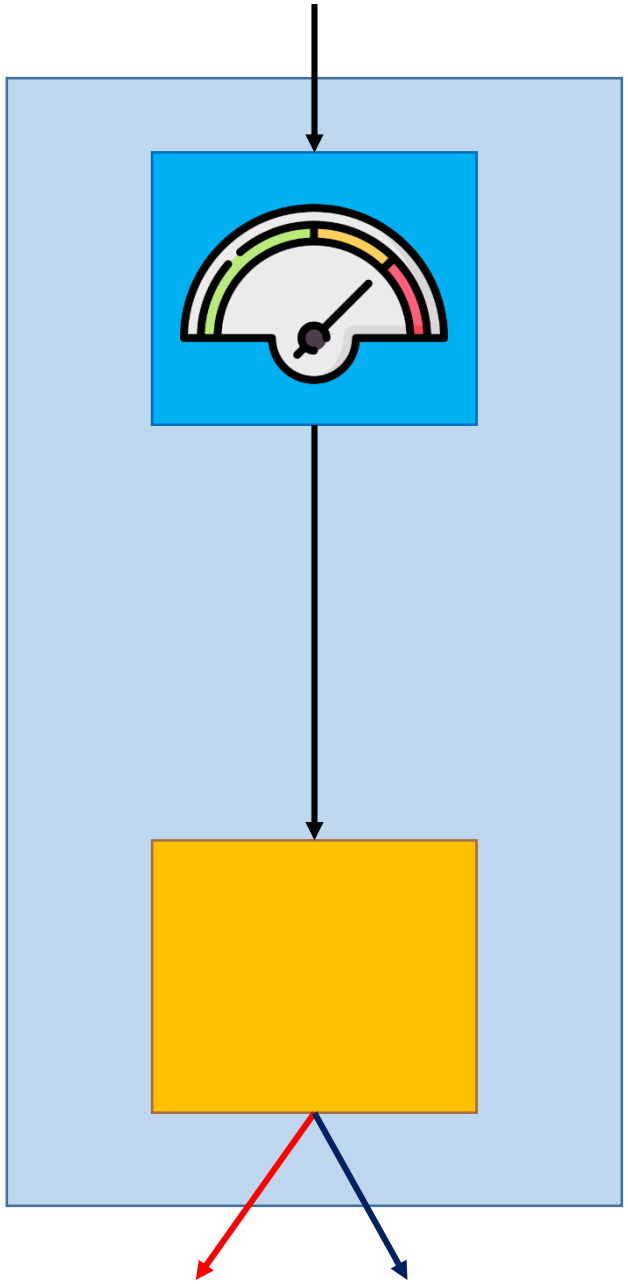
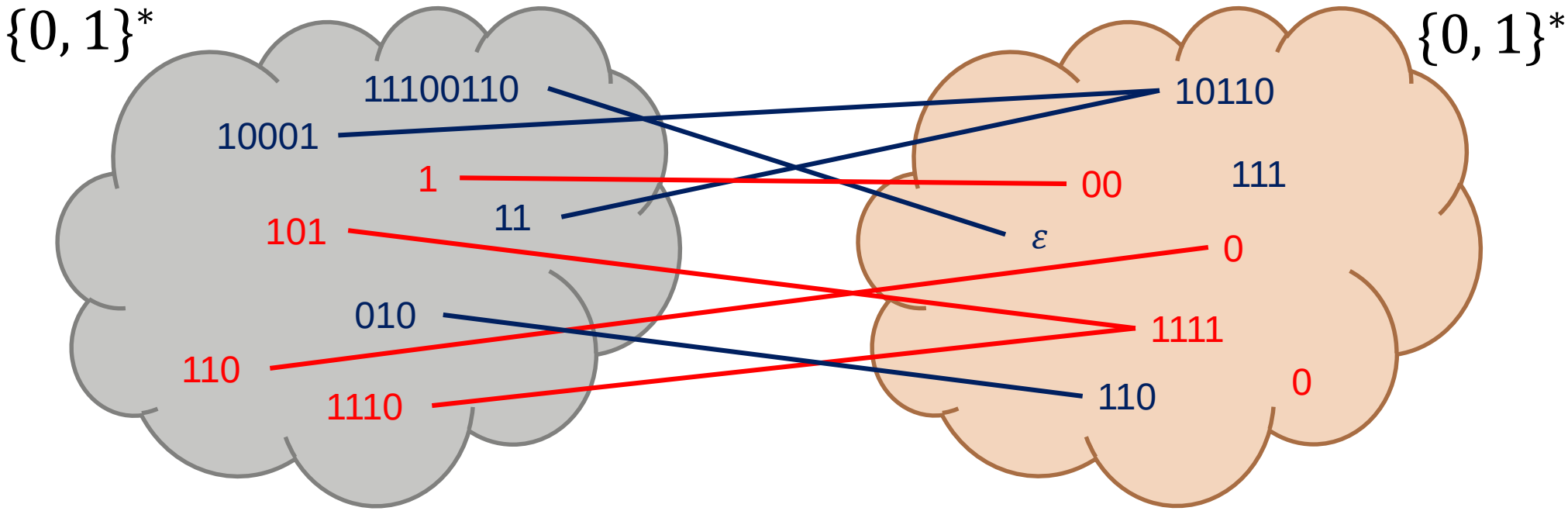


Polynomial-time reductions

Analiza Algoritmilor

Speedometer icon: https://www.flaticon.com/free-icon/speedometer_9581673

Mapping reductions



$f(\blacksquare) = TRUE$

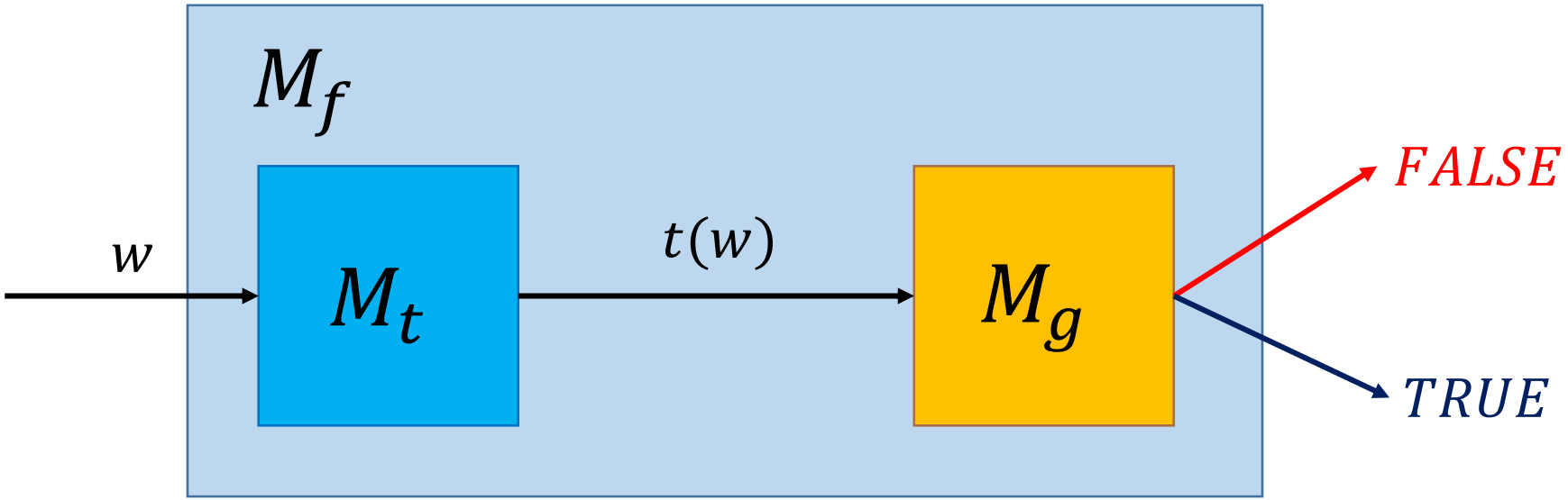
$f(\blacksquare) = FALSE$

$g(\blacksquare) = TRUE$

$g(\blacksquare) = FALSE$

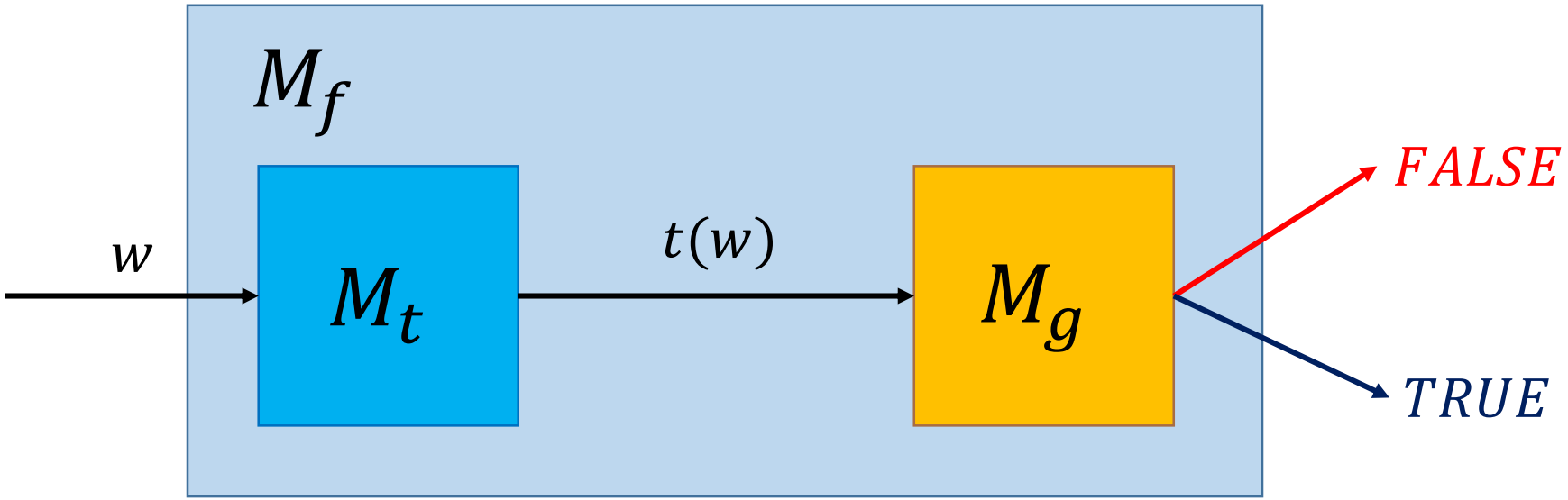
Mapping reductions (2)

$$f \leq_m g \overset{\text{def}}{=} \exists t: \Sigma^* \to \Sigma^*, s.t. \begin{cases} t \text{ is computable} \\ \forall w \in \Sigma^*, f(w) = g(t(w)) \end{cases}$$



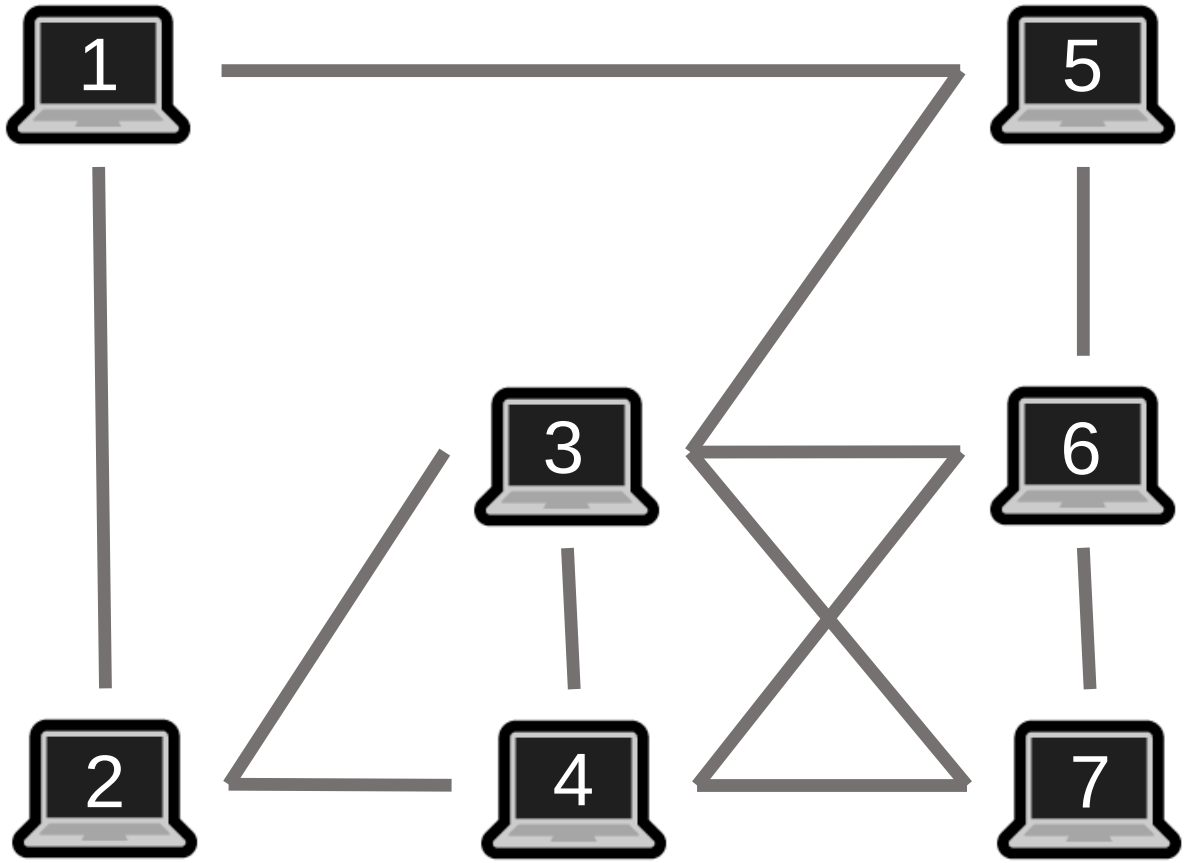
Polynomial-time mapping reductions

$$f \leq_P g \stackrel{\text{def}}{=} \exists t: \Sigma^* \to \Sigma^*, s.t. \begin{cases} t \text{ is computable in polynomial time} \\ \forall w \in \Sigma^*, f(w) = g(t(w)) \end{cases}$$



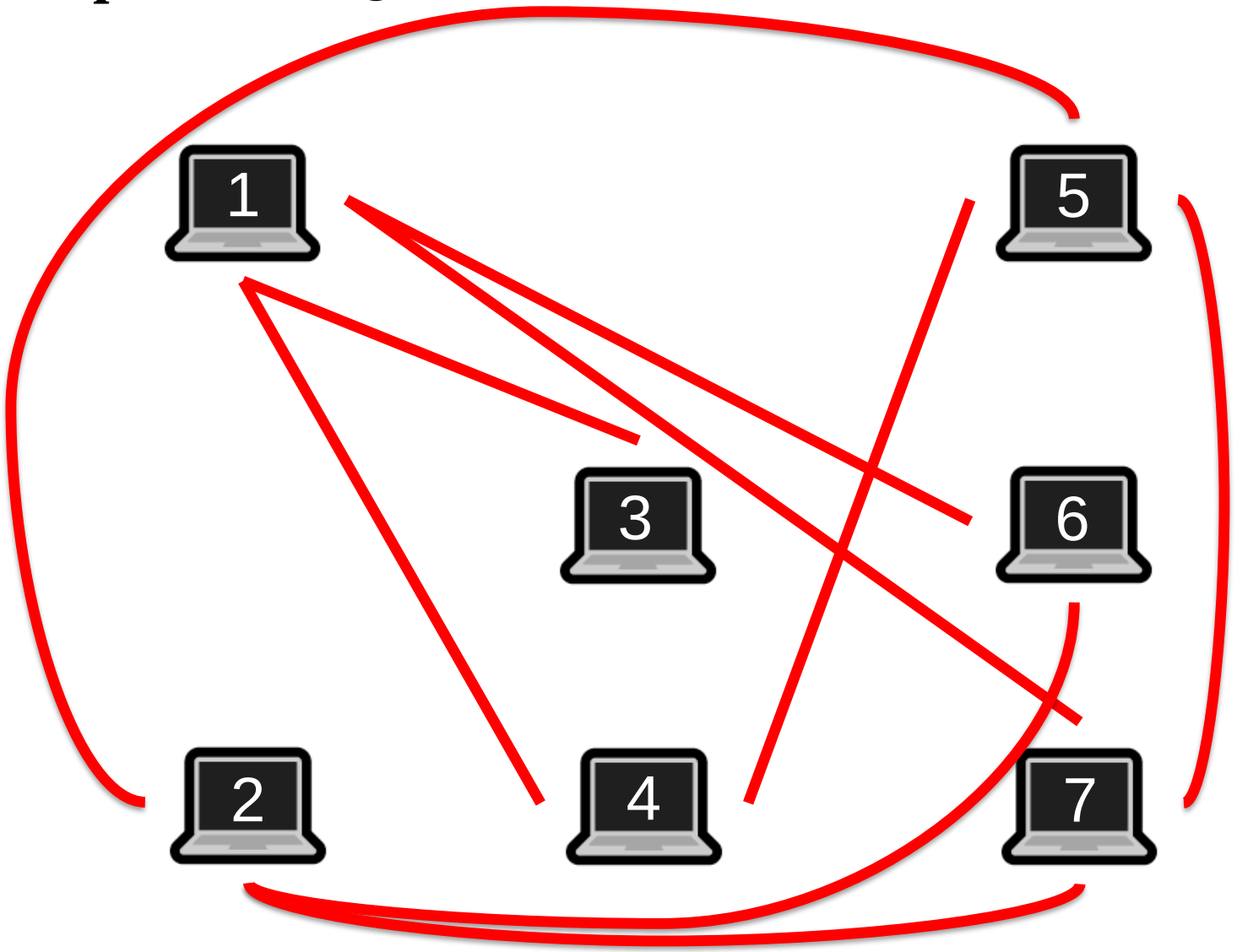
VERTEX COVER $\leq_P$ CLIQUE

Does this graph have a cover of size 4?

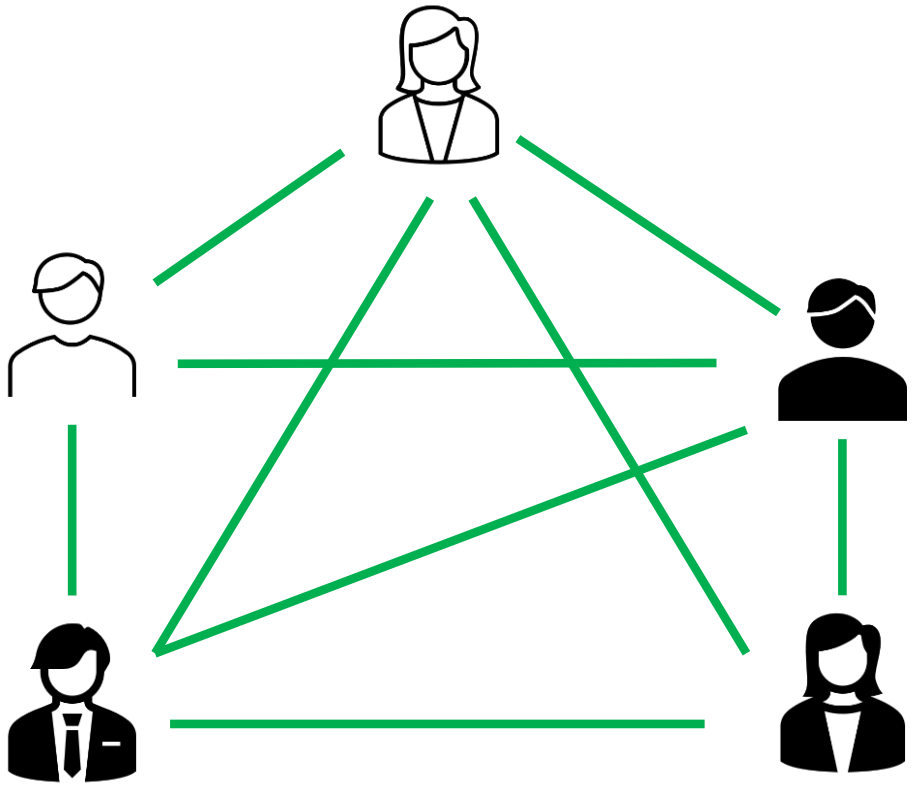


VERTEX COVER $\leq_P$ CLIQUE

Does this graph have a clique of size 7 – 4 (3)?



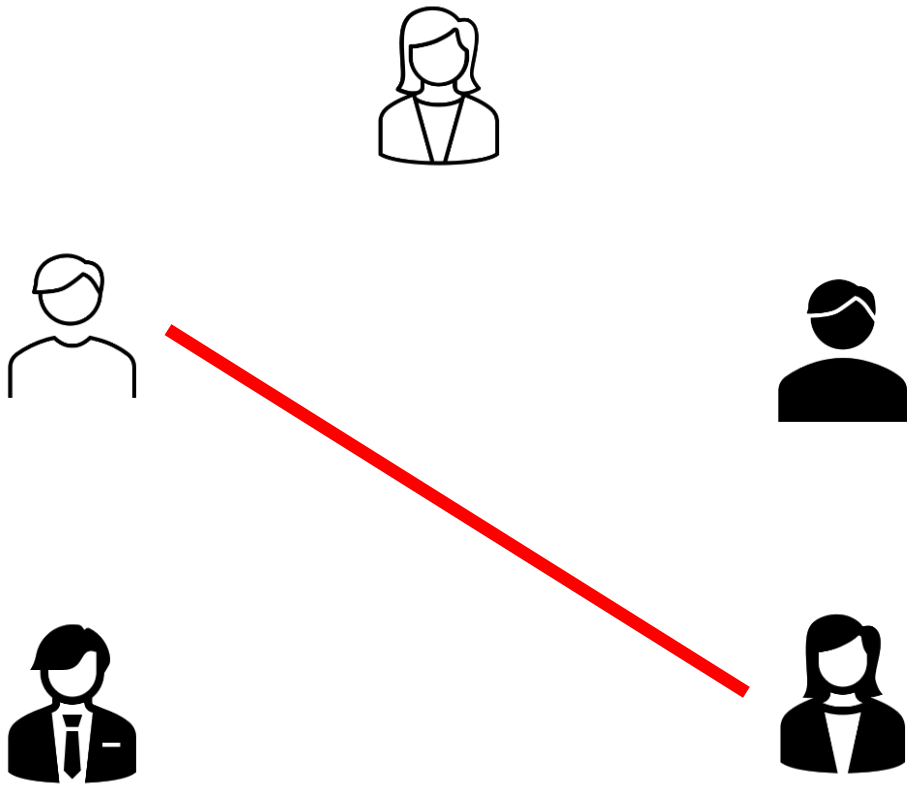
CLIQUE $\leq_P$ VERTEX COVER



Friendship

Does this graph have a clique of size 4?

CLIQUE $\leq_P$ VERTEX COVER



Does this graph have a cover of size 5 – 4 (1)?

$\mathrm{SAT} \leq_P \mathrm{CNF\ SAT}$

$$\phi = \overline{\overline{\overline{(x \vee y)} \wedge (\bar{x} \vee y)}}$$

SAT $\leq_P$ CNF SAT

$$\phi = \overline{\overline{\overline{(x \vee y)}}} \vee \overline{(\bar{x} \vee y)}$$

SAT $\leq_P$ CNF SAT

$$\phi = (x \vee y) \vee \overline{(\bar{x} \vee y)}$$

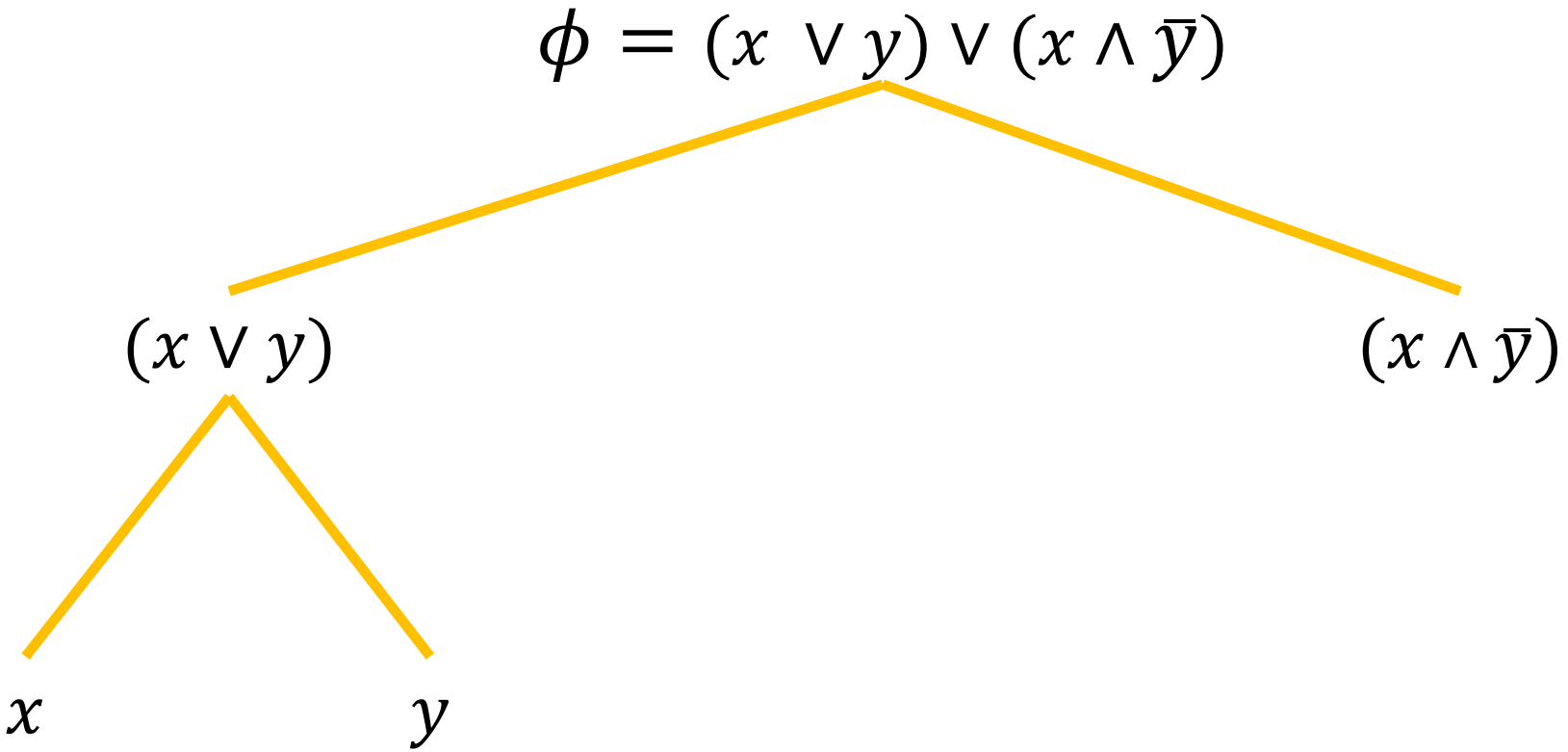
SAT $\leq_P$ CNF SAT

$$\phi = (x \vee y) \vee (\bar{\bar{x}} \wedge \bar{y})$$

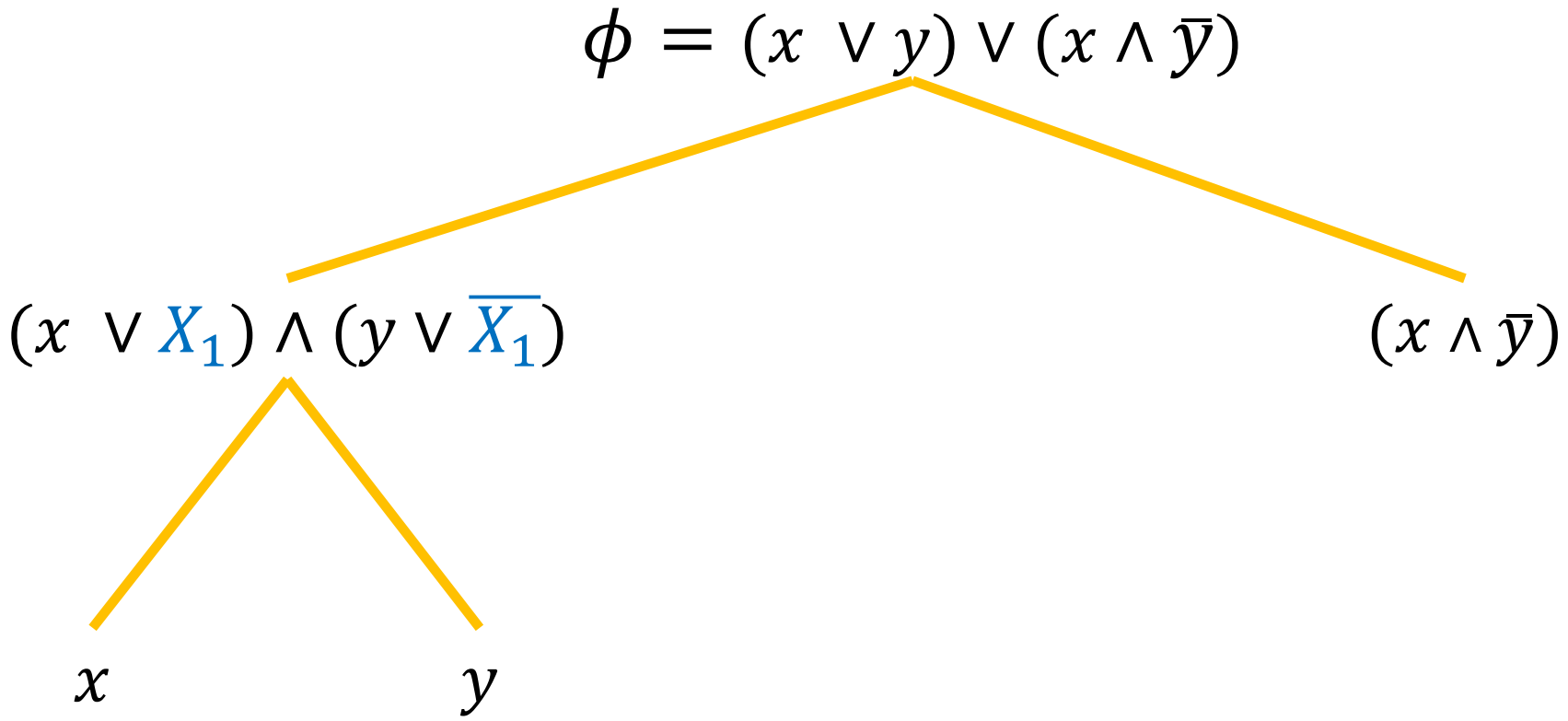
SAT $\leq_P$ CNF SAT

$$\phi = (x \vee y) \vee (x \wedge \bar{y})$$

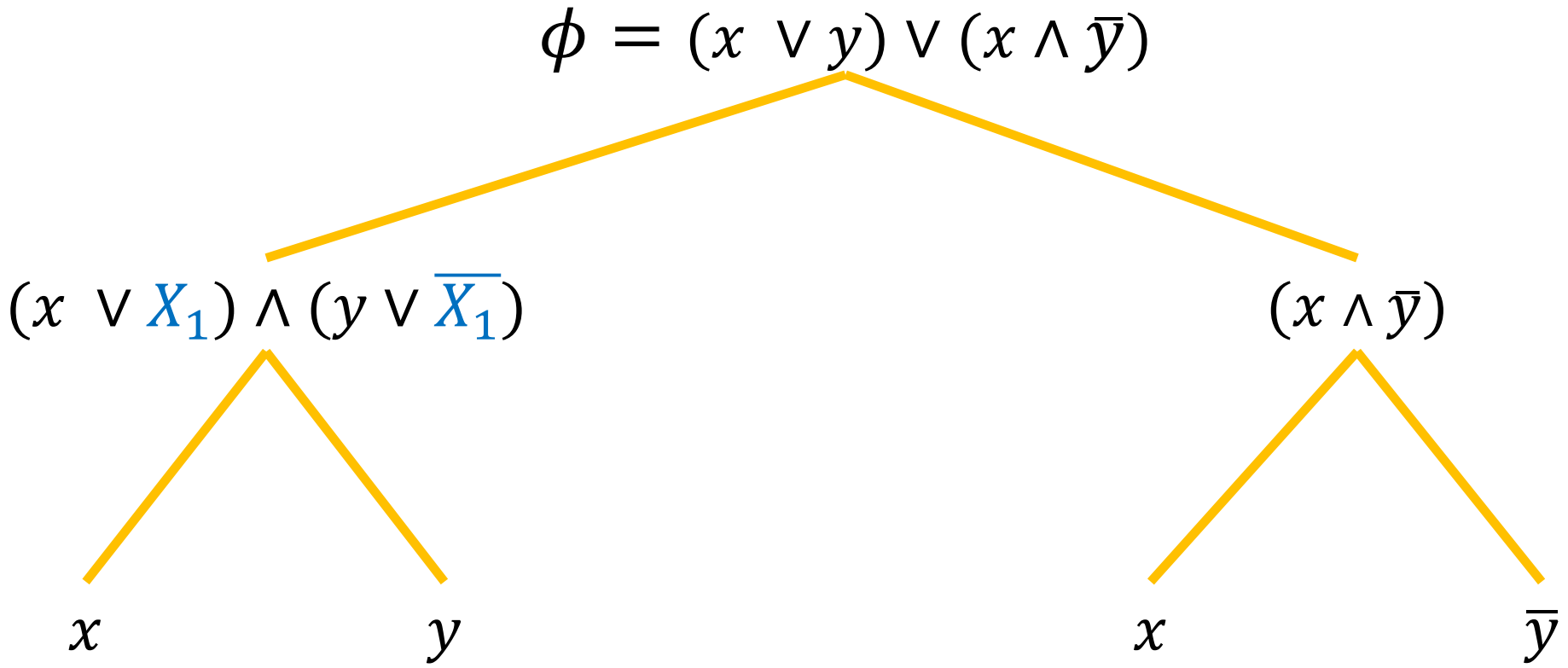
SAT $\leq_P$ CNF SAT



SAT $\leq_P$ CNF SAT

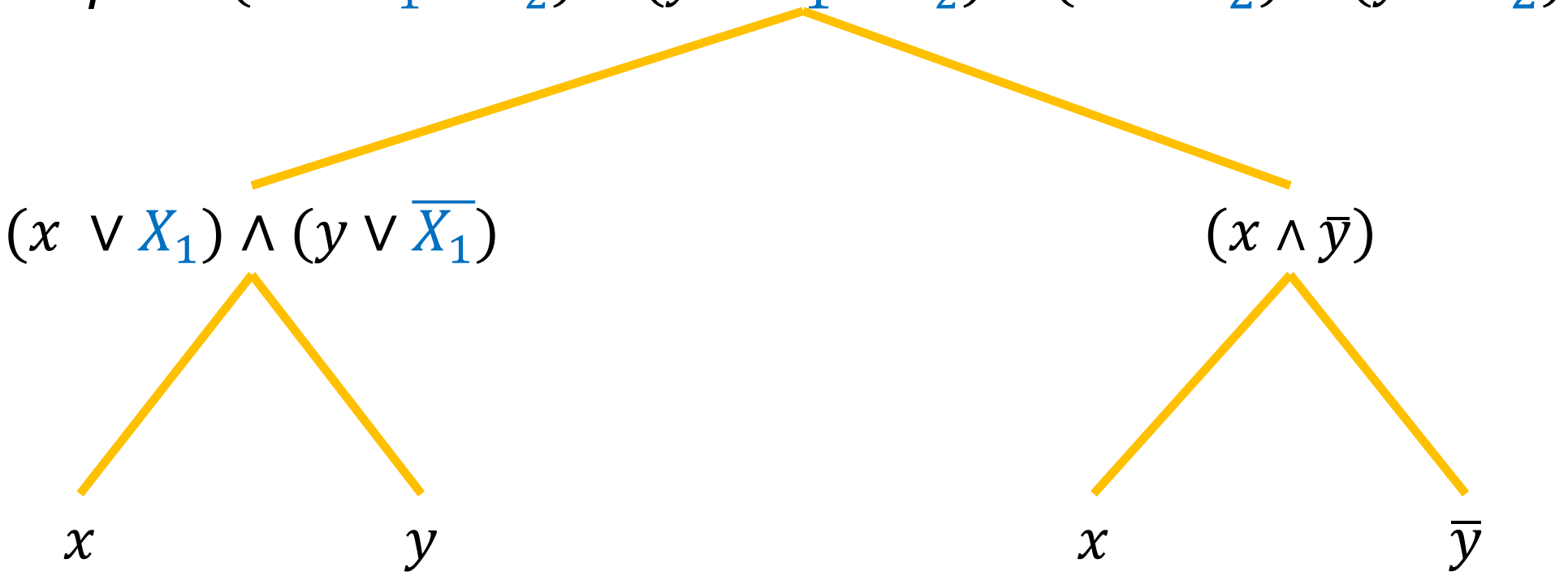


SAT $\leq_P$ CNF SAT



SAT $\leq_P$ CNF SAT

$$\phi = (x \vee X_1 \vee X_2) \wedge (y \vee \overline{X_1} \vee X_2) \wedge (x \vee \overline{X_2}) \wedge (\bar{y} \vee \overline{X_2})$$



CNF SAT $\leq_P$ 3SAT

$$\phi = (x \vee y \vee \bar{t}) \wedge (y \vee \bar{z}) \wedge x \wedge (x \vee y \vee \bar{z} \vee t \vee j \vee k \vee l \vee p)$$

CNF SAT $\leq_P$ 3SAT

$$\phi = (x \vee y \vee \bar{t}) \wedge (y \vee \bar{z}) \wedge x \wedge (x \vee y \vee \bar{z} \vee t \vee j \vee k \vee l \vee p)$$

$(x \vee y \vee \bar{t})$

CNF SAT $\leq_P$ 3SAT

$$\phi = (x \vee y \vee \bar{t}) \wedge (y \vee \bar{z}) \wedge x \wedge (x \vee y \vee \bar{z} \vee t \vee j \vee k \vee l \vee p)$$

$$(x \vee y \vee \bar{t})$$

$$(y \vee \bar{z} \vee X_1) \wedge (y \vee \bar{z} \vee \overline{X_1})$$

CNF SAT $\leq_P$ 3SAT

$$\phi = (x \vee y \vee \bar{t}) \wedge (y \vee \bar{z}) \wedge x \wedge (x \vee y \vee \bar{z} \vee t \vee j \vee k \vee l \vee p)$$

$(x \vee y \vee \bar{t})$

$(y \vee \bar{z} \vee X_1) \wedge (y \vee \bar{z} \vee \overline{X_1})$

$(x \vee X_2 \vee X_3) \wedge (x \vee X_2 \vee \overline{X_3}) \wedge (x \vee \overline{X_2} \vee X_3) \wedge (x \vee \overline{X_2} \vee \overline{X_3})$

CNF SAT $\leq_P$ 3SAT

$$\phi = (x \vee y \vee \bar{t}) \wedge (y \vee \bar{z}) \wedge x \wedge (x \vee y \vee \bar{z} \vee t \vee j \vee k \vee l \vee p)$$

$$(x \vee y \vee \bar{t})$$

$$(y \vee \bar{z} \vee X_1) \wedge (y \vee \bar{z} \vee \overline{X_1})$$

$$(x \vee X_2 \vee X_3) \wedge (x \vee X_2 \vee \overline{X_3}) \wedge (x \vee \overline{X_2} \vee X_3) \wedge (x \vee \overline{X_2} \vee \overline{X_3})$$

$$(x \vee y \vee X_4) \wedge (\overline{X_4} \vee \bar{z} \vee X_5) \wedge (\overline{X_5} \vee t \vee X_6) \wedge (\overline{X_6} \vee j \vee X_7) \wedge (\overline{X_7} \vee k \vee X_8) \wedge (\overline{X_8} \vee l \vee p)$$

CNF SAT $\leq_P$ 3SAT

$$\phi = (x \vee y \vee \bar{t}) \wedge (y \vee \bar{z}) \wedge x \wedge (x \vee y \vee \bar{z} \vee t \vee j \vee k \vee l \vee p)$$

$$(x \vee y \vee \bar{t}) \wedge$$

$$(y \vee \bar{z} \vee X_1) \wedge (y \vee \bar{z} \vee \overline{X_1}) \wedge$$

$$(x \vee X_2 \vee X_3) \wedge (x \vee X_2 \vee \overline{X_3}) \wedge (x \vee \overline{X_2} \vee X_3) \wedge (x \vee \overline{X_2} \vee \overline{X_3}) \wedge$$

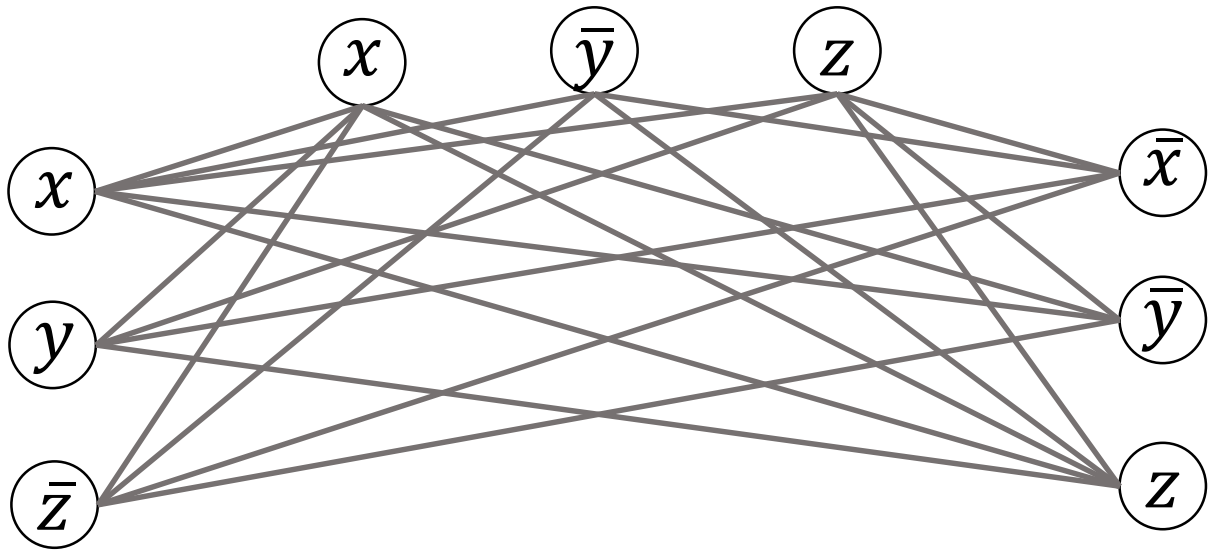
$$(x \vee y \vee X_4) \wedge (\overline{X_4} \vee \bar{z} \vee X_5) \wedge (\overline{X_5} \vee t \vee X_6) \wedge (\overline{X_6} \vee j \vee X_7) \wedge (\overline{X_7} \vee k \vee X_8) \wedge (\overline{X_8} \vee l \vee p)$$

3SAT $\leq_P$ CLIQUE

$$\phi = (x \vee y \vee \bar{z}) \wedge (x \vee \bar{y} \vee z) \wedge (\bar{x} \vee \bar{y} \vee z)$$

3SAT $\leq_P$ CLIQUE

$$\phi = \overbrace{(x \vee y \vee \bar{z})}^{1} \wedge \overbrace{(x \vee \bar{y} \vee z)}^{2} \wedge \overbrace{(\bar{x} \vee \bar{y} \vee z)}^{3}$$



Does this graph has a clique of size 3?