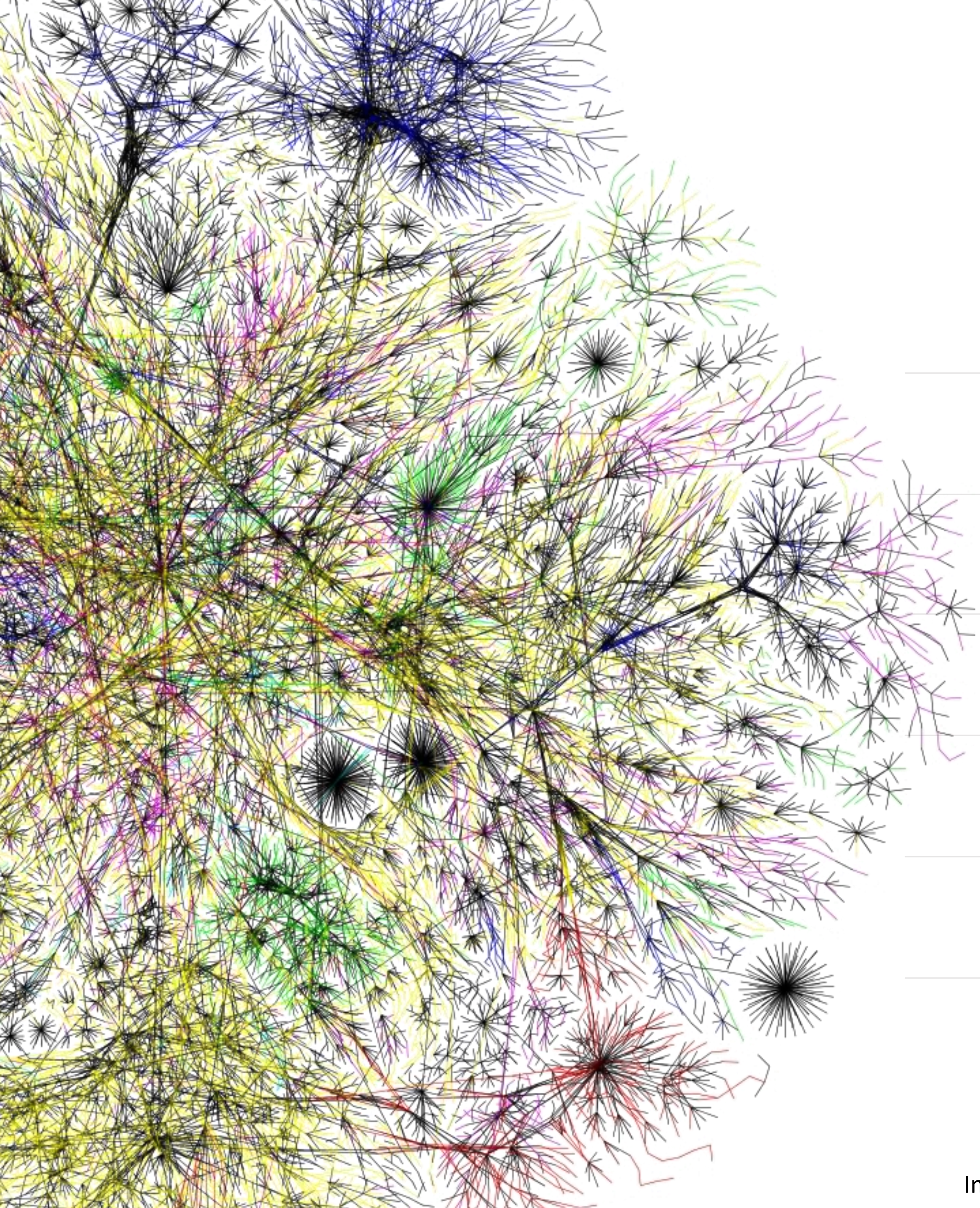


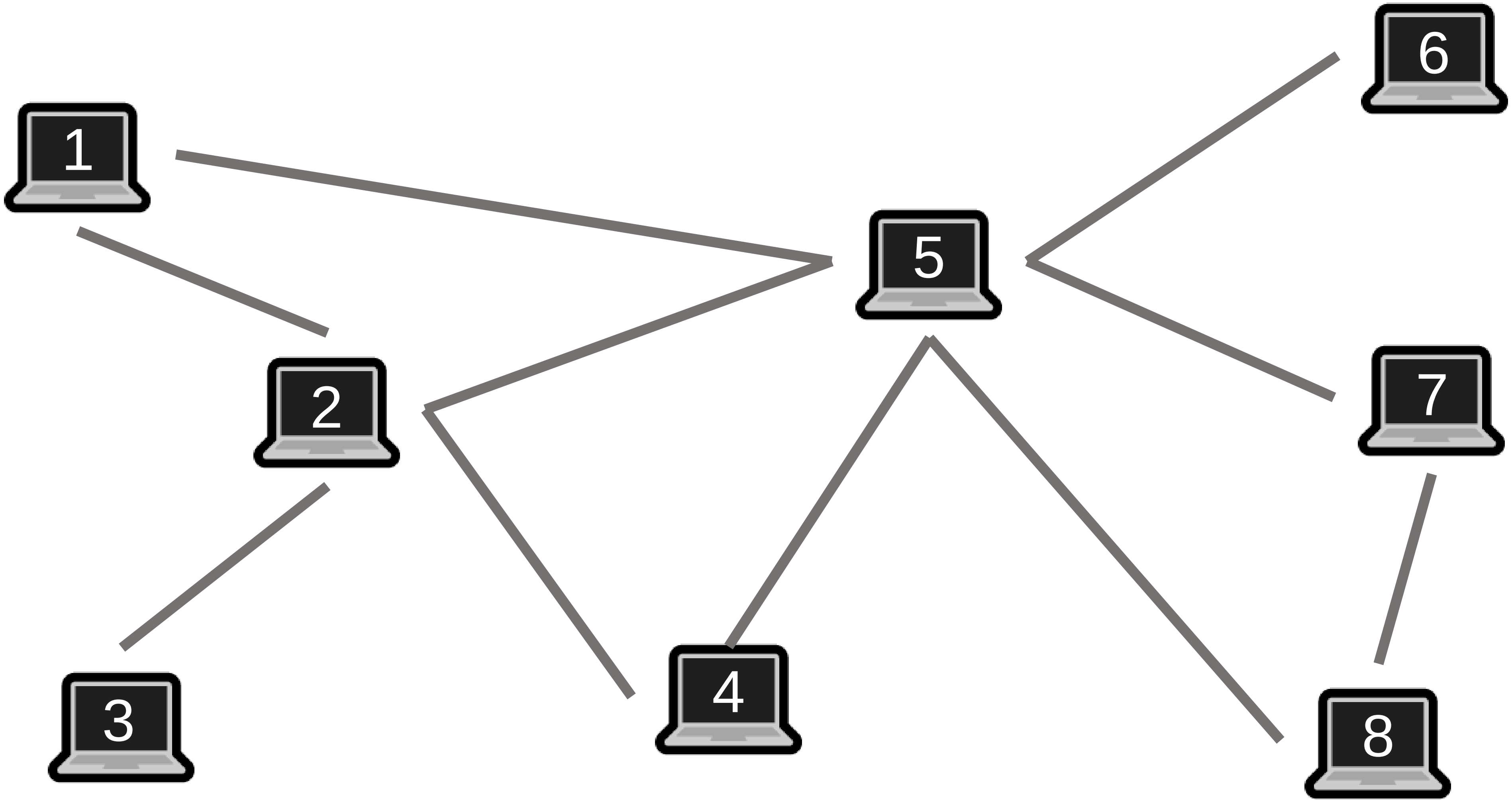
9

“Hard?” problems

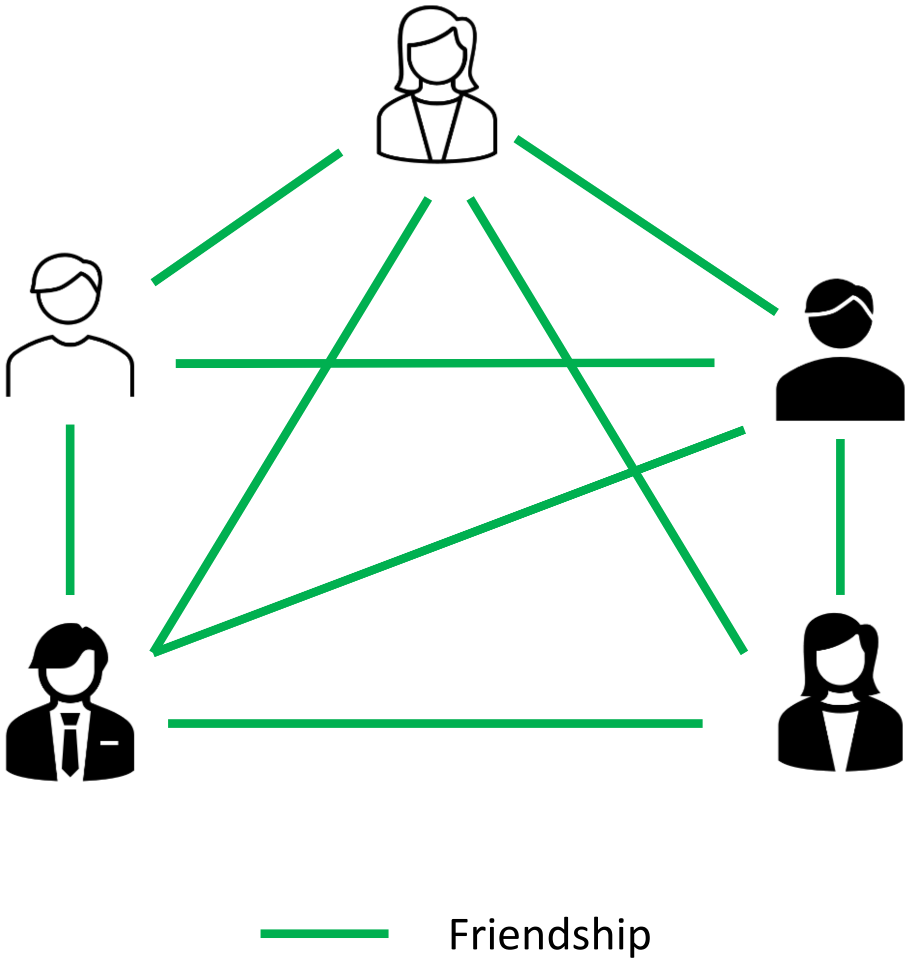
Analiza Algoritmilor

Image source: https://commons.wikimedia.org/wiki/File:Internet_map_1024_-_transparent,_inverted.png

VERTEX COVER

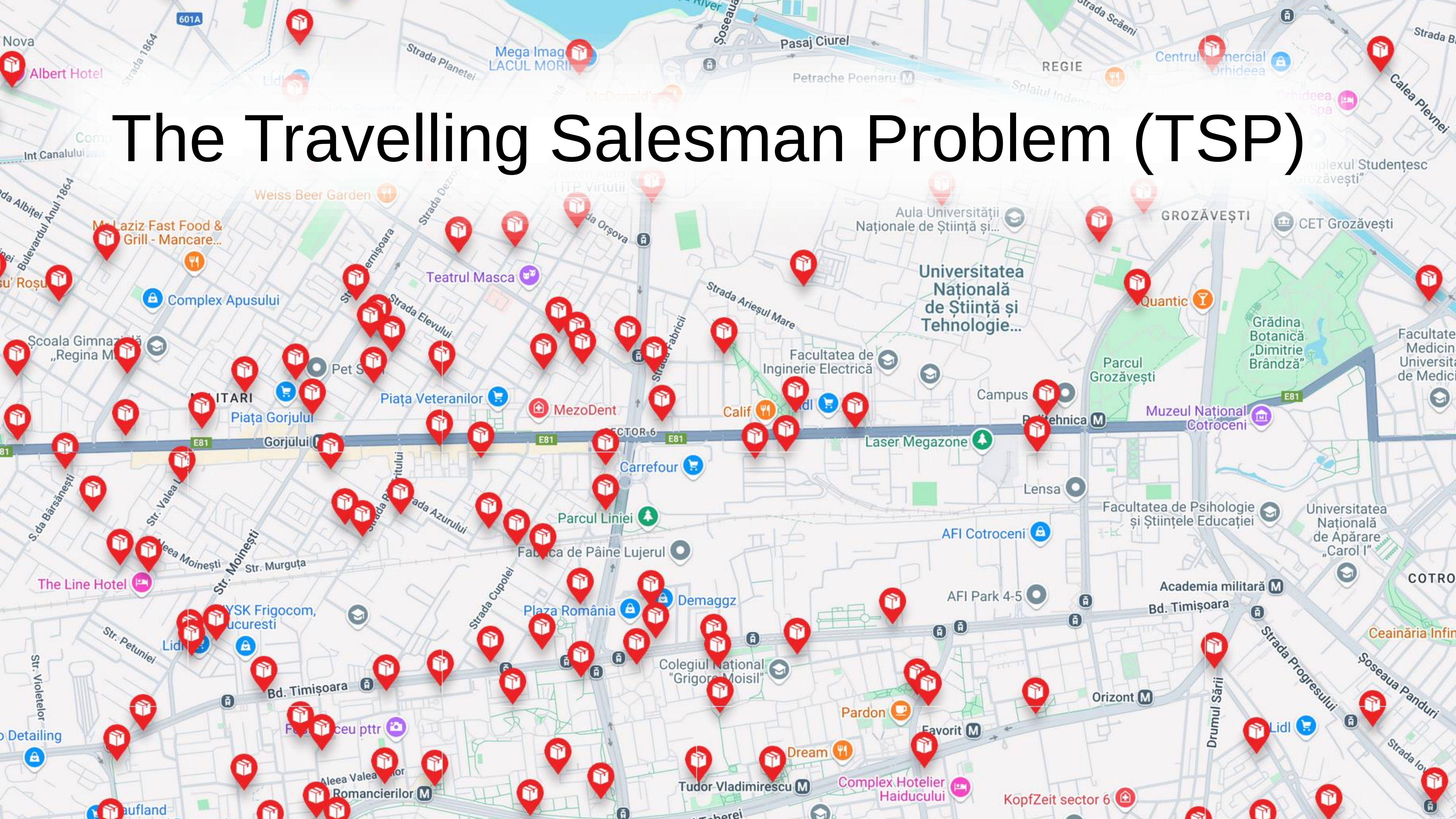


CLIQUE

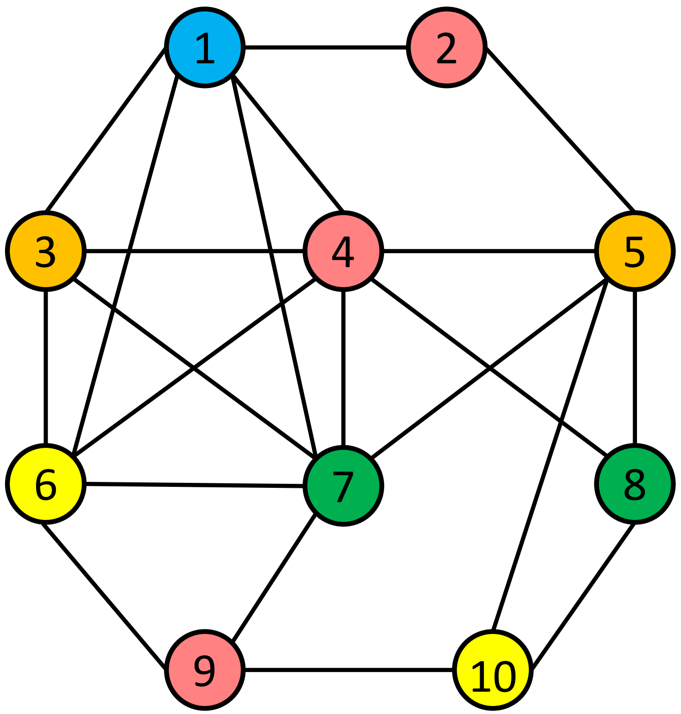


Is there a 4-person friends group (clique)?

The Travelling Salesman Problem (TSP)



COLORING



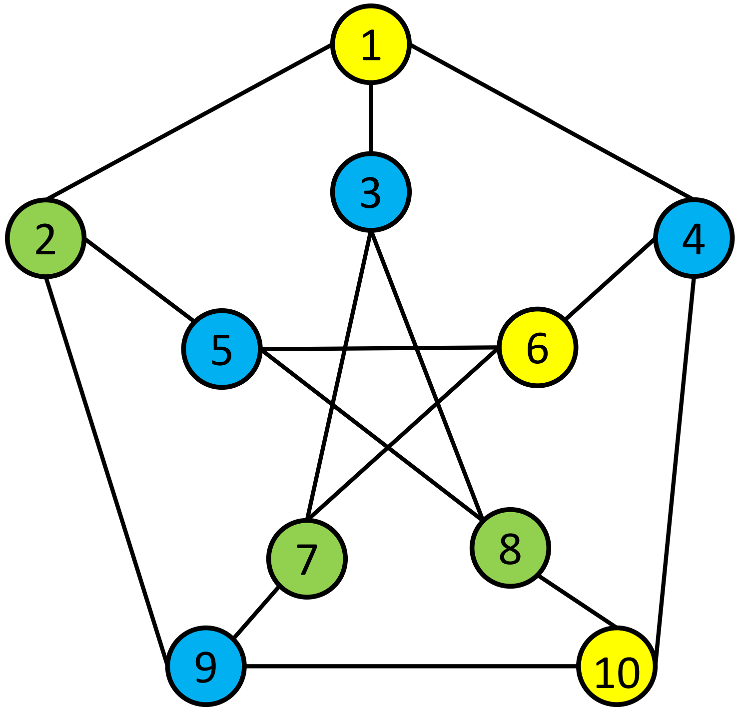
Can we use 5 colors to color the vertices such that no two adjacent vertices are the same color?

YES

Can we use 4 colors to color the vertices such that no two adjacent vertices are the same color?

NO

COLORING



Can we use 3 colors to color the vertices such that no two adjacent vertices are the same color?

YES

COLORING – Register allocation

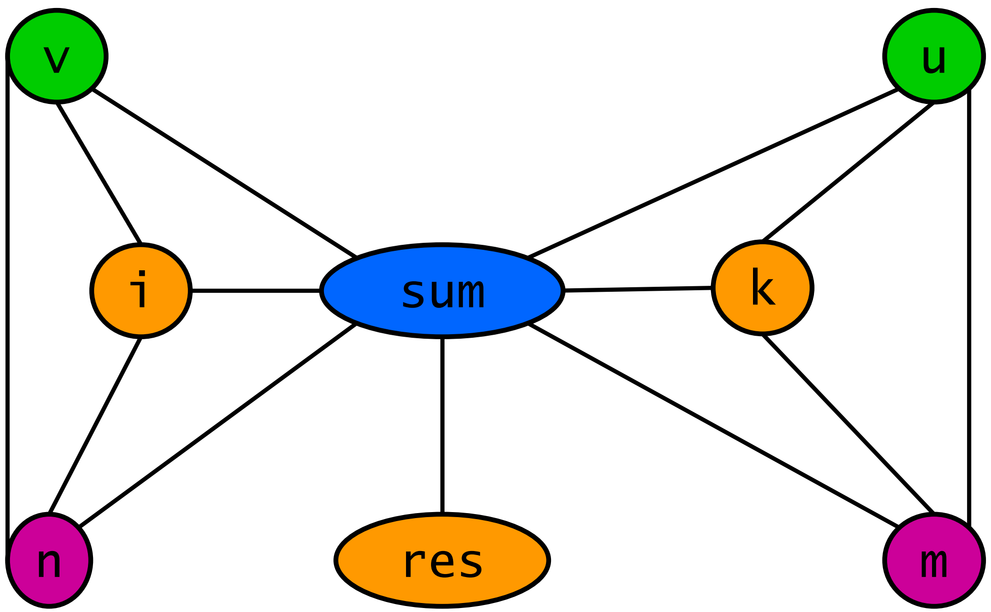
```
int sum = 0;
for (i = 0; i < n; i++)
    sum += v[i];

for (k = 0; k < m; k++)
    sum -= u[k];

int res = sum / m;

print(sum)

print(res)
```

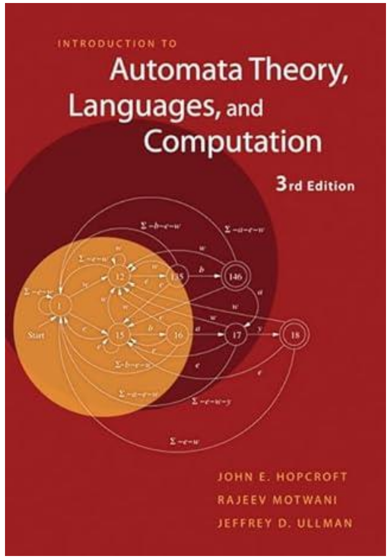


4 registers needed: RAX, RBX, RCX, RDX

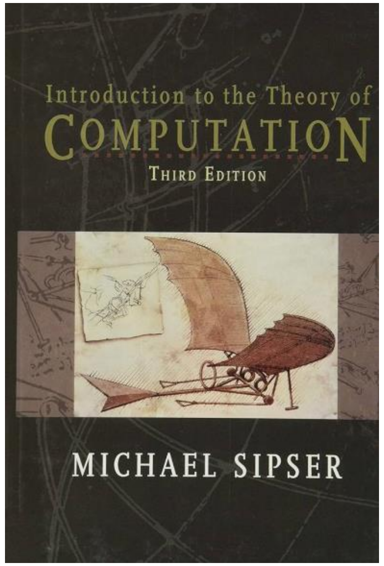
KNAPSACK

Capacity: 2000

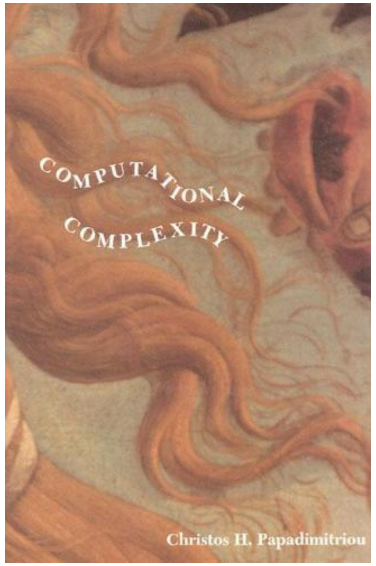
Can we place items in the knapsack such that the total value is greater than 260?



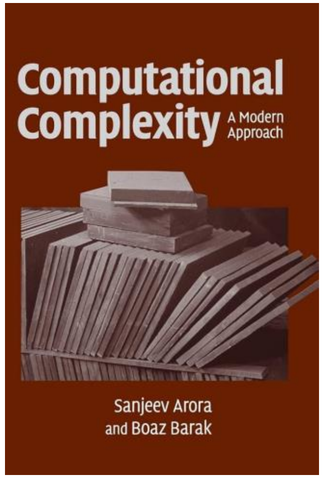
Weight: 560
Value: 93



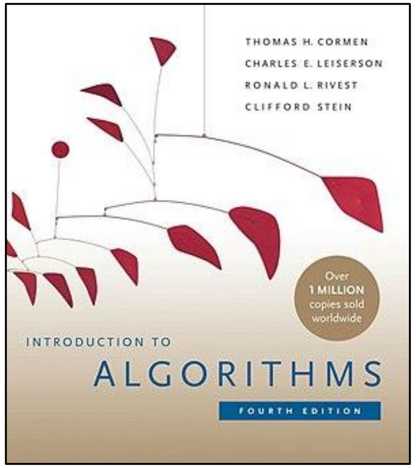
Weight: 504
Value: 90



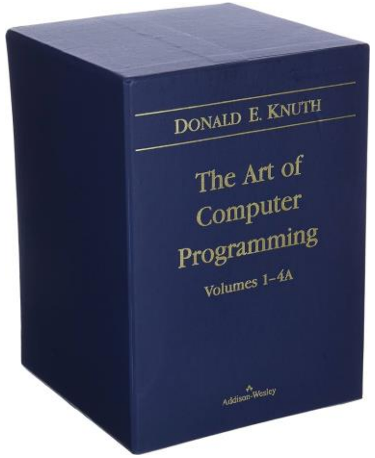
Weight: 523
Value: 104



Weight: 579
Value: 56



Weight: 1312
Value: 140



Weight: 3168
Value: 361

Knapsack image: https://openclipart.org/detail/332879/knapsack

SUBSET SUM

$$S = \{\, 4, 11, 16, 21, 11, 14, 4, 23\} \qquad K = 40$$

Is there $S_1 \subseteq S$, such that:

$$\sum_{e \in S_1} e = K$$

SUBSET SUM – Solution

```
SOLVE_SUBSET_SUM(S, K):
    r1 ← GENERATE_AND_CHECK(∅, 1, FALSE)
    r2 ← GENERATE_AND_CHECK(∅, 1, TRUE)
    return r1 ∨ r2
```

```
GENERATE_AND_CHECK(S1, i, choose):
    if choose
        S1 ← S1 ∪ {S[i]}

    if VALIDATE_SUBSET(S1)
        return TRUE

    r1 ← GENERATE_AND_CHECK(S1, i + 1, FALSE)
    r2 ← GENERATE_AND_CHECK(S1, i + 1, TRUE)
    return r1 ∨ r2
```

```
VALIDATE_SUBSET(S1):
    sum ← 0
    for each e ∈ S1
        sum ← sum + e

    return sum = K
```

PARTITIONING

$$S = \{ 4, 11, 16, 21, 11, 14, 4, 23\}$$

Is there $S_1 \subseteq S$, such that:

$$\sum_{x \in S_1} x = \sum_{y \in S \setminus S_1} y$$

PARTITIONING – Solution

```
SOLVE_SUBSET_SUM(S, K):
    r1 ← GENERATE_AND_CHECK(∅, 1, FALSE)
    r2 ← GENERATE_AND_CHECK(∅, 1, TRUE)
    return r1 ∨ r2
```

```
GENERATE_AND_CHECK(S1, i, choose):
    if choose
        S1 ← S1 ∪ {S[i]}

    if VALIDATE_SUBSET(S1)
        return TRUE

    r1 ← GENERATE_AND_CHECK(S1, i + 1, FALSE)
    r2 ← GENERATE_AND_CHECK(S1, i + 1, TRUE)
    return r1 ∨ r2
```

```
VALIDATE_SUBSET(S1):
    sum1 ← 0
    sum2 ← 0
    for each x ∈ S
        if x ∈ S1
            sum1 ← sum1 + x
        else
            sum2 ← sum2 + x

    return sum1 = sum2
```

SAT

$$\phi = \left(\left(x \vee \bar{y} \vee (\bar{z} \wedge y)\right) \wedge \bar{t}\right) \vee (t \wedge \bar{x})$$

$$\left(\left(TRUE \vee \overline{TRUE} \vee (\overline{TRUE} \wedge TRUE)\right) \wedge \overline{TRUE}\right) \vee (TRUE \wedge \overline{TRUE})$$

$T(x) = TRUE$

$T(y) = TRUE$

$T(z) = TRUE$

$T(t) = TRUE$

SAT

$$\phi = \left(\left(x \lor \bar{y} \lor (\bar{z} \land y)\right) \land \bar{t}\right) \lor (t \land \bar{x})$$

$FALSE$

$T(x) = TRUE$

$T(y) = TRUE$

$T(z) = TRUE$

$T(t) = TRUE$

SAT

$$\phi = \left(\left(x \lor \bar{y} \lor (\bar{z} \land y)\right) \land \bar{t}\right) \lor (t \land \bar{x})$$

$$\left(\left(TRUE \lor \overline{TRUE} \lor (\overline{TRUE} \land TRUE)\right) \land \overline{FALSE}\right) \lor (FALSE \land \overline{TRUE})$$

$T(x) = TRUE$

$T(y) = TRUE$

$T(z) = TRUE$

$T(t) = FALSE$

SAT

$$\phi = \Big(\big(x \vee \bar{y} \vee (\bar{z} \wedge y)\big) \wedge \bar{t}\Big) \vee (t \wedge \bar{x})$$

$TRUE$

$T(x) = TRUE$

$T(y) = TRUE$

$T(z) = TRUE$

$T(t) = FALSE$

CNF SAT

$$\phi = (x \vee y \vee \bar{t}) \wedge (y \vee \bar{z}) \wedge (\bar{y} \vee z \vee t) \wedge (\bar{y} \vee \bar{t})$$

3SAT

$$\phi = (\underbrace{x \vee y \vee \bar{t}}_{3\ variables}) \wedge (\underbrace{y \vee \bar{z} \vee t}_{3\ variables}) \wedge (\underbrace{\bar{y} \vee z \vee t}_{3\ variables})$$