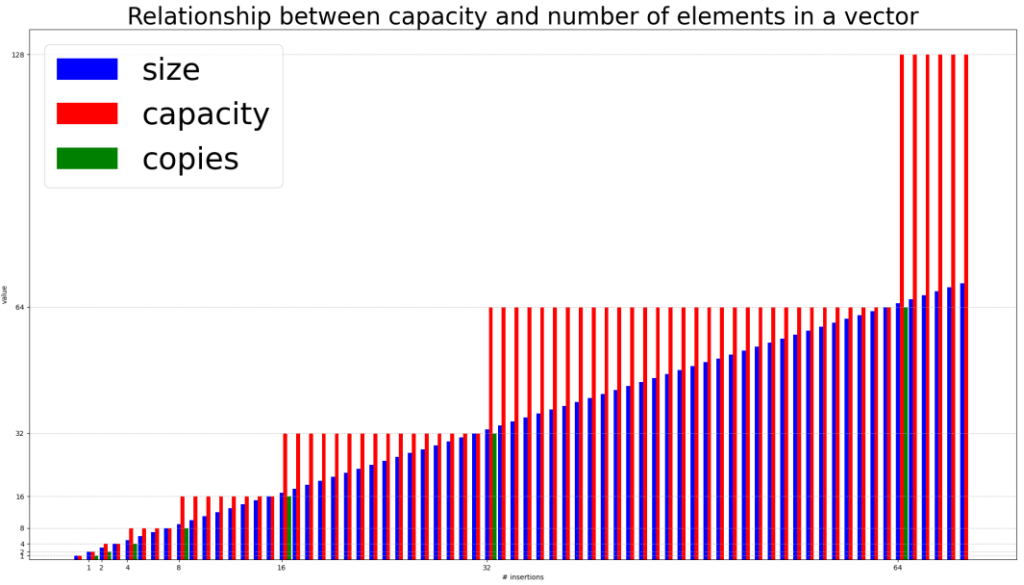


Amortized Analysis

Analiza Algoritmilor

Use-case

We have a sequence of operations (usually on a data structure):

$$op_1, op_2, op_3, op_4, \dots, op_n$$

- the sequence can contain *different* operations
- the *complexity* of the operations may differ
- “worst-case analysis” might be too pessimistic

Incrementing a k-bit binary counter

0	0	1	1	1	1	1	1	1	1	1	1	1	1	1

Worst-case analysis:

- Resource analyzed: *number of flips*
- each increment takes $\mathrm{O}(k)$ flips
- n increment operations will take $O(nk)$ flips

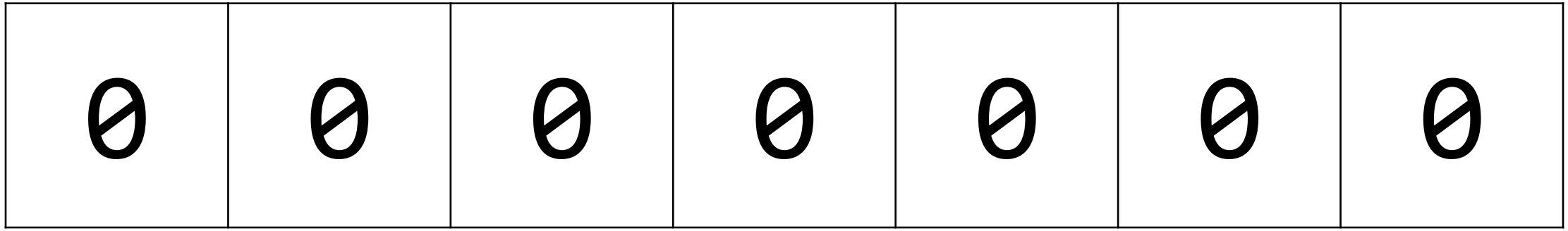
k-bit binary counter revised analysis

- Most of the time, very few bits are affected
- **“Expensive operations”** are *rare*
- *Rare* **expensive operations** and *frequent* **cheap operations** even out

The Aggregate Method

- Compute the worst-case complexity of the entire sequence
- If the sequence cost is $T(n)$ and there are n operations:
  - each operation has an *amortized cost* of $\frac{T(n)}{n}$

The Accounting Method



Coin: https://en.numista.com/catalogue/pieces38278.html

The Accounting Method

0	0	0	0	0	0	0

Coin: https://en.numista.com/catalogue/pieces38278.html

The Accounting Method

0	0	0	0	0	0	1

Coin: https://en.numista.com/catalogue/pieces38278.html

The Accounting Method

0	0	0	0	0	0	1

Coin: https://en.numista.com/catalogue/pieces38278.html

The Accounting Method

0	0	0	0	0	1	0

Coin: https://en.numista.com/catalogue/pieces38278.html

The Accounting Method

0	0	0	0	0	1	0

Coin: https://en.numista.com/catalogue/pieces38278.html

The Accounting Method

0	0	0	0	0	1	1

Coin: https://en.numista.com/catalogue/pieces38278.html

The Accounting Method

0	0	0	0	0	1	1

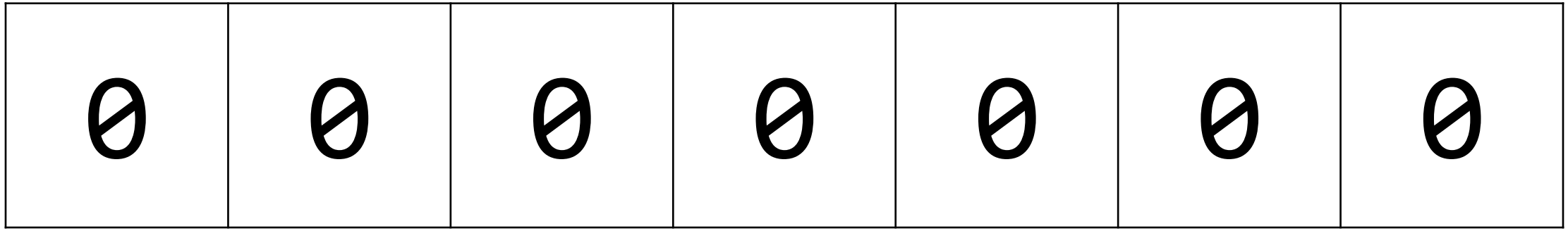
Coin: https://en.numista.com/catalogue/pieces38278.html

The Accounting Method

0	0	0	0	1	0	0

Coin: https://en.numista.com/catalogue/pieces38278.html

The Potential Method



The potential of the counter is the number of “1” bits

Potential: 0

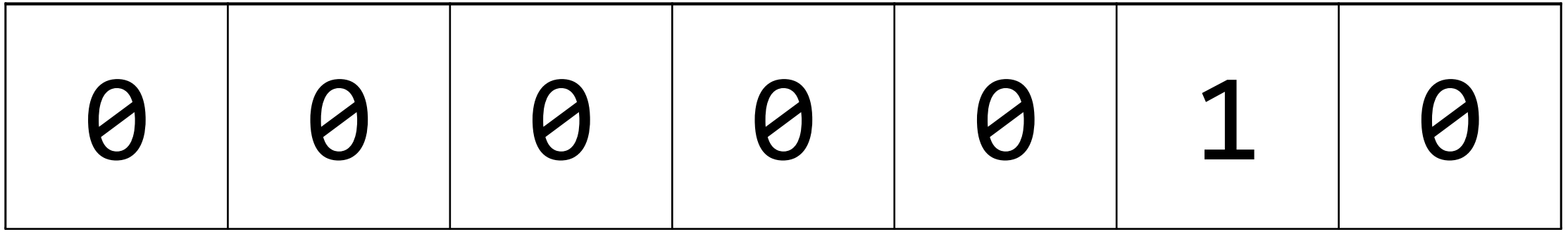
The Potential Method

0	0	0	0	0	0	1

The potential of the counter is the number of “1” bits

Potential: 1

The Potential Method



The potential of the counter is the number of “1” bits

Potential: 1

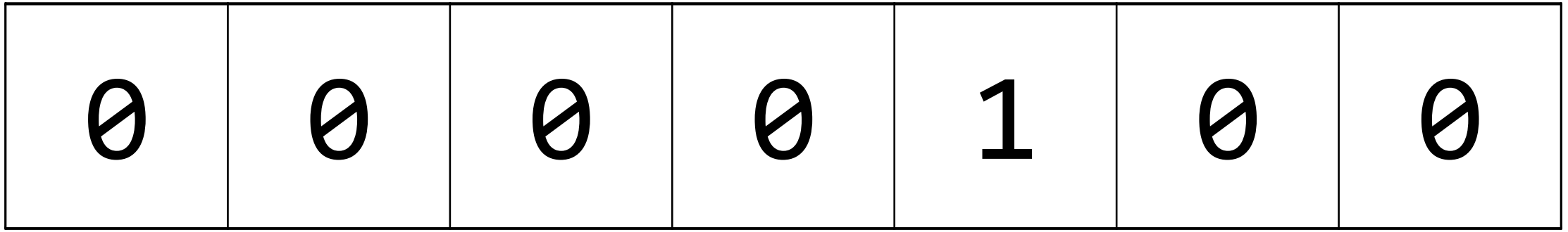
The Potential Method

0	0	0	0	0	1	1

The potential of the counter is the number of “1” bits

Potential: 2

The Potential Method



The potential of the counter is the number of “1” bits

Potential: 1

Summary

- *Frequent* **cheap operations** can amortize the cost of *rare* ***expensive operations***
- Three methods of performing amortized analysis:
  1. **The Aggregate Method:**
     - Determine the cost of the entire sequence and divide by number of operations
  2. The Accounting Method:
     - Associate “credits” to each operations; credits pay for elementary operations
  3. **The Potential Method:**
     - Define a potential function on each state of the structure; the amortized cost is the real one plus the difference in potential