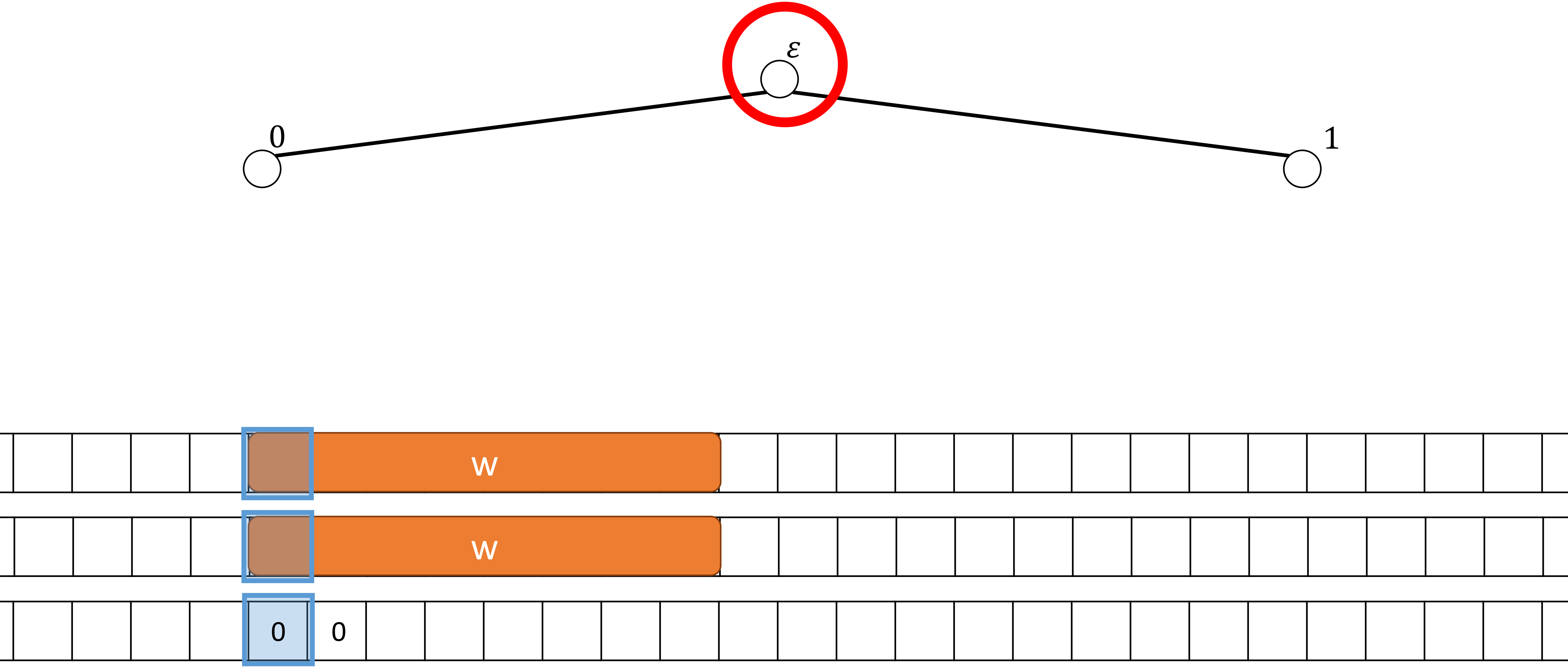
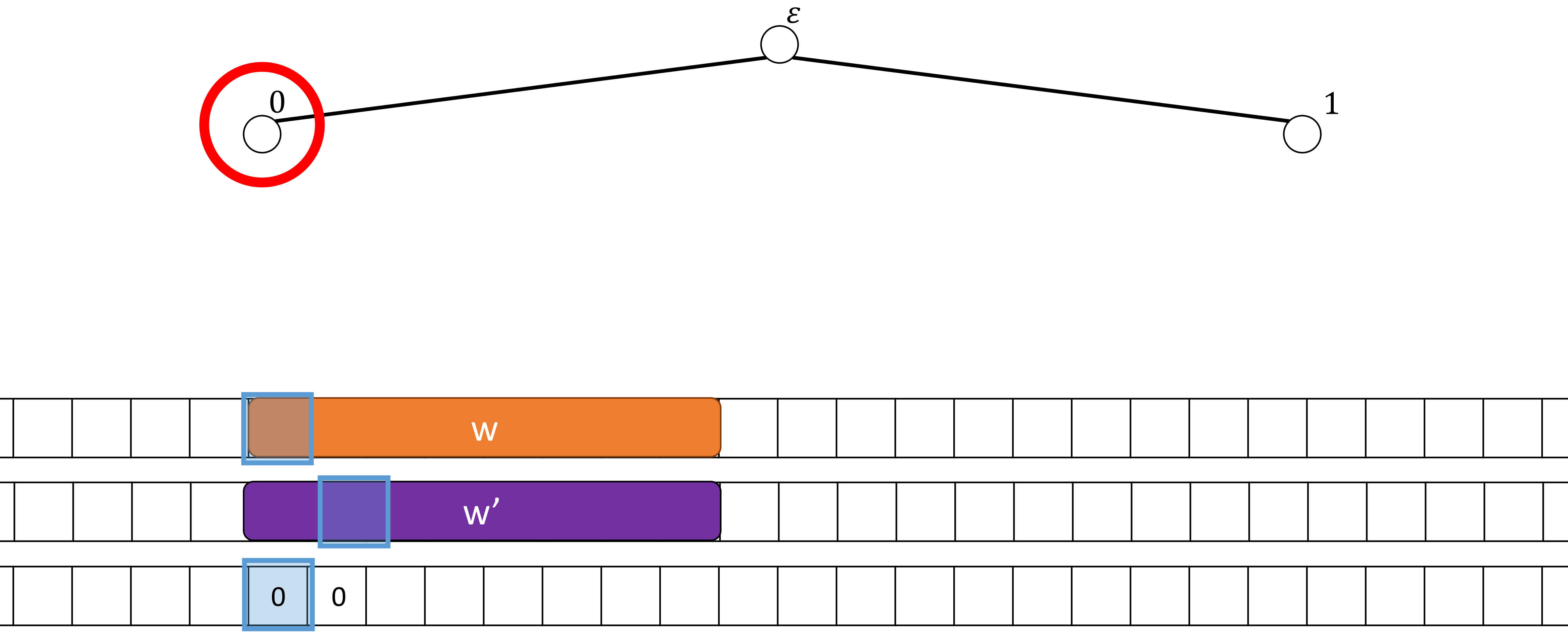


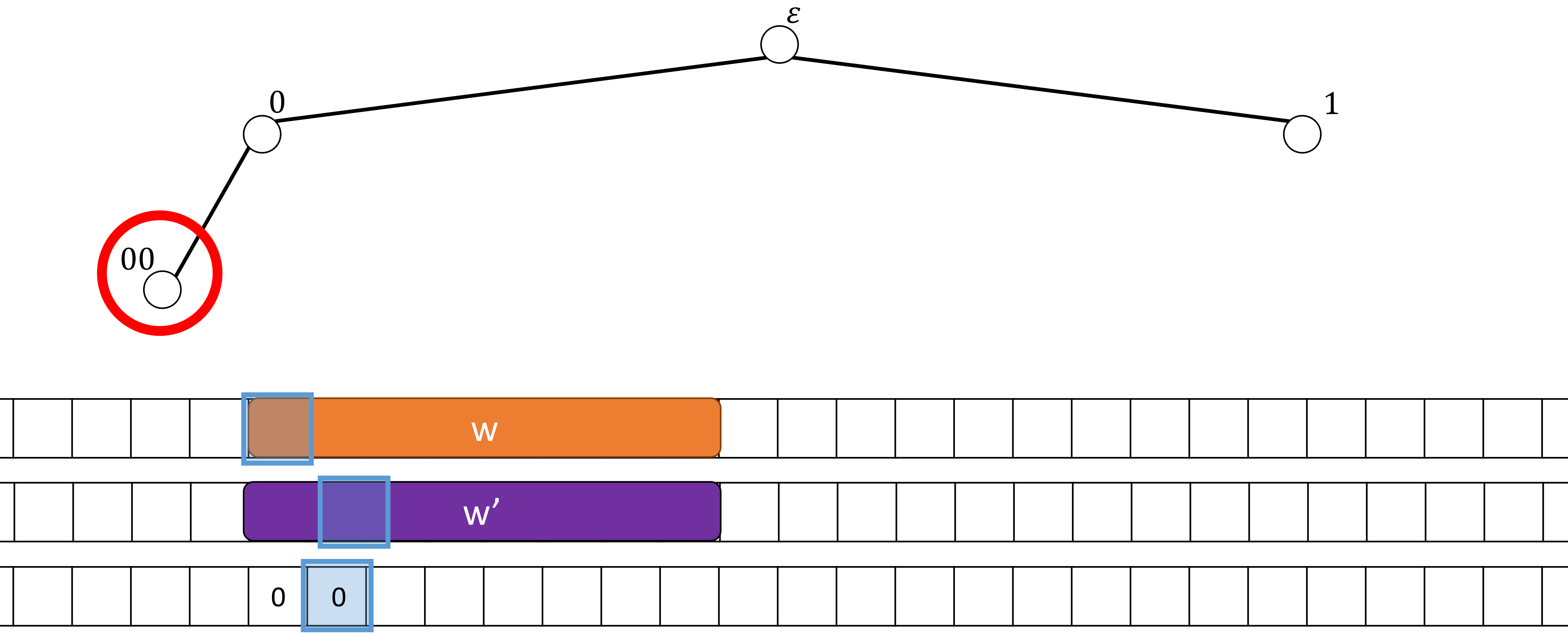
Deterministic simulation



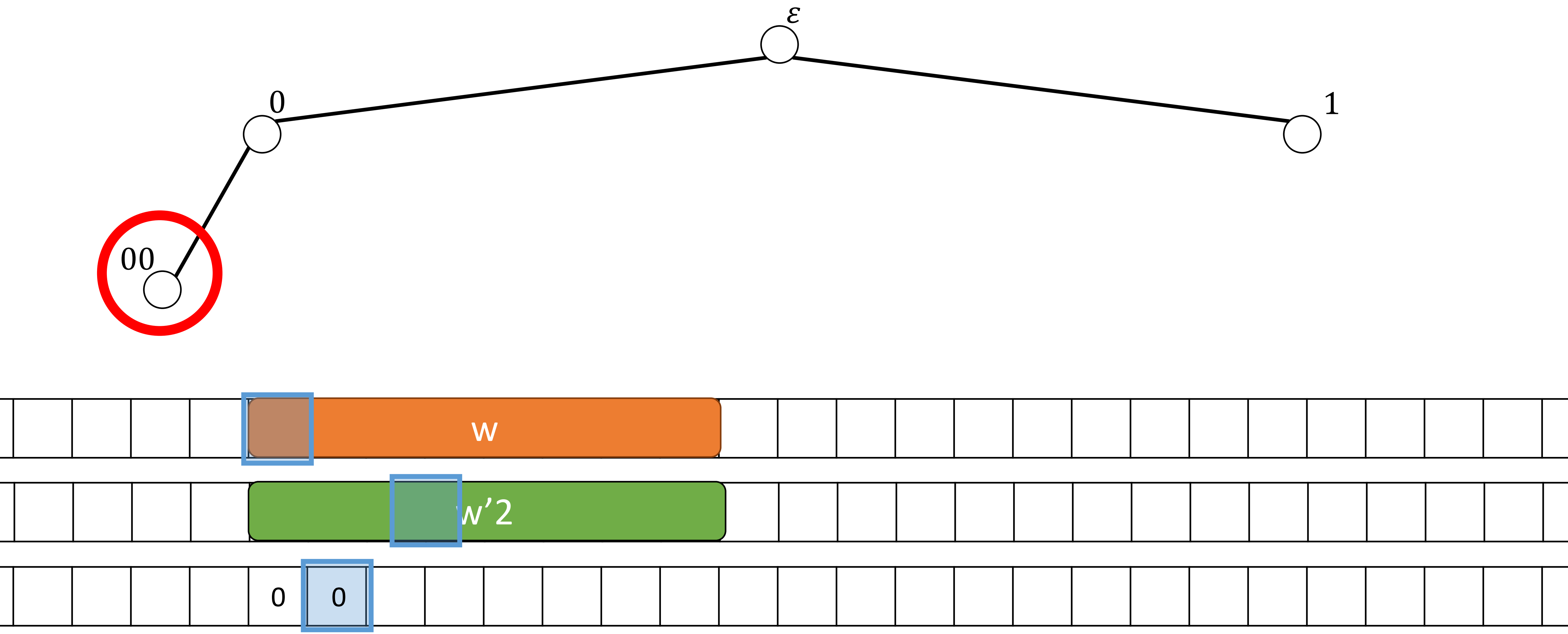
Deterministic simulation



Deterministic simulation



Deterministic simulation



The class $\boldsymbol{P}$

$$\boldsymbol{P} \stackrel{\text{def}}{=} \bigcup_{k \in \mathbb{N}} DTIME(n^k)$$

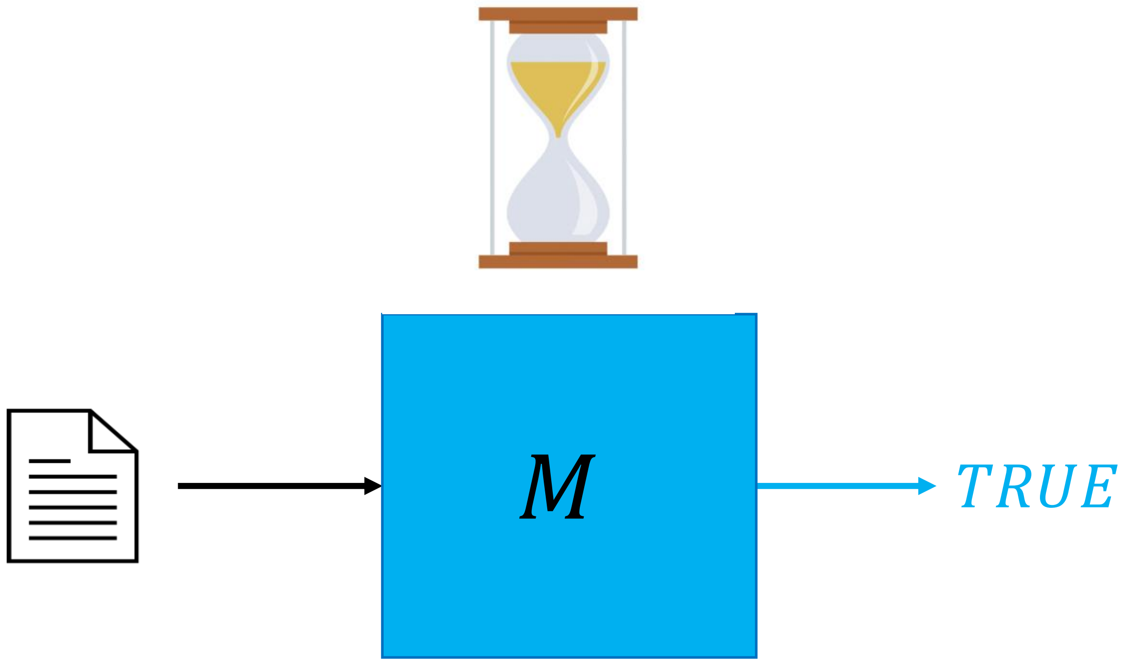
The class of tractable problems

The class $\boldsymbol{NP}$

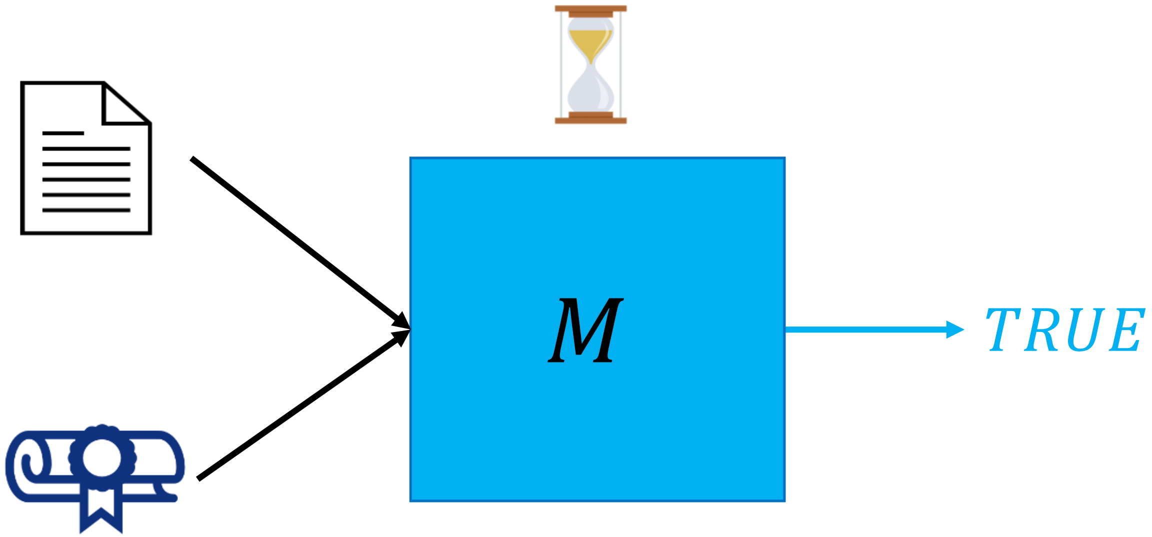
$$\boldsymbol{NP} \overset{\text{def}}{=} \bigcup_{k \in \mathbb{N}} NTIME(n^k)$$

The class of … ?

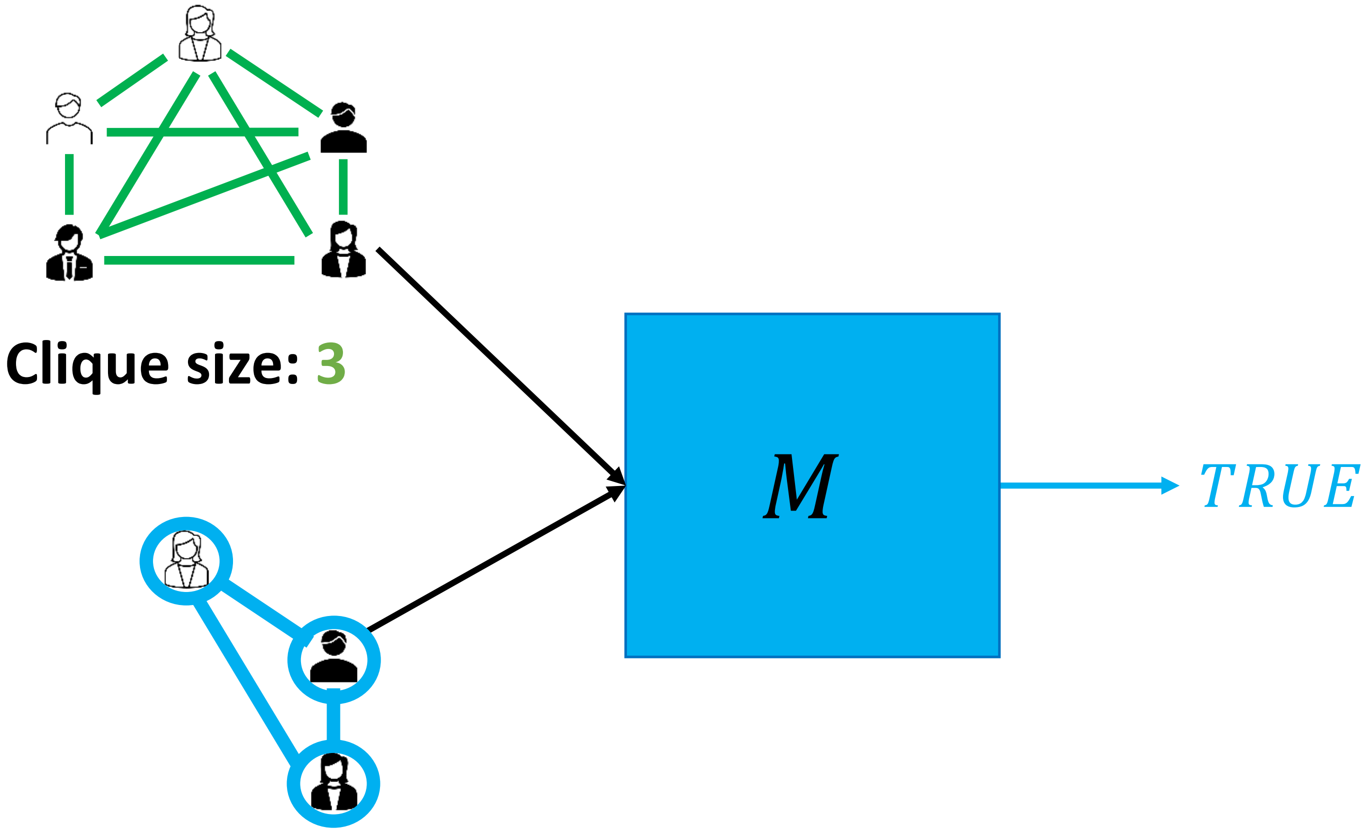
Polynomial certificates



Polynomial certificates

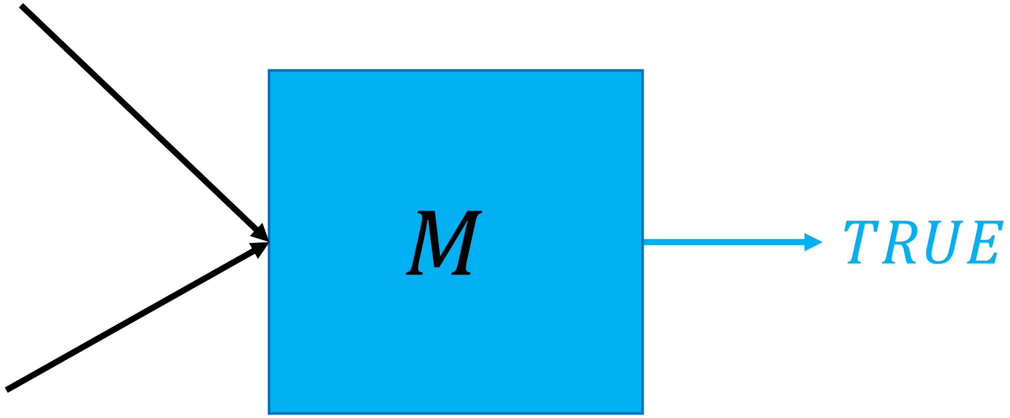


Polynomial certificate for `CLIQUE`



Polynomial certificate for SAT

$$\phi = \left(\left(x \vee \bar{y} \vee (\bar{z} \wedge y)\right) \wedge \bar{t}\right) \vee (t \wedge \bar{x})$$



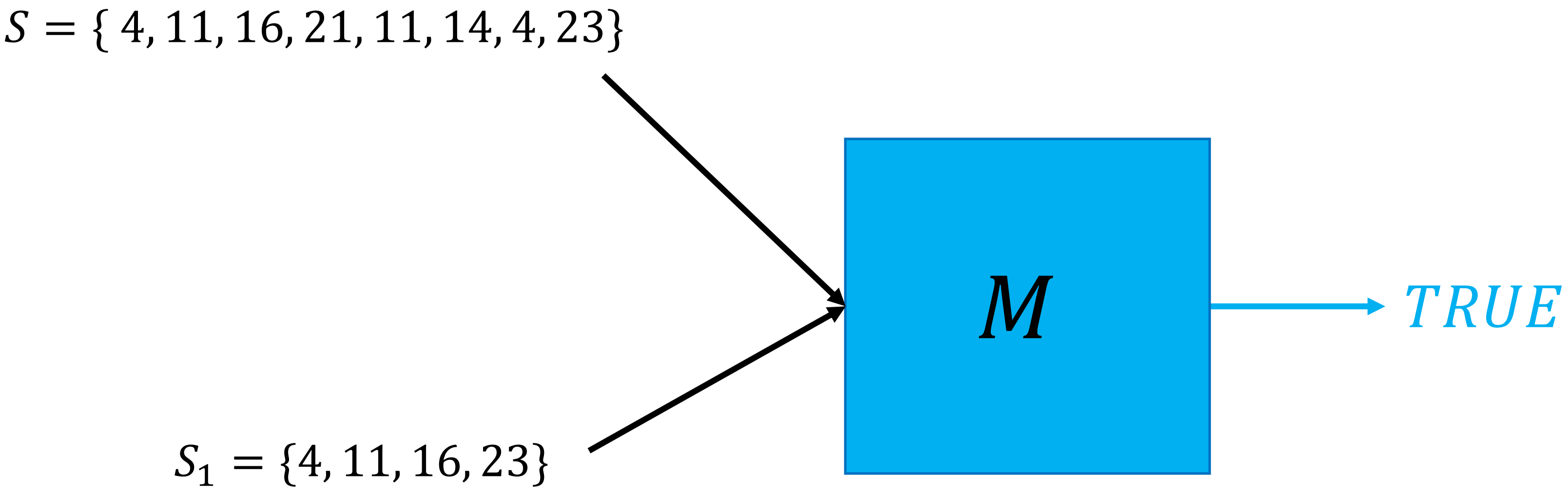
$T(x) = TRUE$

$T(y) = TRUE$

$T(z) = TRUE$

$T(t) = FALSE$

Polynomial certificate for `PARTITIONING`



PARTITIONING – Solution

```
SOLVE_SUBSET_SUM(S, K):
    r1 ← GENERATE_AND_CHECK(∅, 1, FALSE)
    r2 ← GENERATE_AND_CHECK(∅, 1, TRUE)
    return r1 ∨ r2
```

```
GENERATE_AND_CHECK(S1, i, choose):
    if choose
        S1 ← S1 ∪ {S[i]}

    if VALIDATE_SUBSET(S1)
        return TRUE

    r1 ← GENERATE_AND_CHECK(S1, i + 1, FALSE)
    r2 ← GENERATE_AND_CHECK(S1, i + 1, TRUE)
    return r1 ∨ r2
```

```
VALIDATE_SUBSET(S1):
    sum1 ← 0
    sum2 ← 0
    for each x ∈ S
        if x ∈ S1
            sum1 ← sum1 + x
        else
            sum2 ← sum2 + x

    return sum1 = sum2
```

The class $\boldsymbol{NP}$ – alternative definition

$$\boldsymbol{NP} \stackrel{\text{def}}{=} \{\text{problems with polynomial-sized certificates}\}$$

The class of problems with a ***usable solution***

$$\boldsymbol{P \subseteq NP}$$

$$\boldsymbol{P = NP}\ ?$$

$$\boldsymbol{P \neq NP}\ ?$$

It is currently unknown whether $\boldsymbol{P = NP}$...