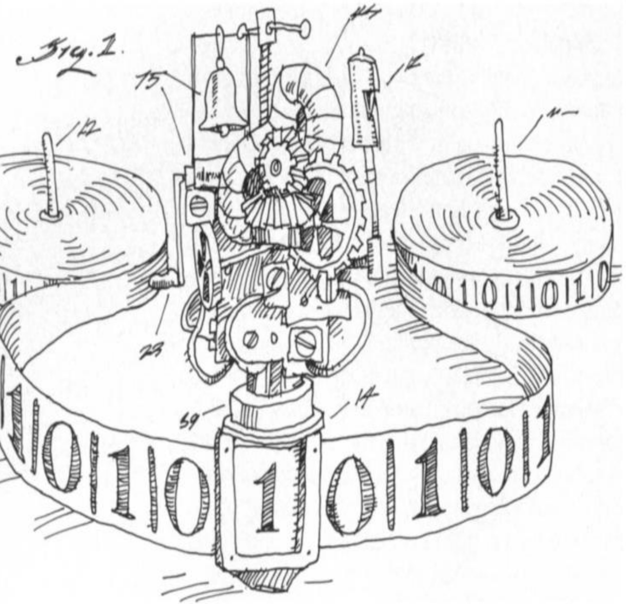
Recap

1. Problems are functions:
    a. From strings to strings
    b. From strings to Booleans (decision problems)
2. Turing Machines:
    a. Increment a number
    b. Check if a number is even
    c. Check if a string starts and ends with the same symbol
    d. Check if a string is palindrome
    e. Check if a string contains the same number of 0s and 1s

3

Computing



Analiza Algoritmilor

Contents

1. Possible results of running a Turing Machine
2. Configurations
3. Solving a problem:
    - Computing
    - Accepting
    - Deciding
4. The Church-Turing thesis

Doing nothing

	0	**1**	$\square$
q_1	$q_1, 0, -$	$q_1, 1, -$	$q_1, \square, -$

Writing 1 forever

	0	1	□
q_1	$q_1, 1, \rightarrow$	$q_1, 1, \rightarrow$	$q_1, 1, \rightarrow$

Flipping forever

	0	1	□
q_1	$q_1, 1, \rightarrow$	$q_1, 0, \rightarrow$	$q_2, \square, \leftarrow$
q_2	$q_2, 1, \leftarrow$	$q_2, 0, \leftarrow$	$q_1, \square, \rightarrow$

Halting on some inputs

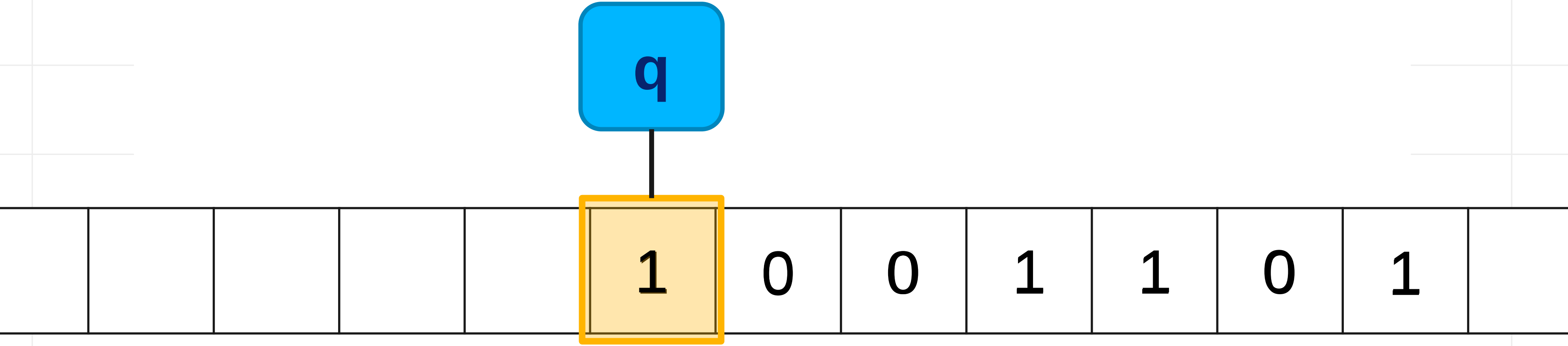
	0	1	$\square$
q_1	$H, 0, -$	$q_2, 1, \rightarrow$	$q_2, 1, \rightarrow$
q_2	$q_2, 1, \rightarrow$	$q_2, 1, \rightarrow$	$q_2, 1, \rightarrow$

When a machine runs...

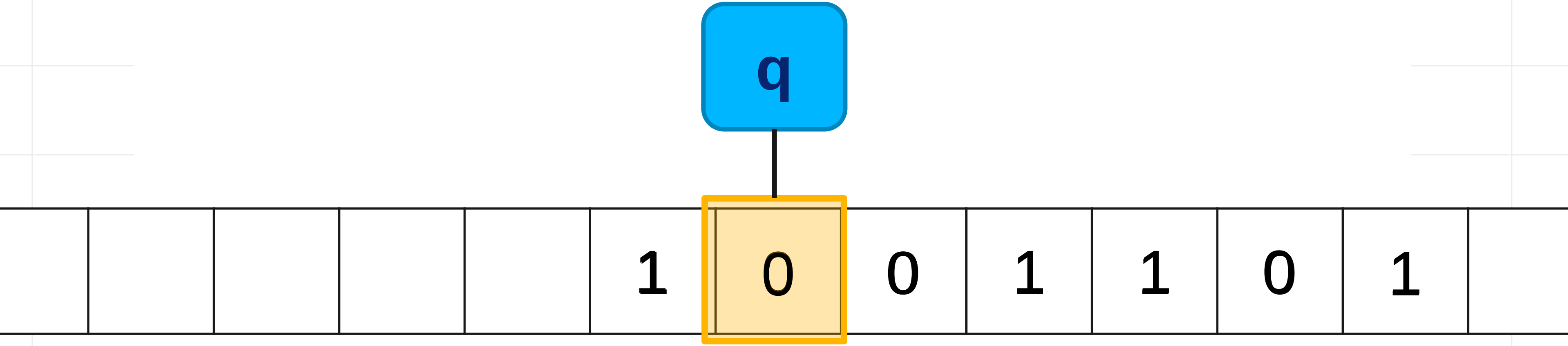
It can halt in one of the final states: Y, N, H

It can go on forever, without halting

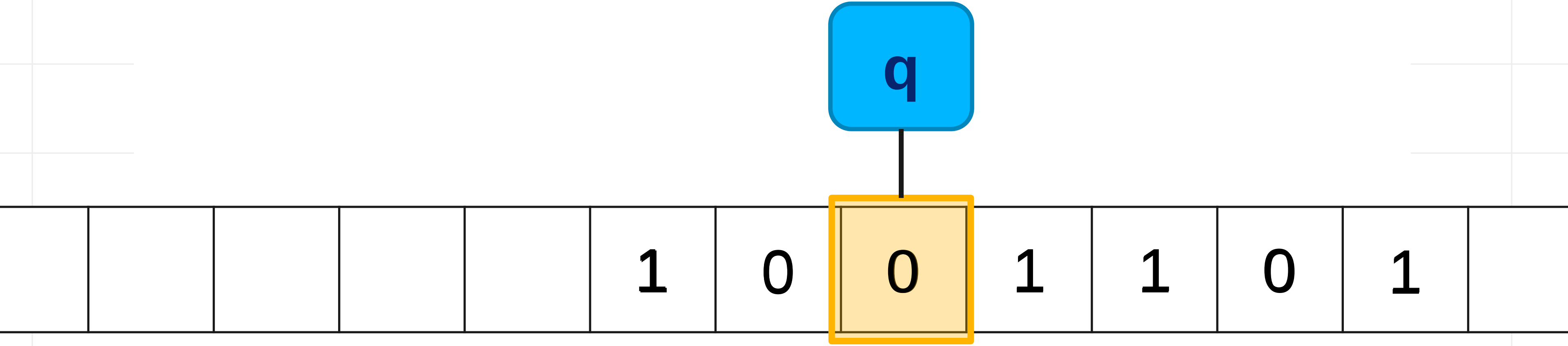
Configurations



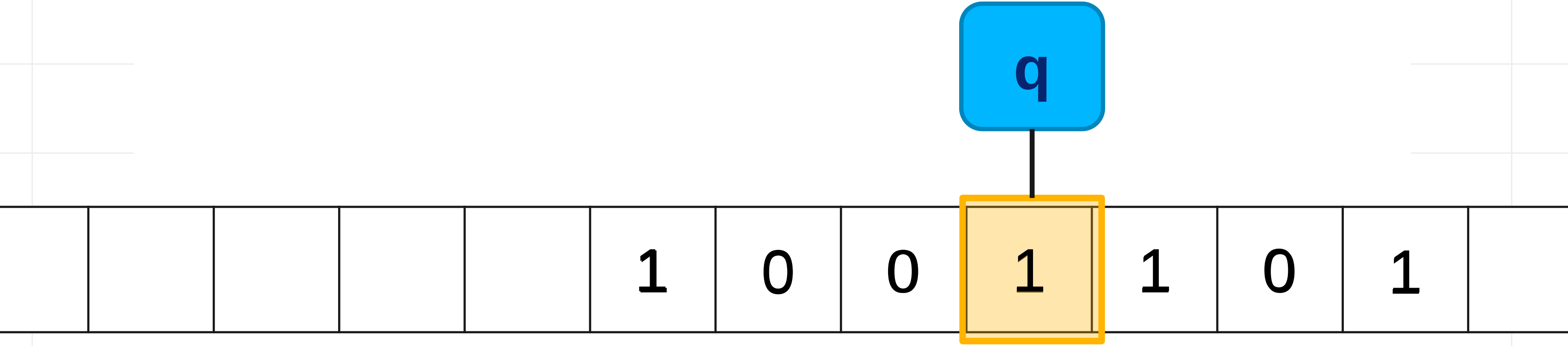
Configurations



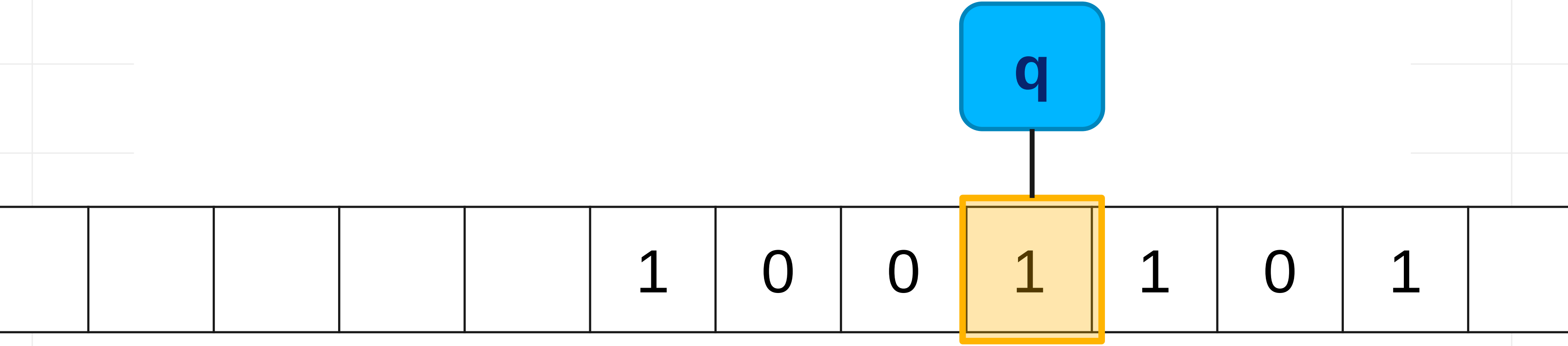
Configurations



Configurations

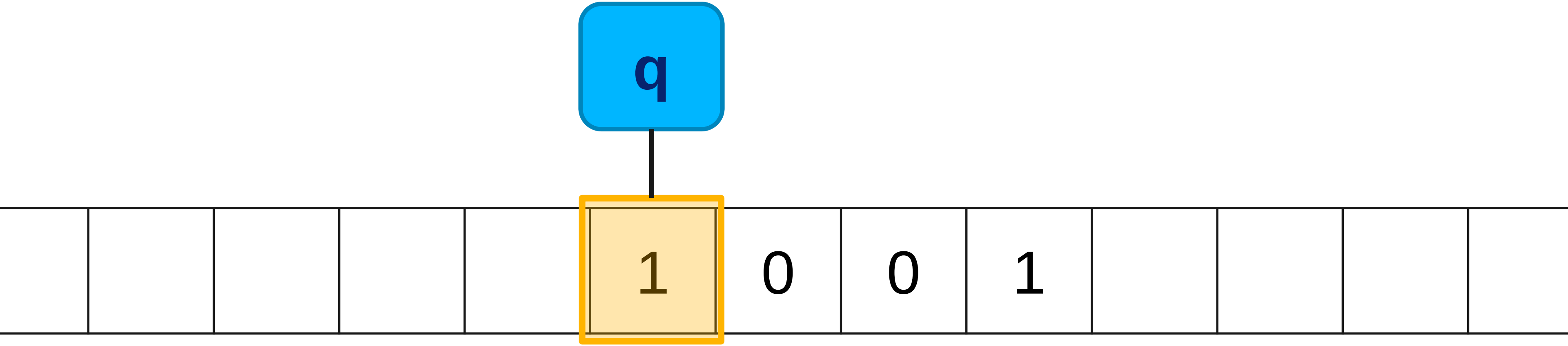


Configurations

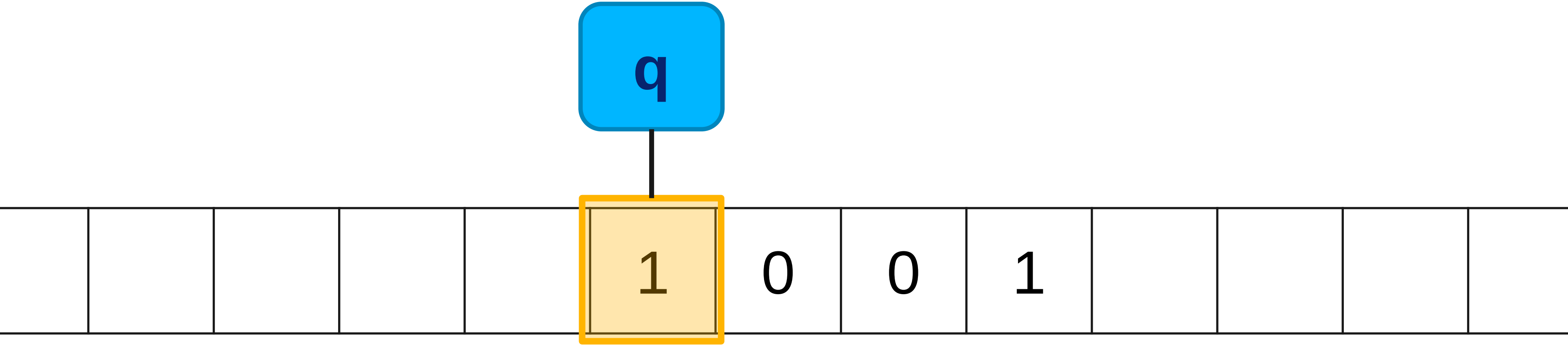


(100, 1101, **q**)

From one configuration to another

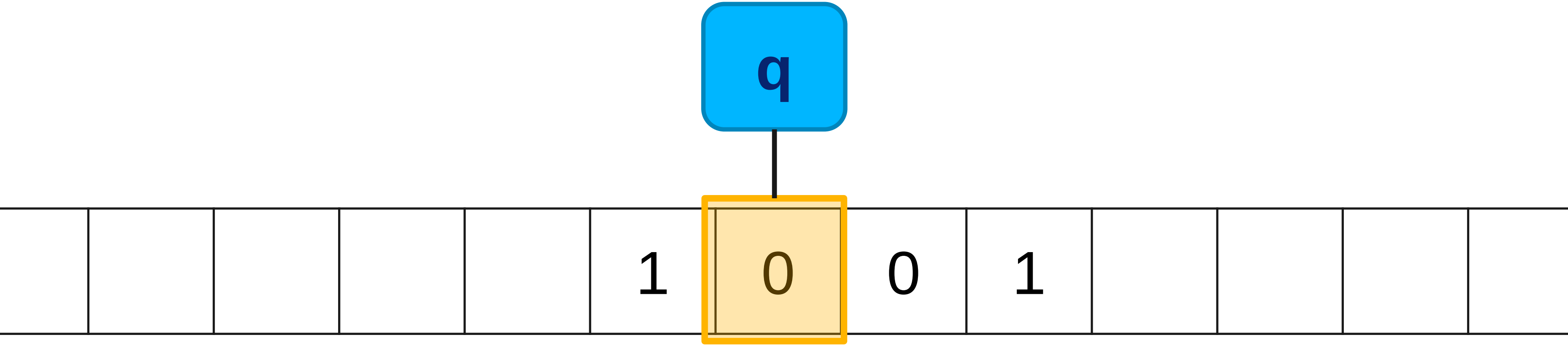


From one configuration to another



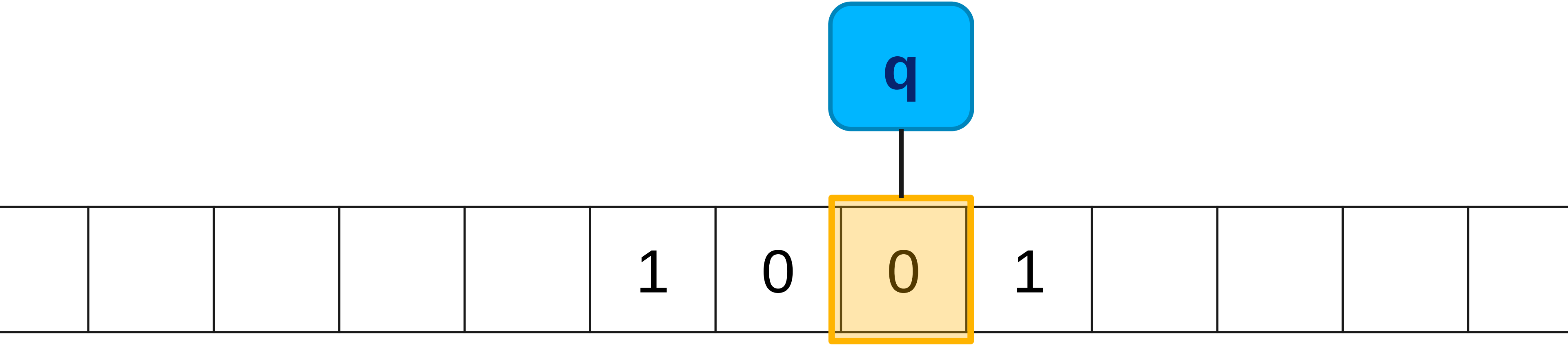
$(\varepsilon, 1001, \mathbf{q})$

From one configuration to another



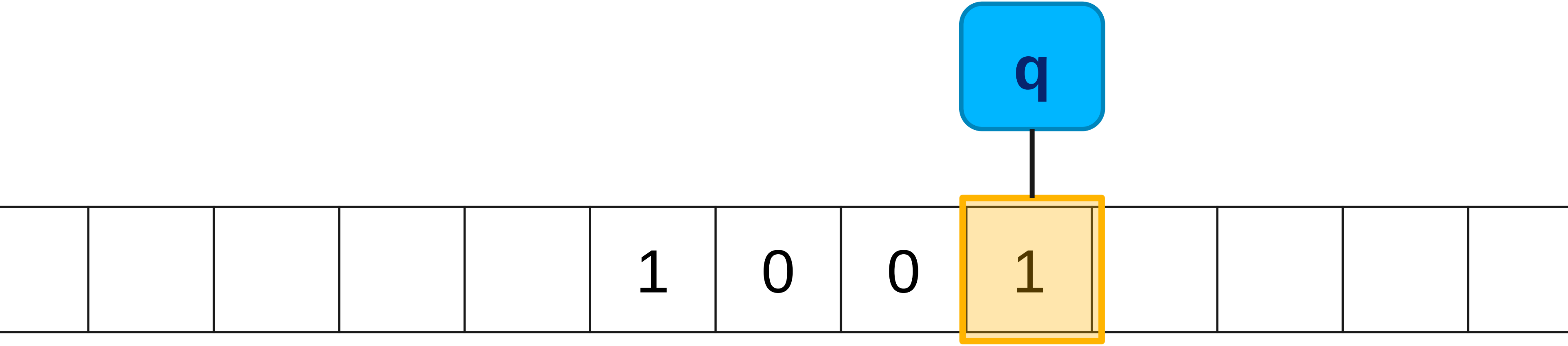
(1, 001, **q**)

From one configuration to another



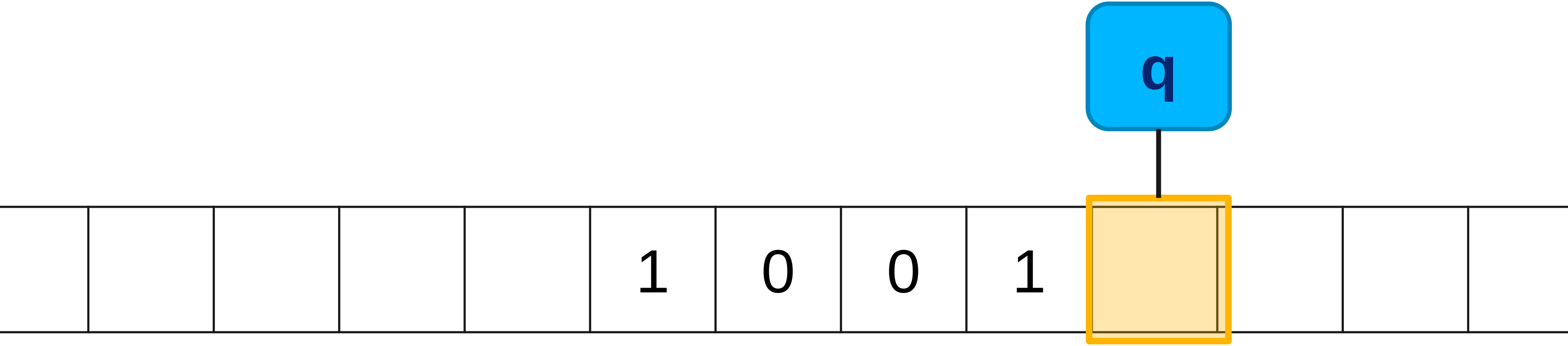
(10, 01, **q**)

From one configuration to another



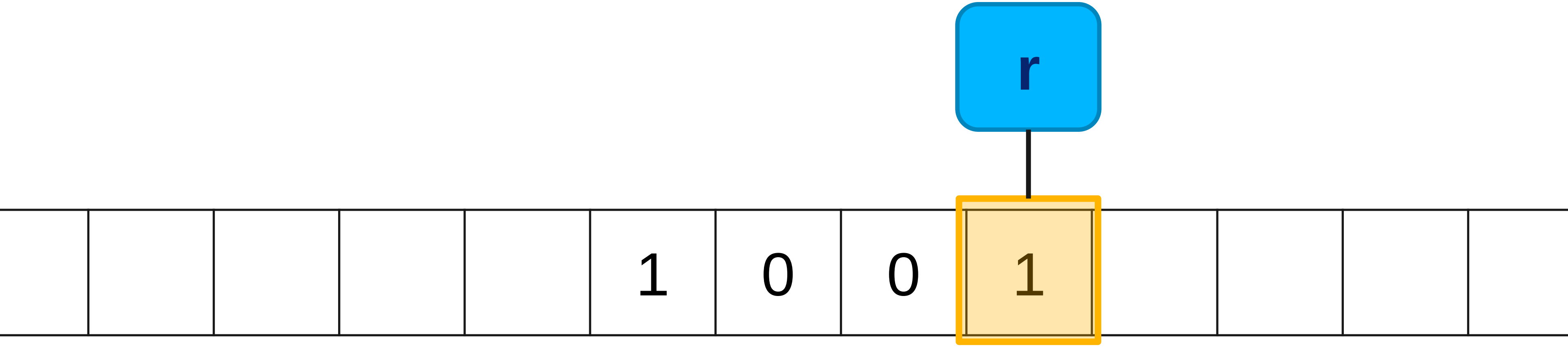
(100, 1, **q**)

From one configuration to another



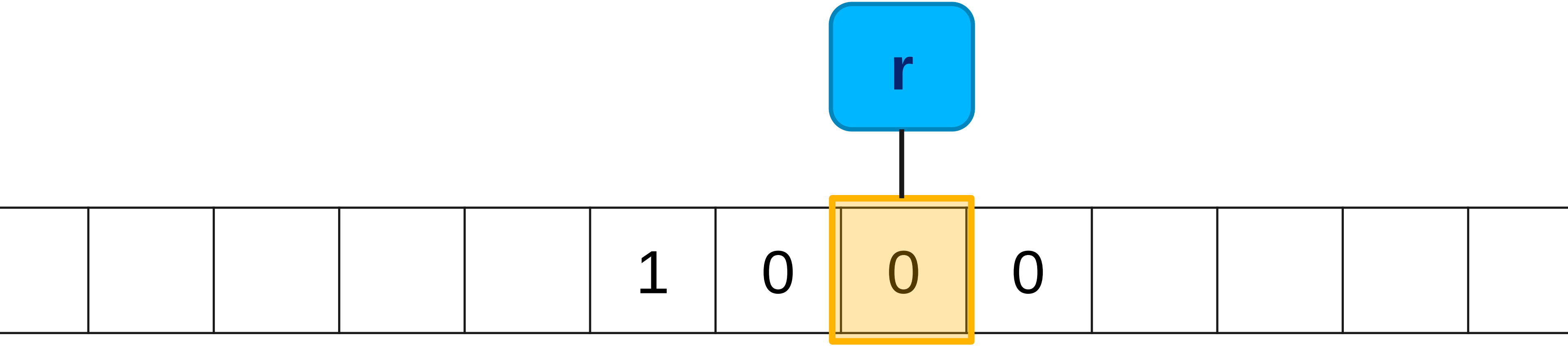
(1001,□, **q**)

From one configuration to another



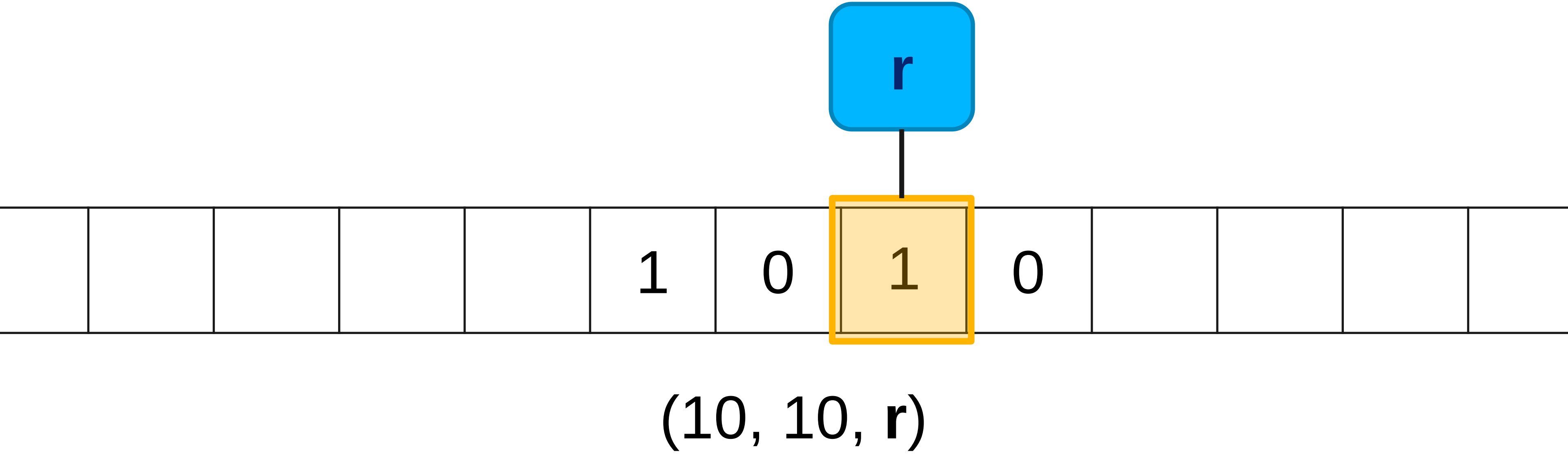
(100, 1, **r**)

From one configuration to another

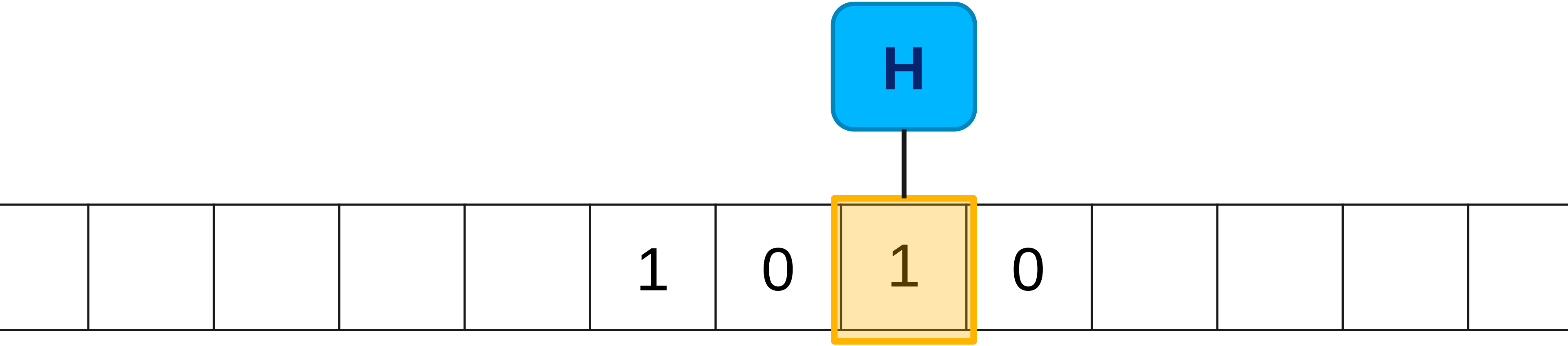


(10, 00, **r**)

From one configuration to another



From one configuration to another



(10, 10, **H**)

One step

$$C \vdash_M C'$$

"From configuration C, machine M goes to configuration C' in one step."

$(\varepsilon, 1001, \mathbf{q})$

$(1, 001, \mathbf{q})$

$(10, 01, \mathbf{q})$

$(100, 1, \mathbf{q})$

$(1001, \square, \mathbf{q})$

$(100, 1, \mathbf{r})$

$(10, 00, \mathbf{r})$

$(10, 10, \mathbf{r})$

$(10, 10, \mathbf{H})$

Zero or more steps

$$C \vdash_M^* C'$$

"From configuration C, machine M goes to configuration C' in zero or more steps."

Computing

$$(\varepsilon, 1001, \mathbf{q}) \quad \vdash_M^* \quad (10, 10, \mathbf{H})$$

"On input 1001, M *computes* output 1010"

$$M[1001] \rightarrow 1010$$

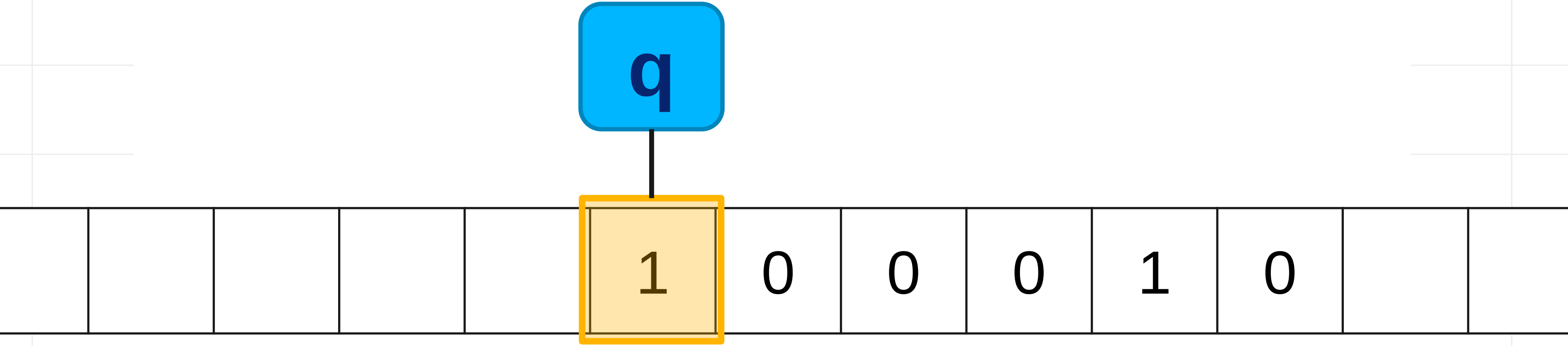
Computable problems

$$f\colon \Sigma_1^* \to \Sigma_2^*$$

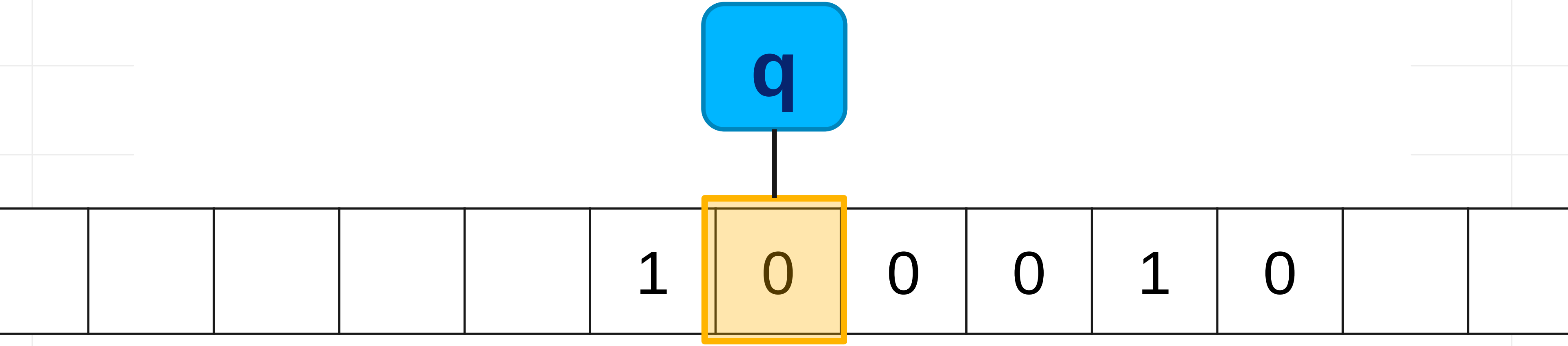
$$M\ computes\ f\ \stackrel{\text{def}}{=}\ \forall w \in \Sigma_1^*, M[w] \to f(w)$$

$$f\ is\ computable\ \stackrel{\text{def}}{=}\ \exists M, s.t.\ M\ computes\ f$$

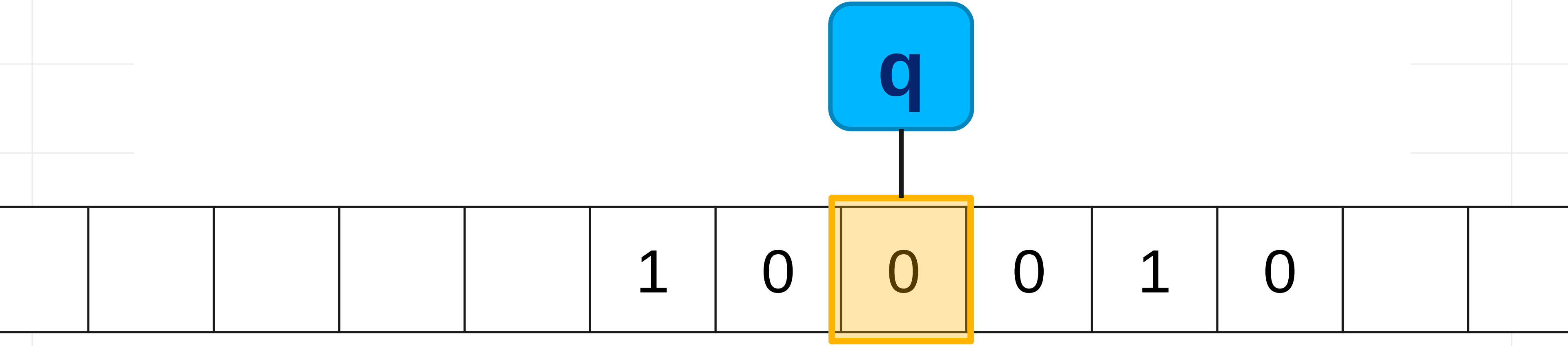
Accepting an input



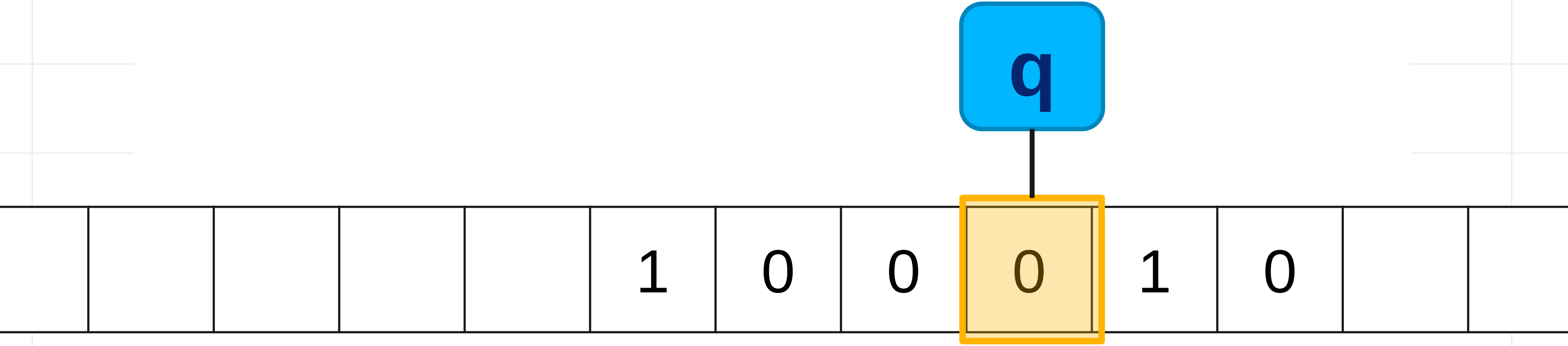
Accepting an input



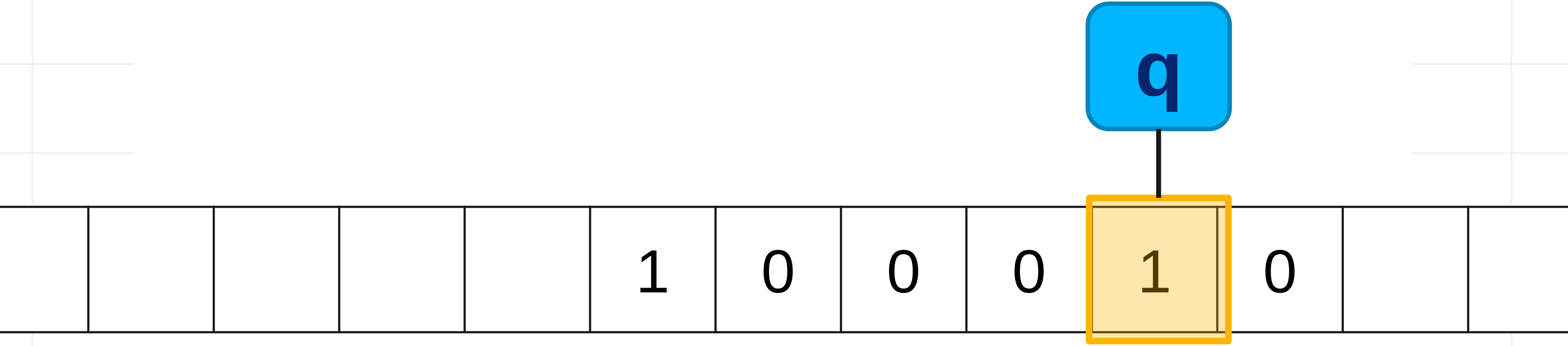
Accepting an input



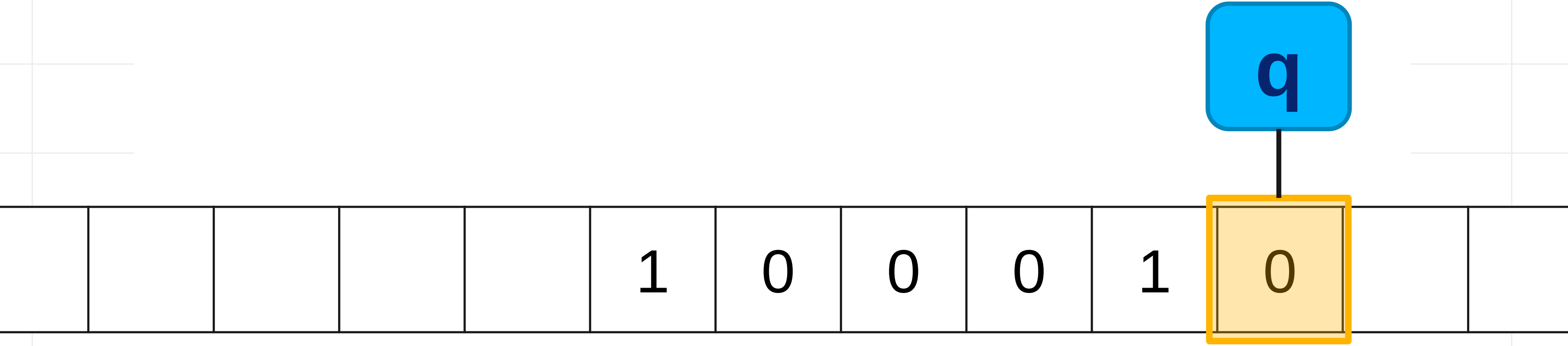
Accepting an input



Accepting an input



Accepting an input



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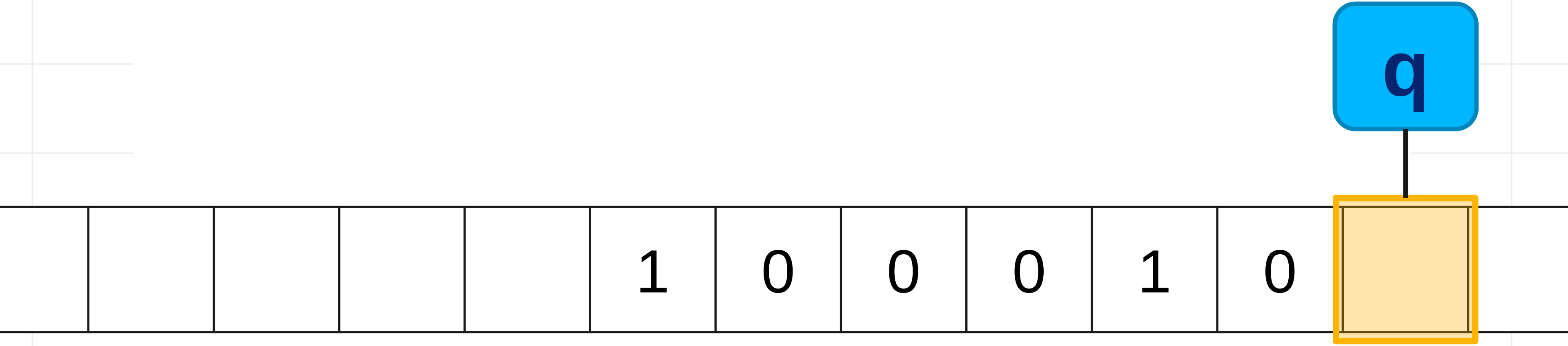

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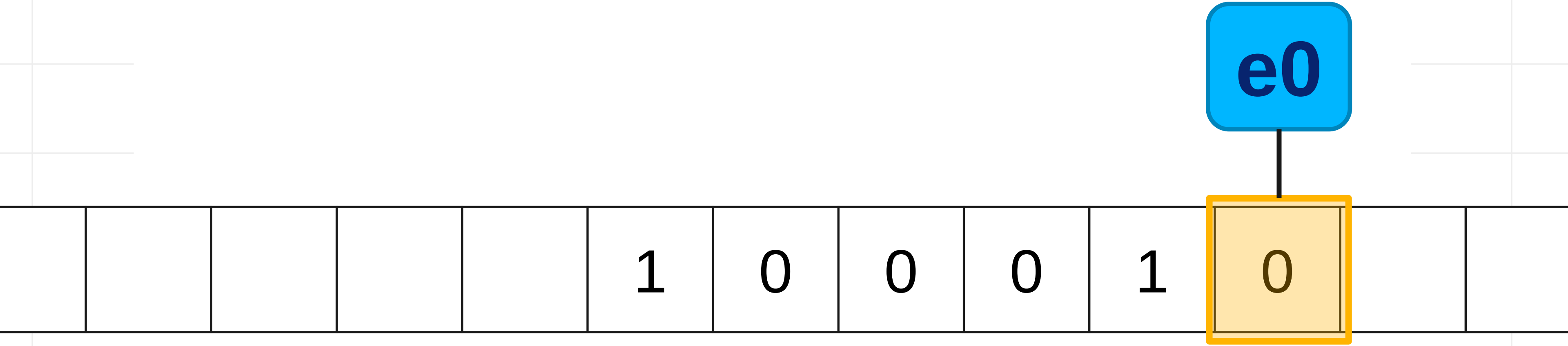

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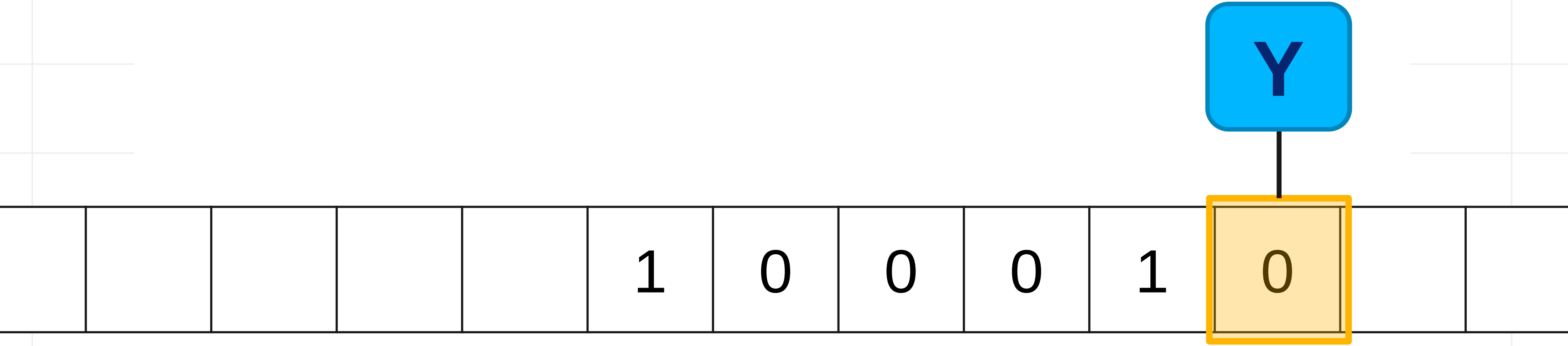
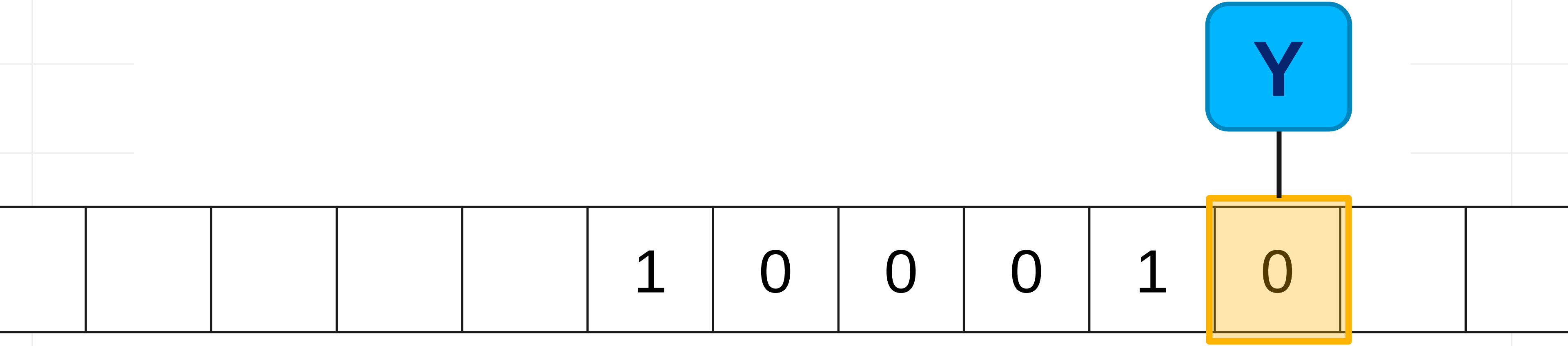


Accepting an input



$$M[100010] \to TRUE$$

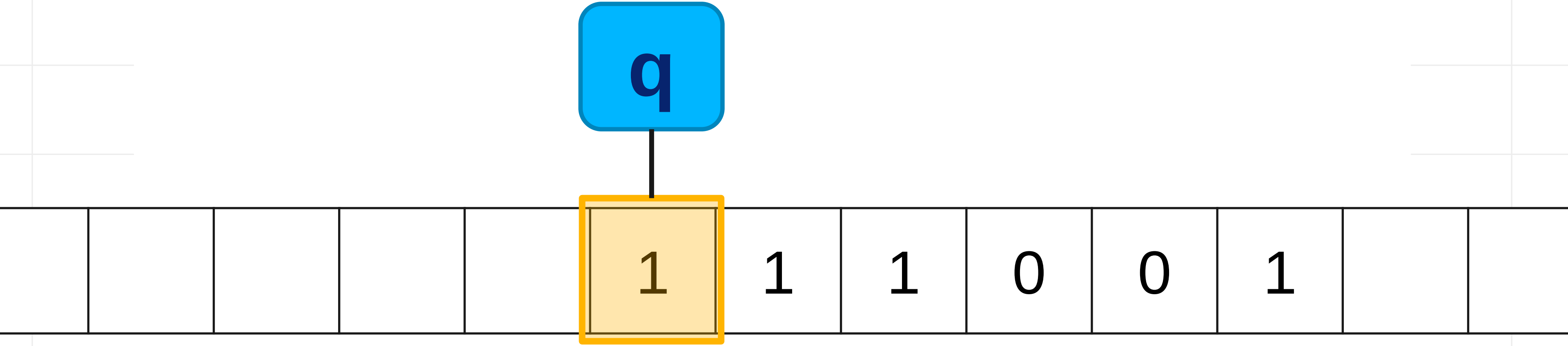
Acceptable problems

$$f: \Sigma^* \to \{FALSE, TRUE\}$$

$$M\ accepts\ f \stackrel{\text{def}}{=} \forall w \in \Sigma^*, (M[w] \to TRUE) \Leftrightarrow (f(w) = TRUE)$$

$$f\ is\ acceptable \stackrel{\text{def}}{=} \exists M, s.t.\ M\ accepts\ f$$

Rejecting an input



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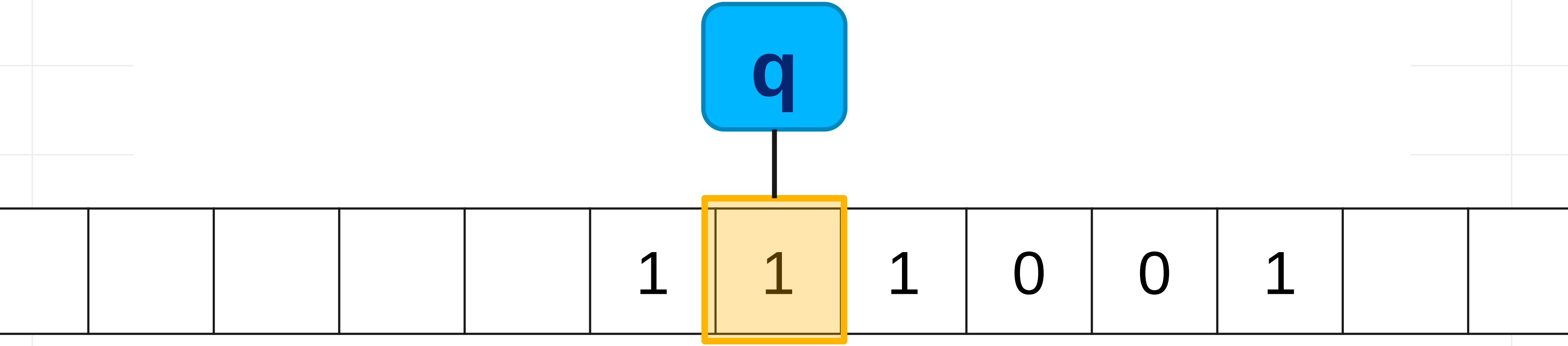

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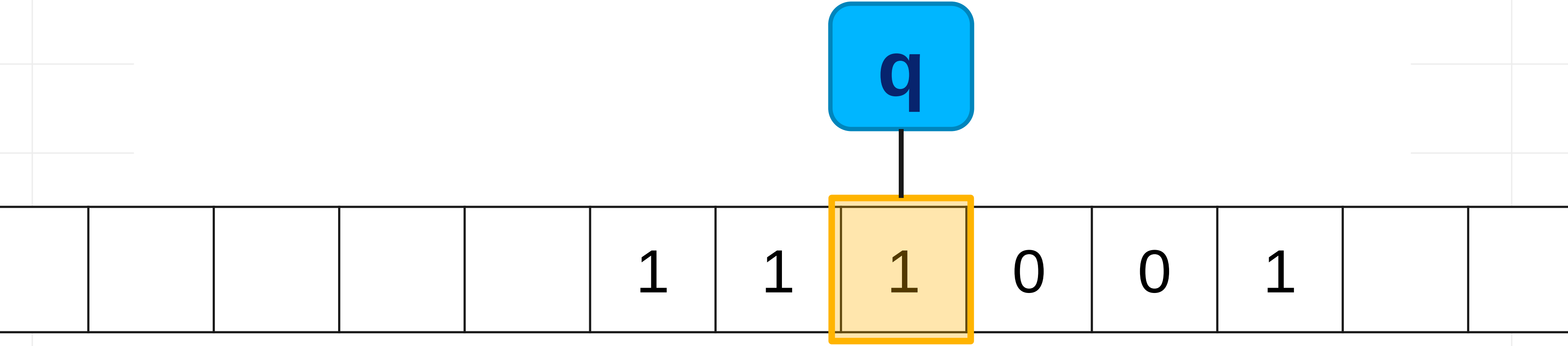

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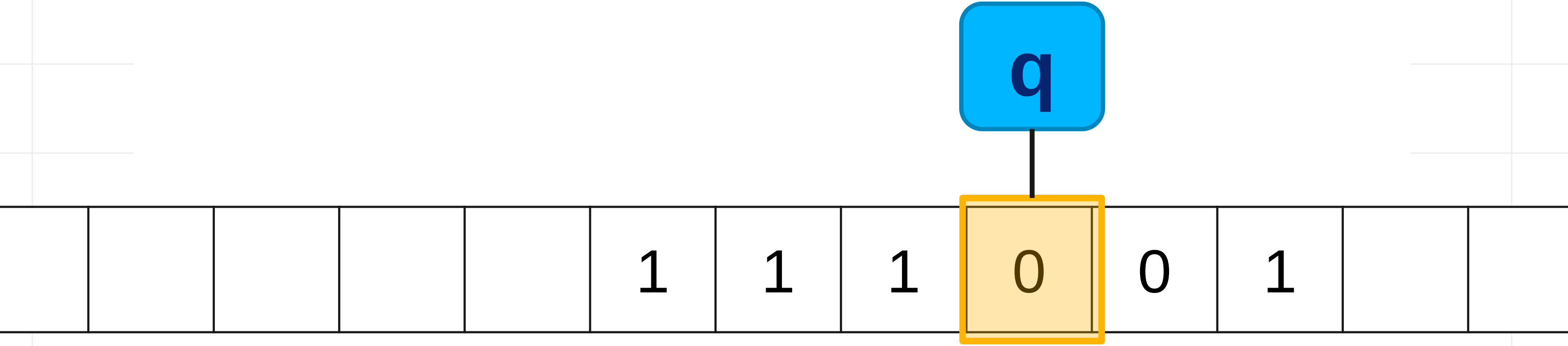

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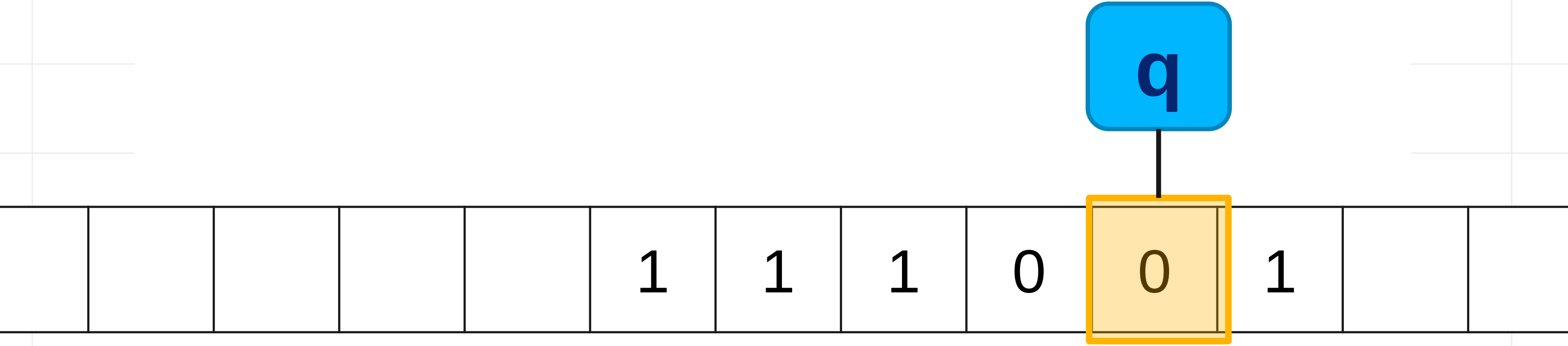

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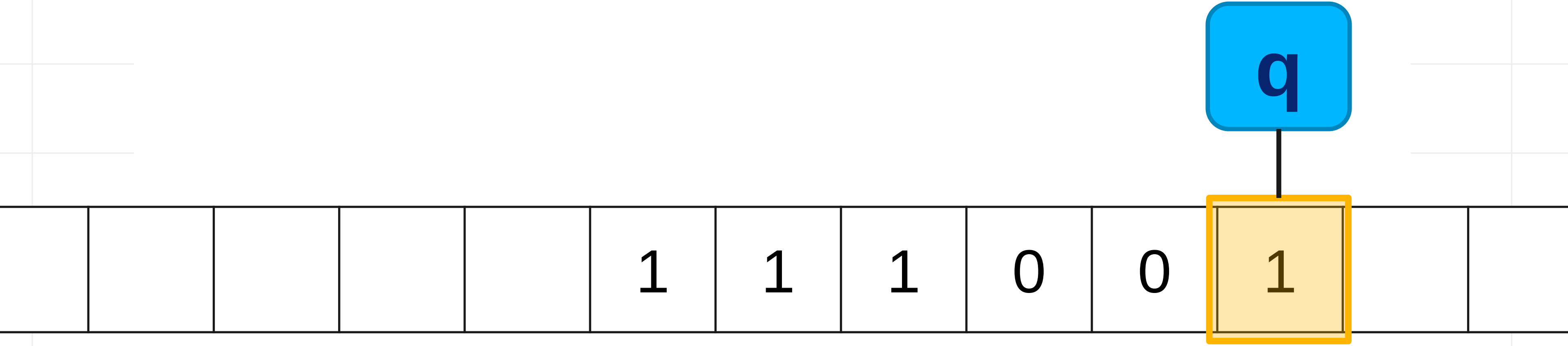

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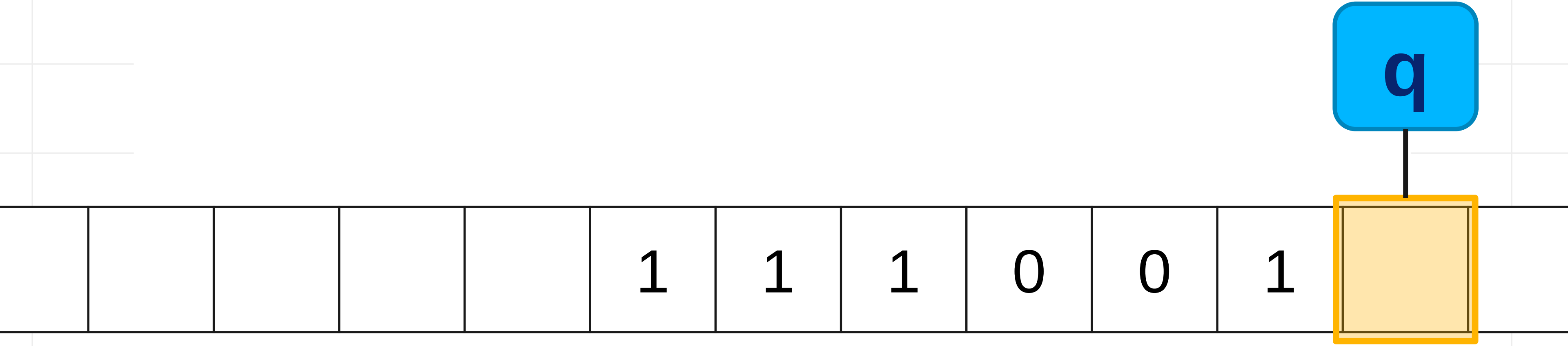

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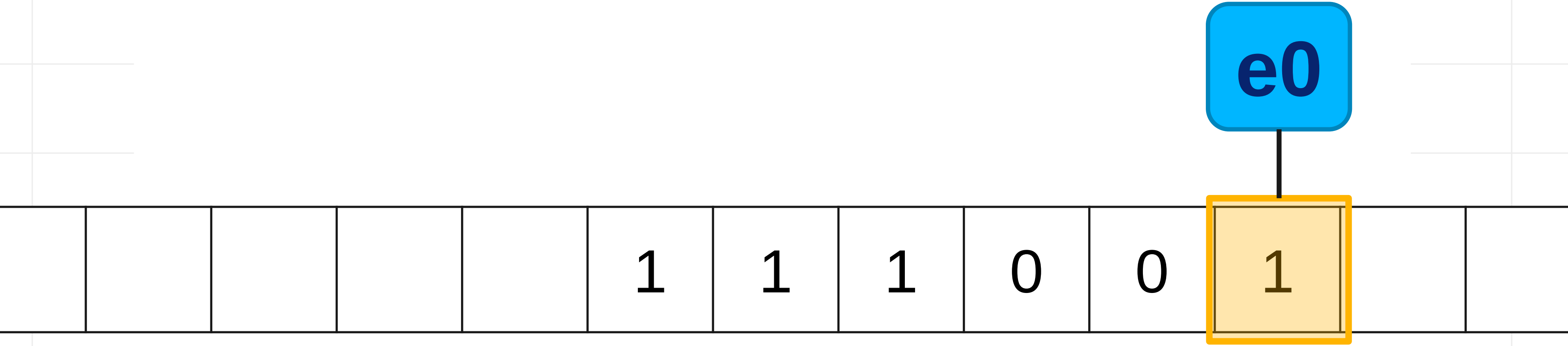

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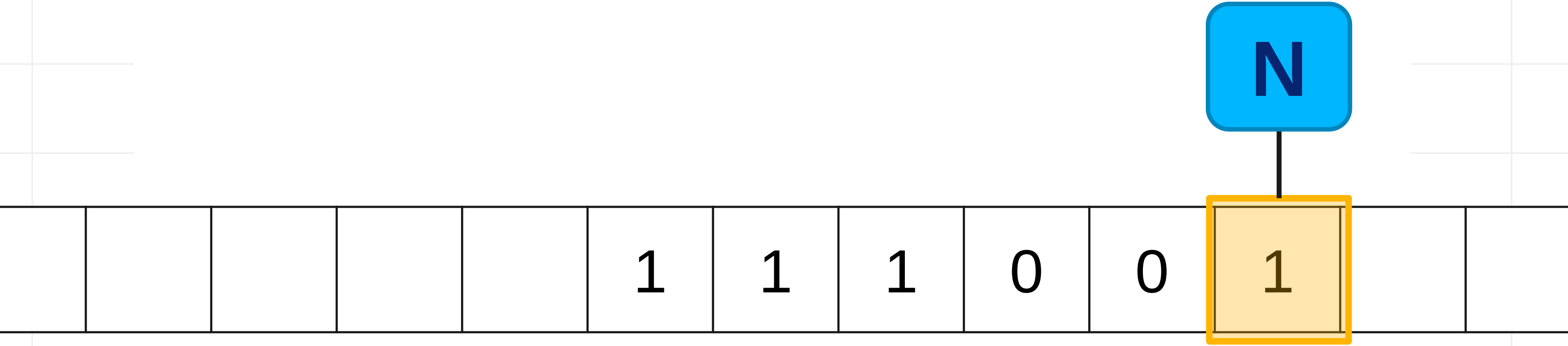

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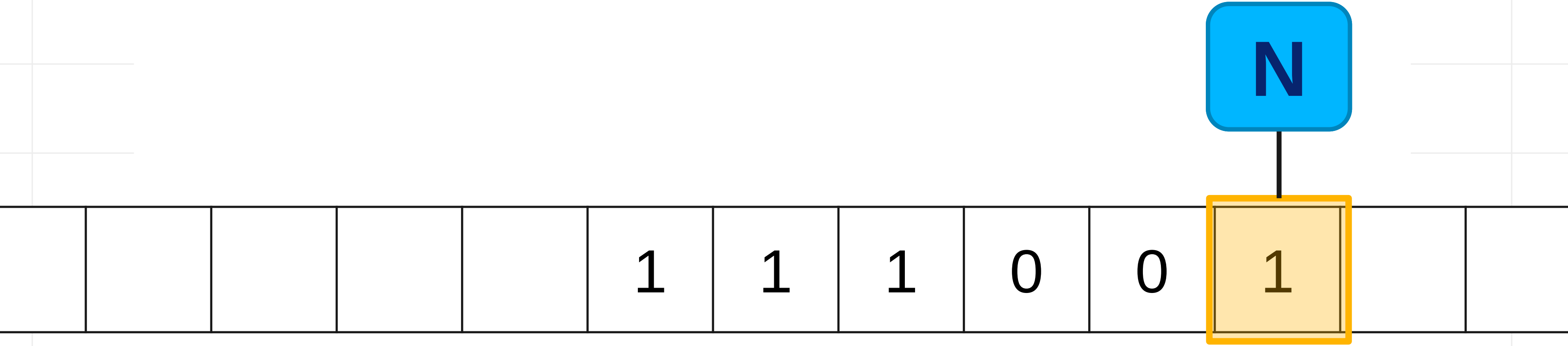


$$M[111001] \rightarrow FALSE$$

Decidable problems

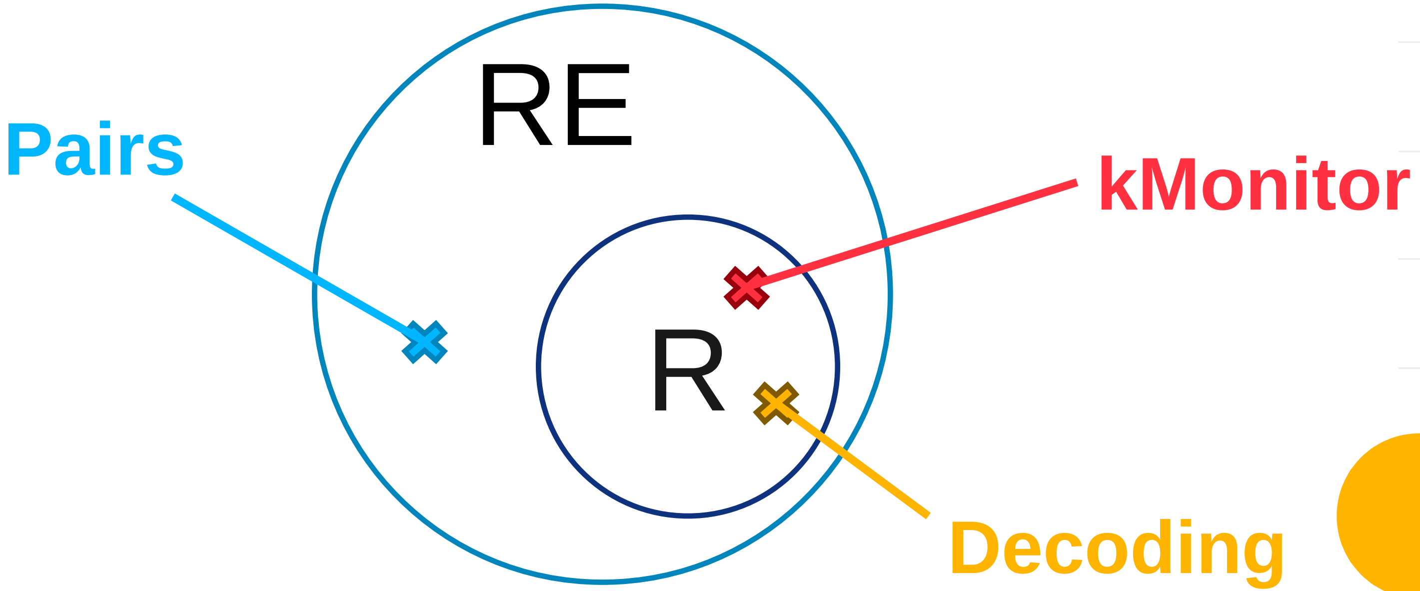
$$f: \Sigma^* \rightarrow \{FALSE, TRUE\}$$

$$M \ decides \ f \stackrel{\text{def}}{=}$$
$$\forall w \in \Sigma^*, (M[w] \rightarrow TRUE \Leftrightarrow f(w) = TRUE) \land$$
$$(M[w] \rightarrow FALSE \Leftrightarrow f(w) = FALSE)$$

$$f \ is \ decidable \stackrel{\text{def}}{=} \exists M, s.t. M \ decides \ f$$

The sets RE, R

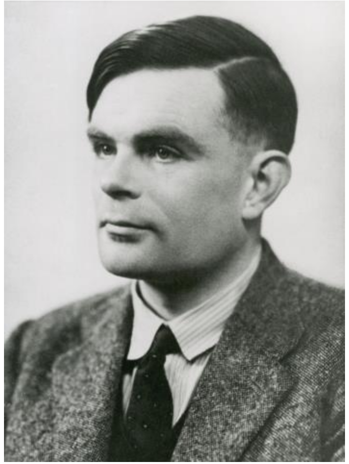
$RE = \{f \mid f \text{ is acceptable}\}$ $\quad$ $R = \{f \mid f \text{ is decidable}\}$



The Church-Turing thesis

Alonzo Church
(1903 – 1995)

"The problems solvable by algorithms are those computable by Turing machines."



Alan Turing
(1912 – 1954)

All roads lead to R

Recursive functions	Queue automata
λ-calculus	Counter machines
Turing machines	Post canonical system
Markov algorithms	Wang B-machine
RAM machines	Uniform boolean circuits