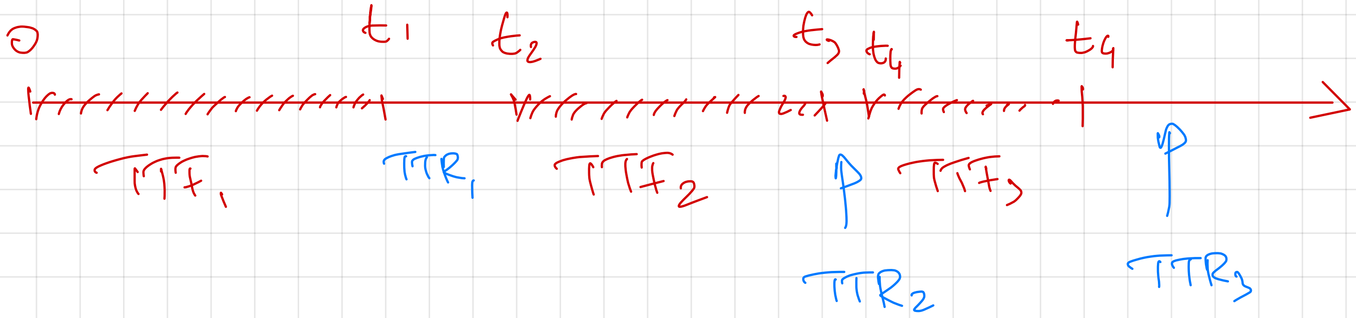
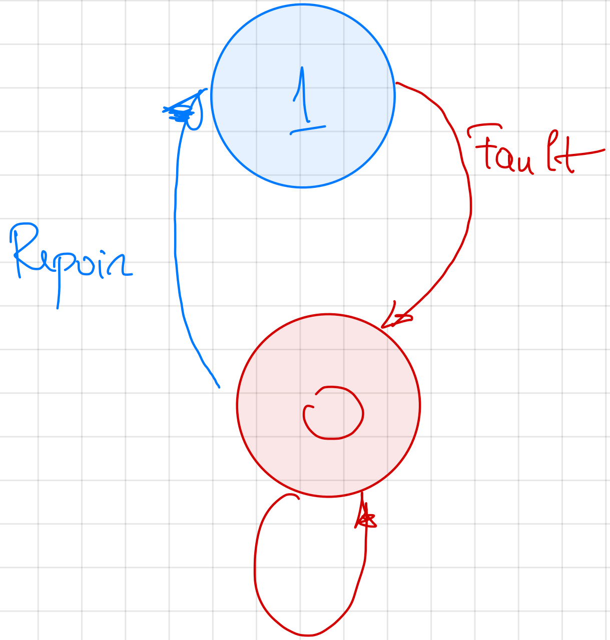
Toleranță la defecte Hardware

Fiabilitate: $R(t) = P(ok\ @\ t)$





Media timpului de bună funcționare (MTBF)

$$MTBF = \sum_{i=1}^{n} \frac{TTF_i}{n}$$

Media timpului de reparare (MTR)

$$MTR = \sum_{i=1}^{n} \frac{TTR_i}{n}$$

Disponibilitatea (Availability)

$$A = \frac{MTBF}{MTBF + MTR} \ \%$$

		Down time / an
$A = 90\%$	"one nine"	36,5 zile
$A = 99\%$	"two nines"	3,65 zile
$A = 99{,}99\%$	"four nines"	52,56 min
$A = 99{,}9999\%$	"six nines"	31,5 s

Refresher teoria probabilităţii

$$0 \le P(A) \le 1 \quad - \text{eveniment } A$$

$$P(\bar{A}) = 1 - P(A)$$

A şi B sunt dependente $\rightarrow P(A|B)$

$$P(A \cdot B) = P(A|B) \cdot P(B)$$

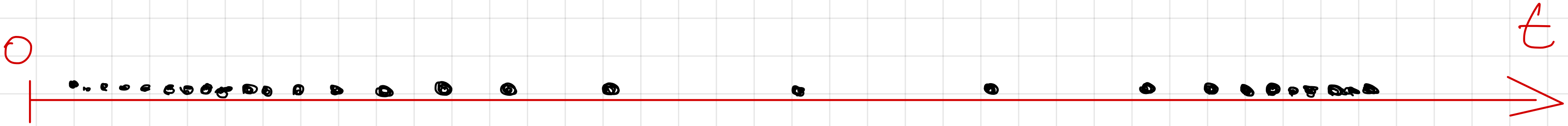
$$P(A|B) = \frac{P(A \cdot B)}{P(B)}, \quad P(B) > 0$$

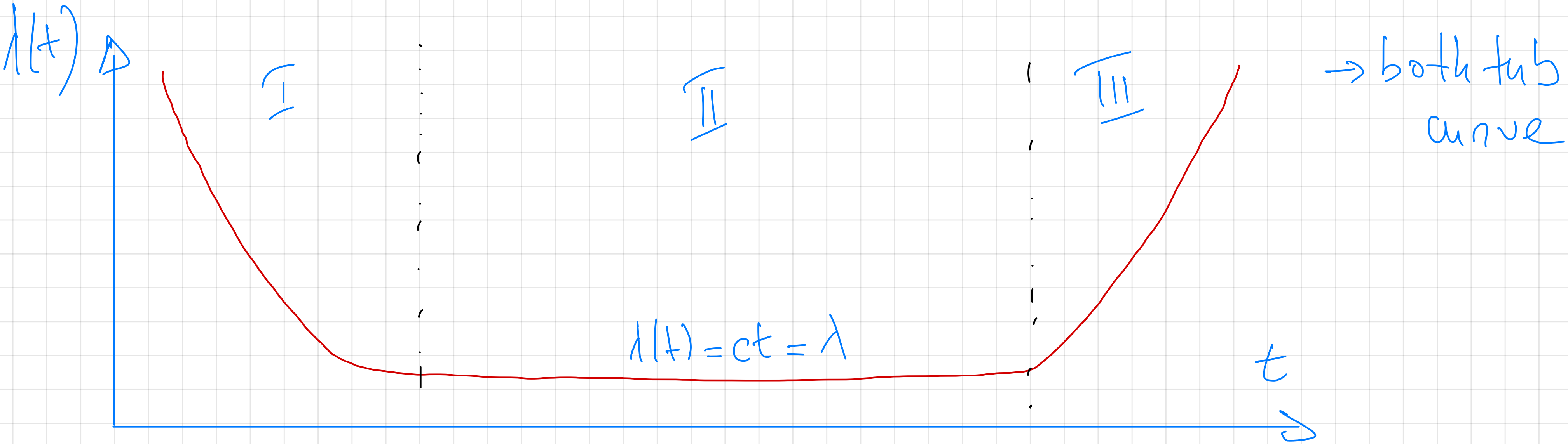
A şi B sunt mutual independente : $P(A|B) = P(A)$

$$P(A \cdot B) = P(A) \cdot P(B)$$

$$P(A+B) = P(A) + P(B) \cancel{- P(A \cdot B)}$$

Intensitatea defectiunilor





MIL-HDBK-217E

$$\lambda = \pi_L \, \pi_Q \left(C_1 \pi_T \pi_V + C_2 \pi_E \right)$$

π_L – learning factor

π_Q – quality factor

π_T – temperature factor

π_V – voltage stress factor

π_E – environmental shock factor

C_1, C_2 – factori complexitate

$f(t)$ – funcția de densitate de probabilitate

$F(t)$ – funcția distribuție cumulativă de probabilitate

$$f(t) = \frac{dF(t)}{dt} \qquad F(t) = \int_0^t f(z)\,dz$$

$$f(t) \geq 0 \ (\forall)\, t \geq 0 \quad \text{și} \quad \int_0^\infty f(t)\,dt = 1$$

$$F(t) = Prob(t \leq T)$$

$$R(t) = Prob(t > T) = 1 - F(t)$$

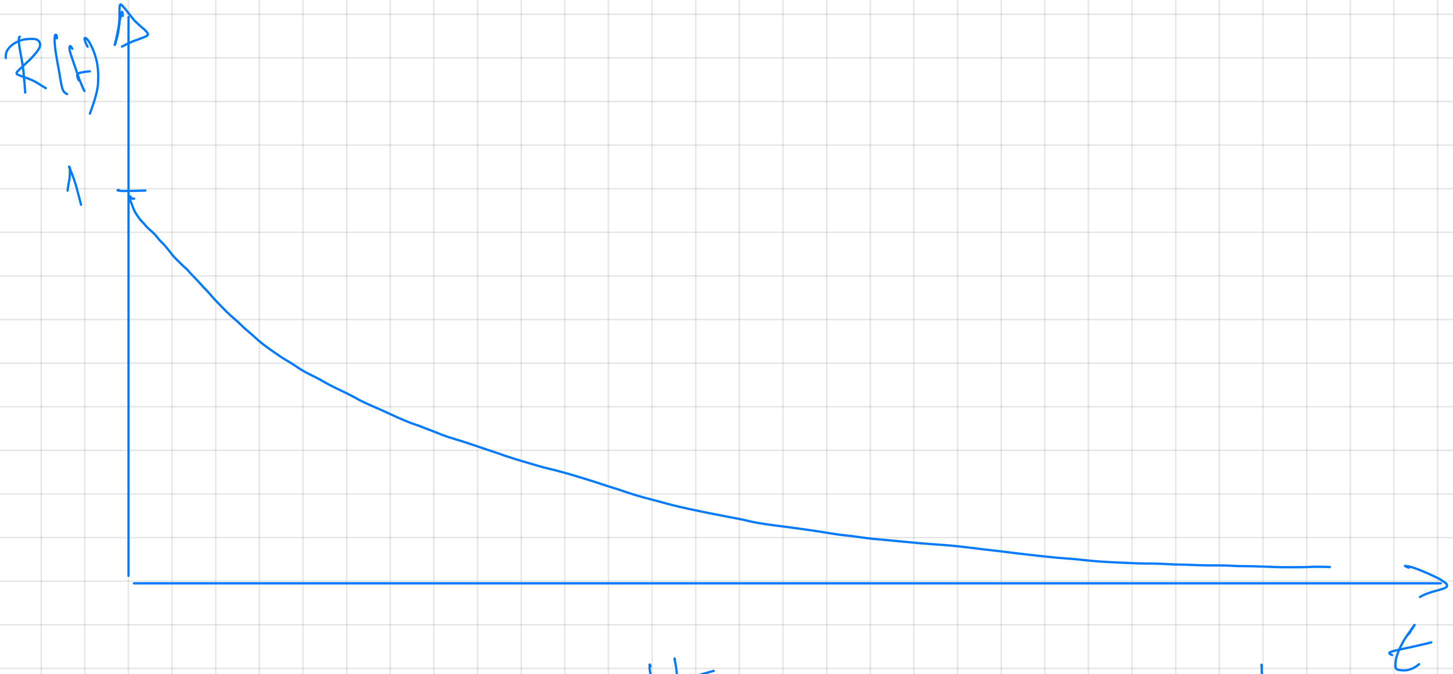
$$\lambda(t) = \frac{f(t)}{1 - F(t)}$$

$$f(t) = \frac{dF(t)}{dt} = \frac{d(1 - R(t))}{dt} = -\frac{dR(t)}{dt}$$

$$\Rightarrow \lambda(t) = \frac{-\frac{dR(t)}{dt}}{R(t)} = -\frac{1}{R(t)}\frac{dR(t)}{dt}$$

$$\lambda = -\frac{1}{R(t)} \frac{dR(t)}{dt} \iff \int -\lambda \, dt = \int \frac{dR(t)}{R(t)} \implies -\lambda t = \ln(R(t)) \implies$$

$$\implies R(t) = e^{-\lambda t}$$



$$F(t) = 1 - e^{-\lambda t} \qquad f(t) = \lambda e^{-\lambda t} \qquad \forall\, t \geq 0$$

Valoarea așteptată a unei variabile aleatoare

$$E[X] = p_1 x_1 + p_2 x_2 + \dots \; p_k x_k$$

x – var. aleatoare $\quad x \in \{x_1, x_2 \dots x_k\}$

Ex: zar cu 6 fețe $\quad x \in \{1, 2, 3, 4, 5, 6\}$

$$E[X] = \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \dots \frac{1}{6} \cdot 6 = 3,5$$

$f(x)$

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$$

fiabilitate $\rightarrow$ X = timpul $\quad t \geq 0$

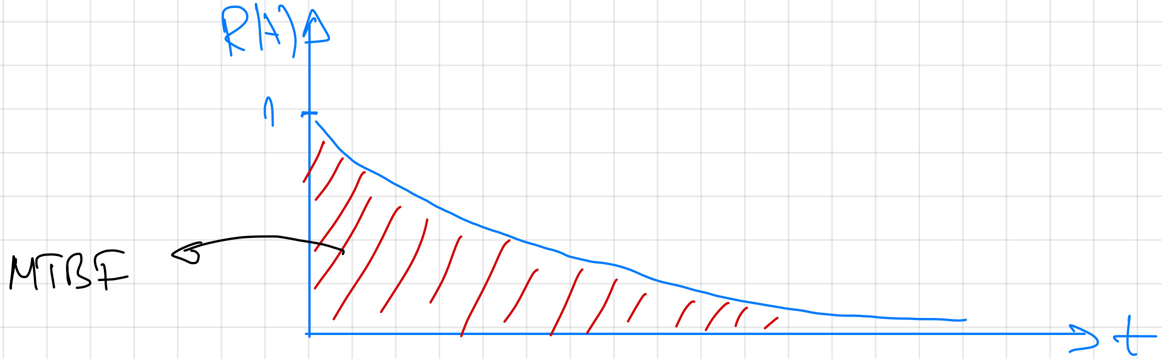
$$E[T] = \int_0^{\infty} t f(t)\, dt = MTBF$$

$$MTBF = \int_0^{\infty} t f(t)\, dt = \int_0^{\infty} t \cdot \left(- \frac{dR(t)}{dt} \right) dt = -\int_0^{\infty} t R'(t)\, dt = -t R(t) \Big|_0^{\infty} +$$

$$+\int_0^\infty R(t)\,dt$$

$$MTBF = \cancel{-tR(t)\Big|_0^\infty} + \int_0^\infty R(t)\,dt = \int_0^\infty R(t)\,dt$$

$$MTBF = \int_0^\infty R(t)\,dt$$



$$MTBF = \int_0^\infty e^{-\lambda t}\,dt =$$

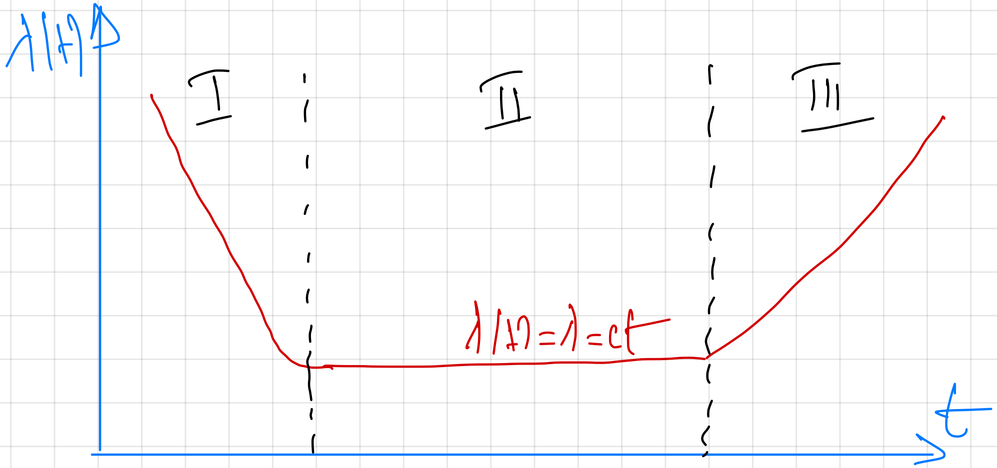
$$\int e^x\,dx = e^x + c \qquad \int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$= -\frac{1}{\lambda}e^{-\lambda t}\Big|_0^\infty = \cancel{-\frac{1}{\lambda}e^{-\infty}} + \frac{1}{\lambda}e^0 = \frac{1}{\lambda}$$

doco: $\lambda(t) = ct \Rightarrow MTBF = \frac{1}{\lambda}$

Distributia Weibull

$$f(t) = \lambda \beta t^{\beta-1} e^{-\lambda t^{\beta}}$$



$$\lambda(t) = \lambda \beta t^{\beta-1}$$

$\beta \in [0, 1) \rightarrow \lambda(t)$ este $\searrow$ (mortalitate infantilă I)

$\beta = 1 \rightarrow \lambda(t) = \lambda = ct$ (intervalul II)

$\beta > 1 \rightarrow \lambda(t)$ este $\nearrow$ (intervalul III)

$$R(t) = e^{-\lambda t^{\beta}}$$

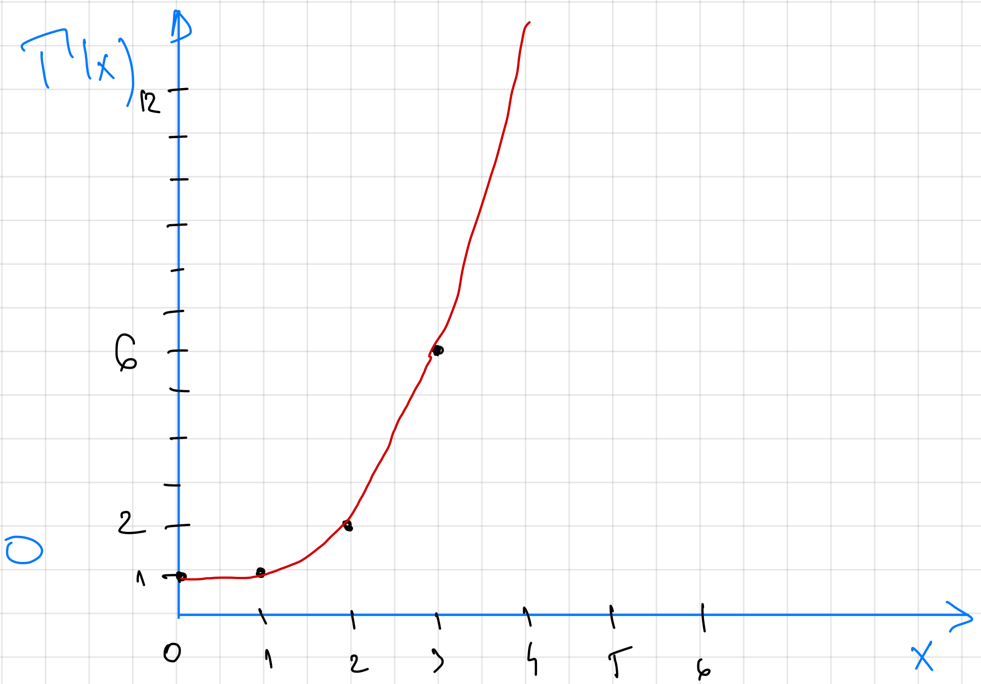
$$MTBF = \int_0^{\infty} R(t)\,dt = \int_0^{\infty} e^{-\lambda t^{\beta}}\,dt = \frac{\Gamma(\beta^{-1})}{\beta \lambda^{\beta-1}}$$

$$\Gamma(x) = \int_0^\infty y^{x-1} e^{-y}\, dy$$

$$\Gamma(x) = (x-1)\,\Gamma(x-1) \quad \forall\ x>1$$

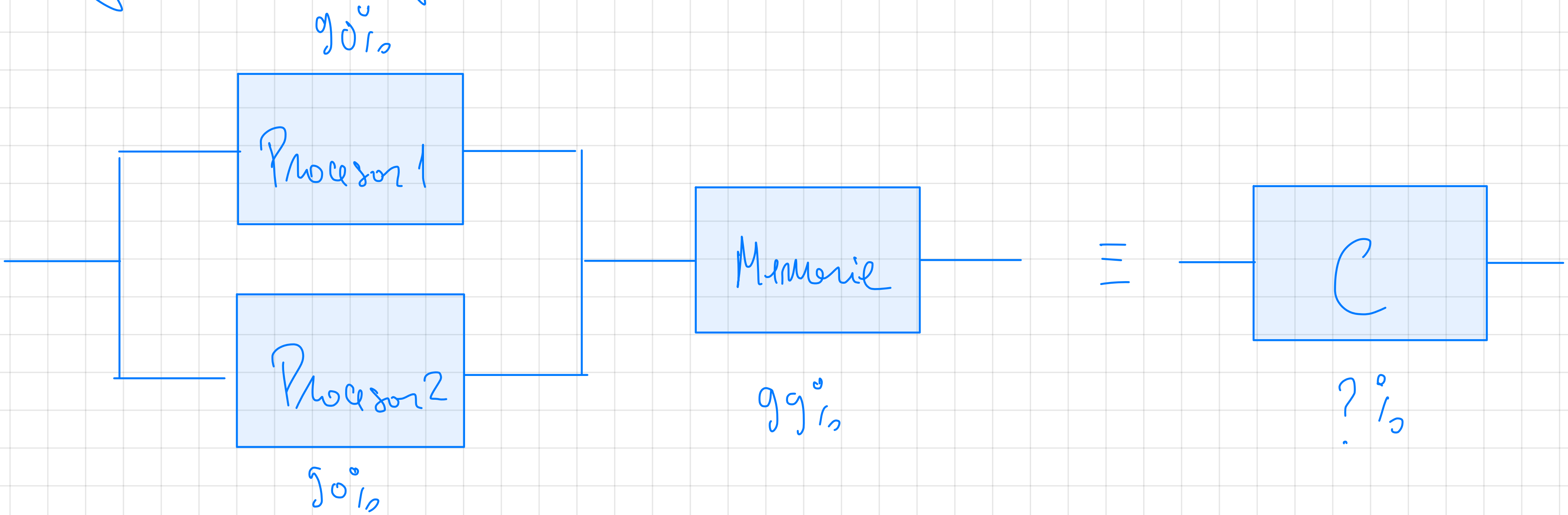
$$\Gamma(0) = \Gamma(1) = 1$$

$$\Gamma(n) = \Gamma(n-1)! \quad (\forall)\ n \in \mathbb{N},\ n \neq 0$$



Estimarea fiabilității

Diagrama de fiabilitate

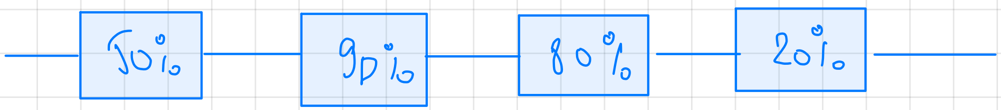


Strutture canoniche

Strutture serie

$$R_S = R_1 \cdot R_2 \cdot R_3 \cdot \ldots R_n = \prod_{i=1}^{n} R_i$$

$$R_S(t) = \prod_{i=1}^{n} R_i(t)$$



$$R_S = 0{,}5 \cdot 0{,}9 \cdot 0{,}8 \cdot 0{,}2 = 0{,}072$$
$$= 7{,}2\,\%$$

$$R(t) = e^{-\lambda t}$$

$$R_S(t) = \prod_{i=1}^{n} R_i(t) = \prod_{i=1}^{n} e^{-\lambda t} = e^{-n \lambda t} = 99{,}99\,\%$$

$$R_s(t) = \prod_{i=1}^{n} R_i(t) \qquad Q(t) = 1 - R(t)$$

$$R_s(t) = \prod_{i=1}^{n} (1 - Q_i(t)) = 1 - (Q_1 + Q_2 + \dots Q_n) + (Q_1 Q_2 + \dots Q_{n-1} Q_n) - \dots$$

R_i este mare $(\geq 90\%) \Rightarrow Q_i$ este mic $< 10\%$

$$R_s(t) \simeq 1 - (Q_1 + Q_2 + \dots Q_n) = 1 - \sum_{i=1}^{n} Q_i$$

$$Q_s = 1 - R_s(t) = \sum_{i=1}^{n} Q_i$$

$$R_i(t) = e^{-\lambda_i t}$$

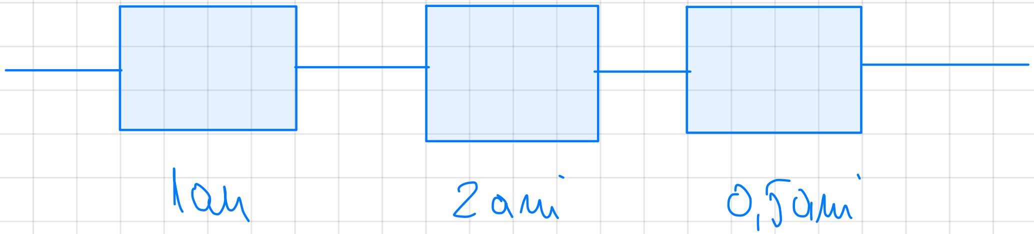
$$R_s(t) = \prod_{i=1}^{n} R_i(t) = \prod_{i=1}^{n} e^{-\lambda_i t} = e^{-\overbrace{\sum_{i=1}^{n} \lambda_i}^{\lambda_s} t} = e^{-\lambda_s t}$$

$$\lambda_s(t) = \sum_{i=1}^{n} \lambda_i(t)$$

$$MTBF_S = \int_0^\infty R_S(t)\,dt = \int_0^\infty e^{-\lambda_S t}\,dt = \frac{1}{\lambda_S} = \frac{1}{\lambda_1+\lambda_2+\dots+\lambda_n}$$

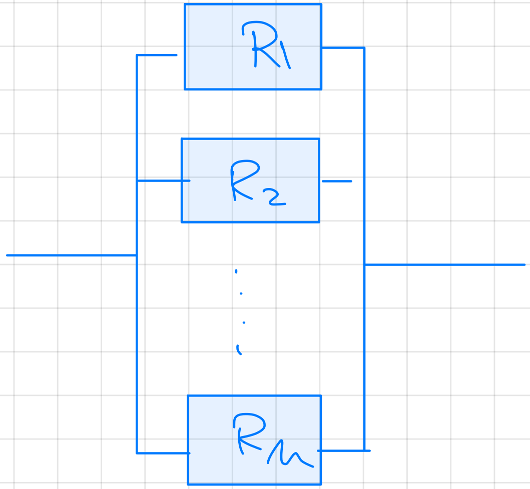
$$MTBF_i = \frac{1}{\lambda_i} \Rightarrow \lambda_i = \frac{1}{MTBF_i} \Rightarrow$$

$$MTBF_S = \frac{1}{\frac{1}{MTBF_1}+\frac{1}{MTBF_2}+\dots+\frac{1}{MTBF_n}} = \frac{1}{\sum_{i=1}^{n}\frac{1}{MTBF_i}}$$



$$MTBF_S = \frac{1}{\frac{1}{1}+\frac{1}{2}+\frac{1}{0{,}5}} = \frac{1}{3+\frac{1}{2}} = \frac{2}{7}\text{ ani} = 0{,}28\text{ ani}$$

Struktura paralel



$$R_p(t) = 1 - Q_p(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

$$Q_p(t) = Q_1 \cdot Q_2 \cdot Q_3 \dots \cdot Q_n = \prod_{i=1}^{n} Q_i$$

$$R_p(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

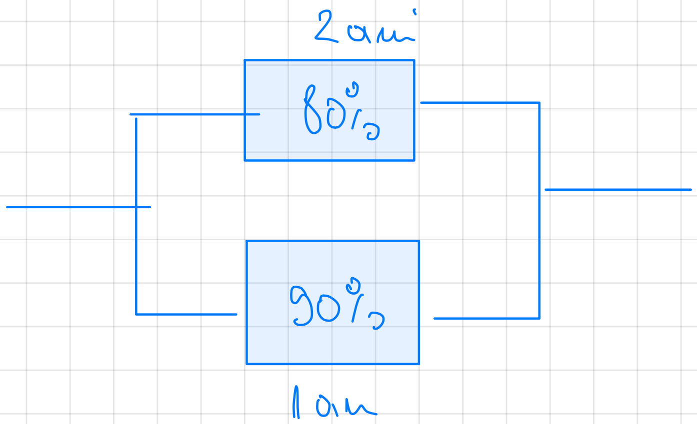
oboča $R_1 = R_2 = \dots R_n = R = e^{-\lambda t}$

$$R_p(t) = 1 - (1 - R(t))^n = 1 - (1 - e^{-\lambda t})^n =$$

$$MTBF_p = \int_0^\infty R_p(t)\,dt = \int_0^\infty \left(1 - (1 - e^{-\lambda t})^n\right) dt = \frac{n}{\lambda} - \frac{n}{2\lambda} + \dots + (-1)^{n+1} \frac{1}{n\lambda}$$

$$MTBF_p = \frac{1}{\lambda} \sum_{i=1}^{n} \frac{1}{i}$$

$$\lambda_p = \frac{\lambda}{\sum_{i=1}^{n} \frac{1}{i}}$$



$$R_p = 1-(1-R_1)(1-R_2) = 1-(1-0{,}8)(1-0{,}9)$$

$$= 1-0{,}2\cdot 0{,}1 = 1-0{,}02 = 0{,}98 = 98\%$$

$$R_1 = e^{-\lambda_1 t} = e^{-\frac{1}{2}t} \quad (\lambda_1 = 0{,}5)$$

$$R_2 = e^{-\lambda_2 t} = e^{-t} \quad (\lambda_2 = 1)$$

$$MTBF_p =$$

$$R_p = 1-(1-e^{-0{,}5t})(1-e^{-t}) = 1-\left(1-e^{-t}-e^{-0{,}5t}+e^{-1{,}5t}\right) =$$

$$= e^{-t}+e^{-0{,}5t}-e^{-1{,}5t}$$

$$MTBF_p = \int_0^\infty \left(e^{-t}+e^{-0{,}5t}-e^{-1{,}5t}\right)dt = \int_0^\infty e^{-t}dt+\int_0^\infty e^{-0{,}5t}dt-\int_0^\infty e^{-1{,}5t}dt =$$

$$= \frac{1}{1}+\frac{1}{0{,}5}-\frac{1}{1{,}5} \quad \left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1+\lambda_2}\right)$$

$$= 3-\frac{2}{3} = \frac{7}{3}\text{ ani} = 2{,}3\ldots\text{ ani}$$

$$R_{TOTAL} = (R_1 \parallel R_2)\, R_3$$

$$R_1 \parallel R_2 = 1 - (1 - R_1)(1 - R_2)$$

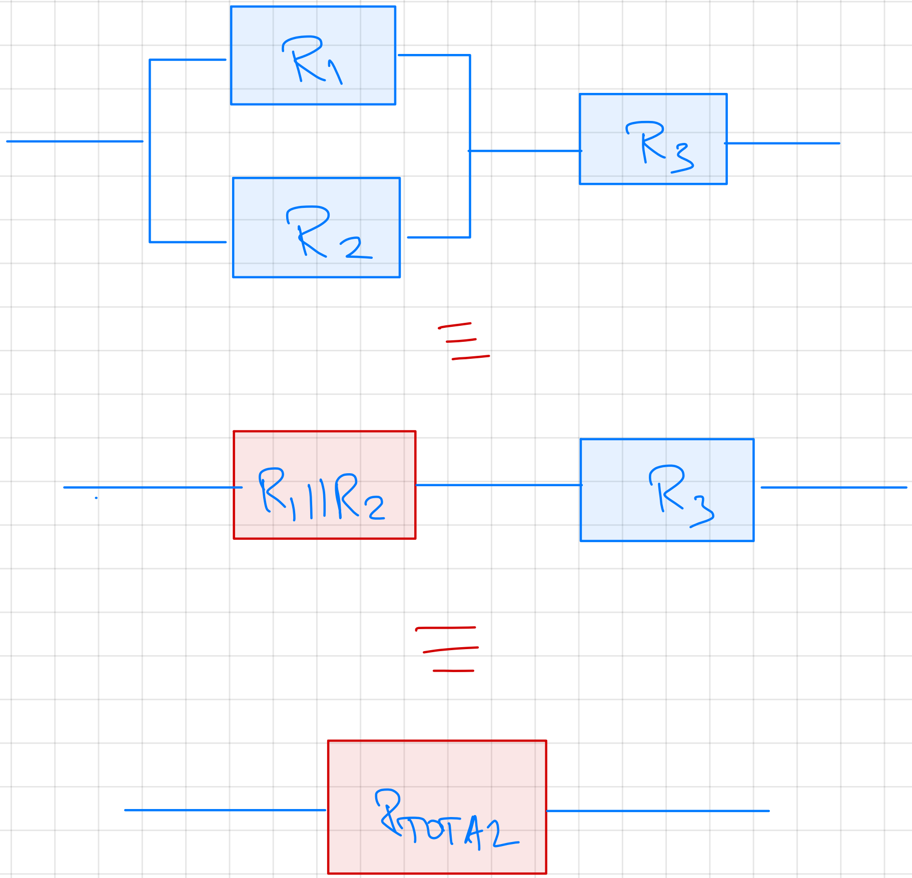
$$= 1 - (1 - R_2 - R_1 + R_1 R_2)$$

$$= R_1 + R_2 - R_1 R_2$$

$$R_{TOTAL} = R_1 R_3 + R_2 R_3 - R_1 R_2 R_3$$

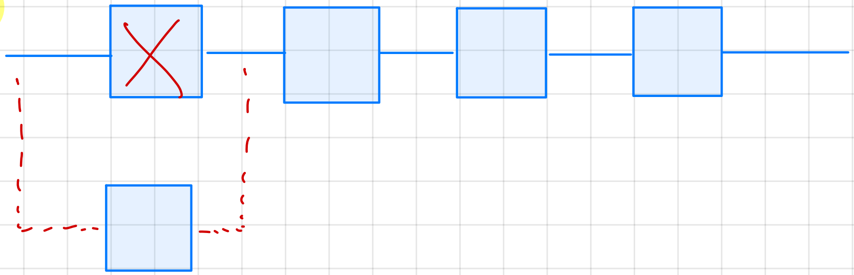
$R_1 = 70\%$ $\quad R_2 = 80\%$ $\quad R_3 = 40\%$

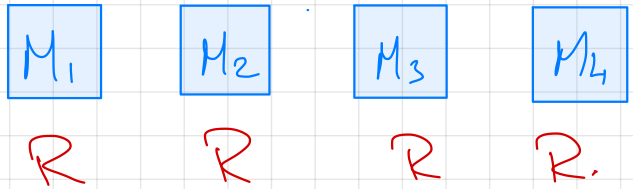
$$R_{TOTAL} = 0{,}28 + 0{,}32 - 0{,}224 = \,..$$



Fiabilitatea structurilor n din m

$$R_{2|4} = R^4 + 4 \cdot R^3(1-R) + 6R^2(1-R)^2$$





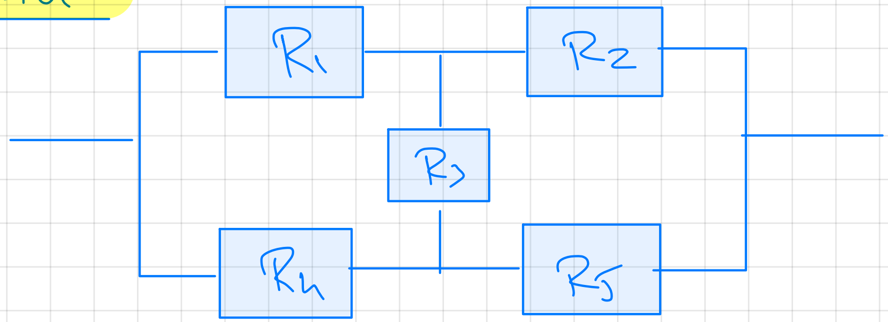
în general:

$$R_{n|m} = \sum_{n=i}^{m} C_m^i R^i (1-R)^{m-i}$$

dacă $n = m \Rightarrow R_{TOTAL} = R^m$ - fiabilitatea serie

$n = 1 \Rightarrow R_{TOTAL} = 1 - (1-R)^m$ - fiabilitatea paralel

Fiabilitatea structurilor nedecompozabile



$R_{TOTAL} = ?$

Cazul 1: R_3 funcționează ($R_3 = 1$)

$$R_T(S|R_3) = \dots$$

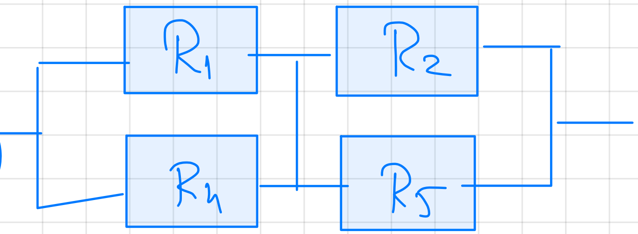
Cazul 2: R_3 nu funcționează ($R_3 = 0$)

$$R_T(S|\overline{R_3}) = \dots$$

$$R_{TOTAL} = R_T(S|R_3)\cdot R_3 + R_T(S|\overline{R_3})\cdot(1-R_3)$$

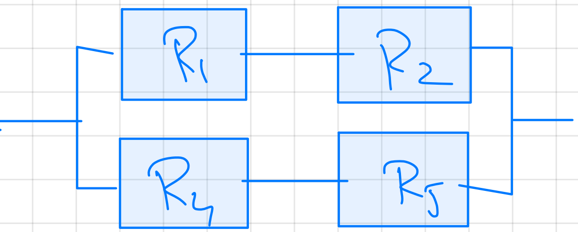
Cazul 1: $R_3 = 1$

$$R_T(S|R_3) = (R_1 \| R_4)\cdot(R_2 \| R_5) = (R_1+R_4-R_1R_4)(R_2+R_5-R_2R_5)$$



Cazul 2: $R_3 = 0$

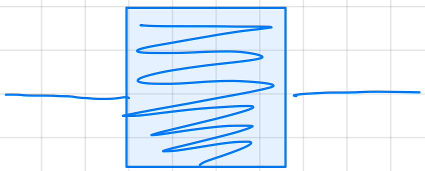
$$R_T(S|\overline{R_3}) = (R_1R_2) \| (R_4R_5) = R_1R_2+R_4R_5-R_1R_2R_4R_5$$



$$R_{TOTAL} = (R_1 + R_4 - R_1 R_4)(R_2 + R_5 - R_2 R_5) R_3 + (R_1 R_2 + R_4 R_5 - R_1 R_2 R_4 R_5)(1 - R_3)$$

$$R_1 = R_2 = \dots R_5 \Rightarrow R_{TOTAL} = (2R - R^2)^2 \cdot R + (2R^2 - R^4)(1 - R) =$$

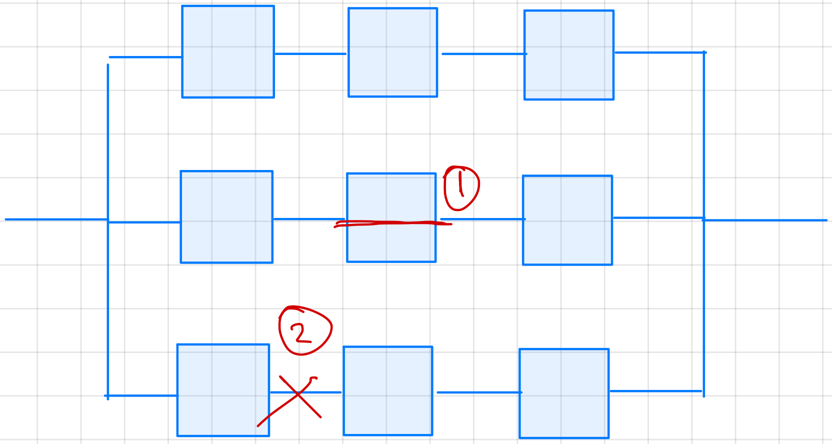
$$= R^2 (2R^3 - 5R^2 + 2R + 2) \quad \dots$$



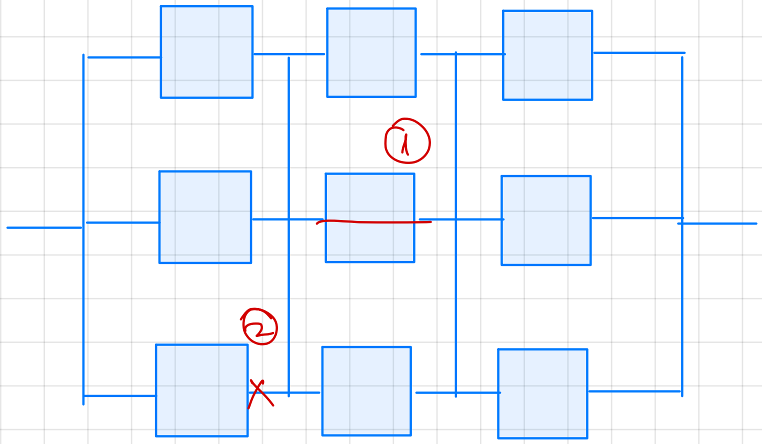
Celulă fotovoltaică

Moduri de defectare:
- defect prin întrerupere 
- defect prin scurtcircuit 

Serie-paralel



Paralel-serie



1 scurtcircuit: pierdem 1 celulă din 9

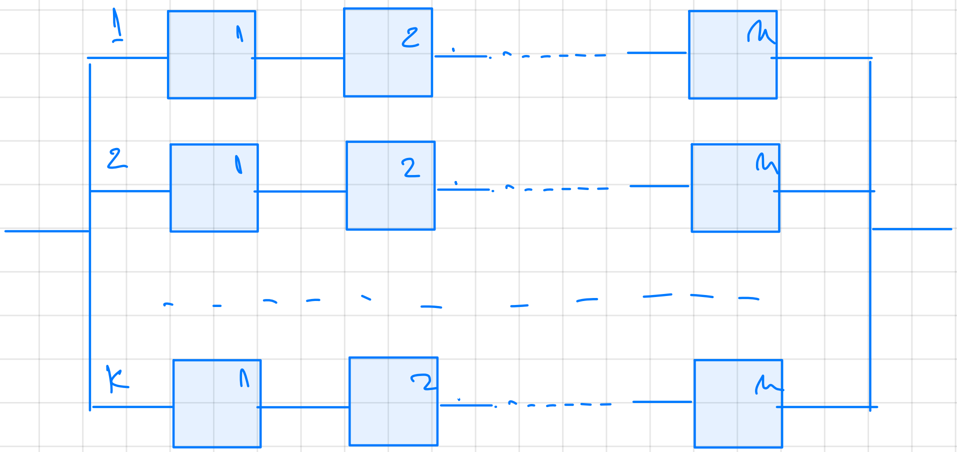
2 întrerupere: pierdem 3 celule din 9

1 scurtcircuit: pierdem 3/9

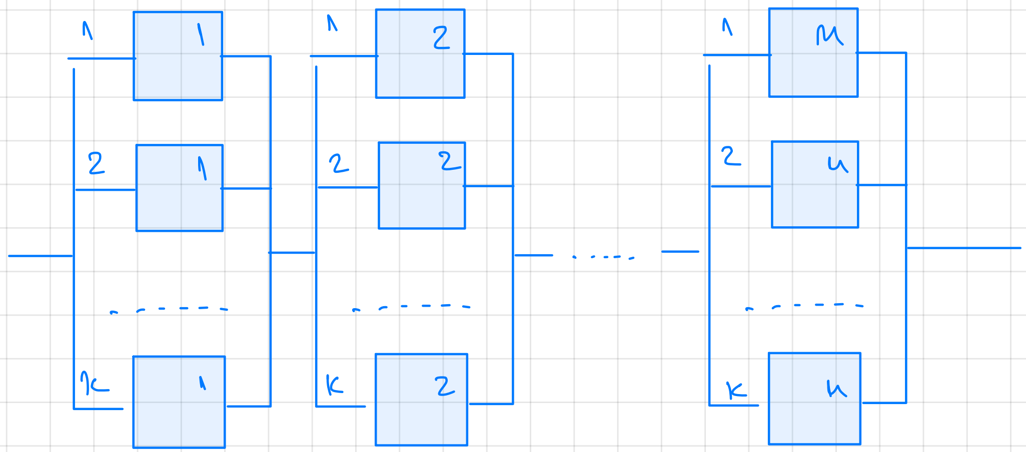
2 întrerupere: pierdem 1/9

Caz general k × n module cu fiabilități identice R

serie - paralel



paralel - serie

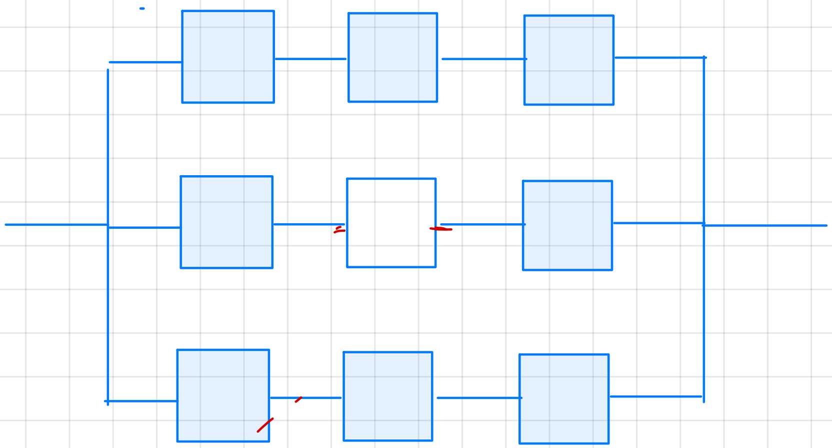


$$R_{SP} = \underbrace{R^n \| R^n \| \ldots \| R^n}_{k \text{ ori}} = 1 - (1 - R^n)^k$$

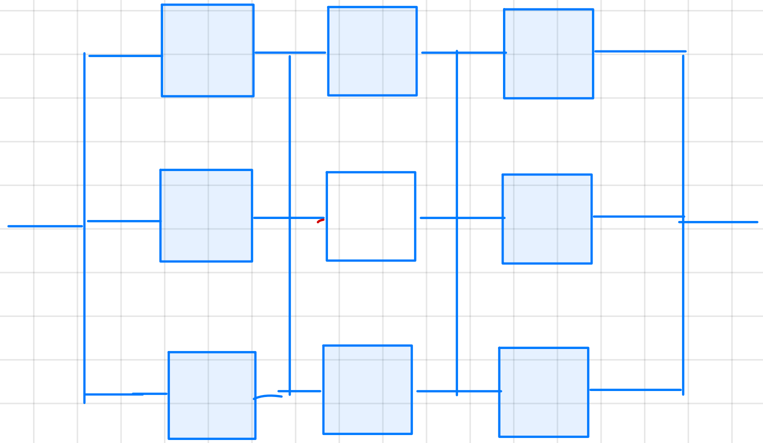
$$R_{PS} = \left[1 - (1 - R)^k\right]^n$$

Exemple :

S - P



P - S

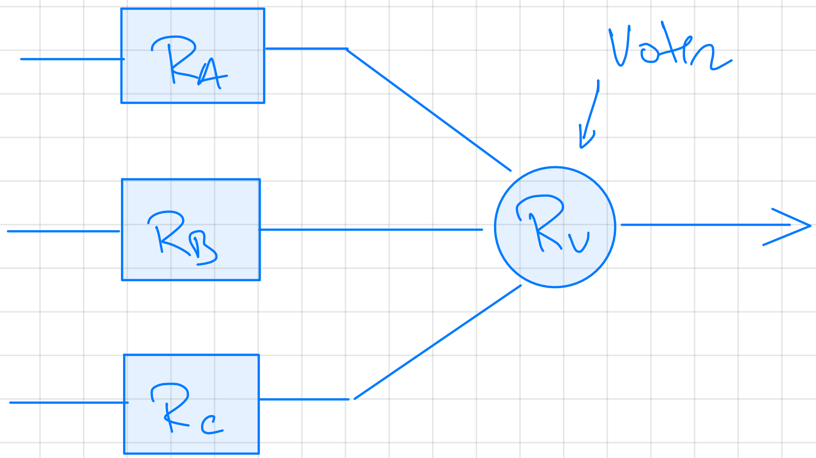


$R = 90\%$ $\quad n = 3$

$R_{SP} = 0,9^3 \,||\, 0,9^3 \,||\, 0,9^3 = 1 - (1 - 0,9^3)^3 = 1 - (1 - 0,729)^3 = 0,98 \; (98\%)$

$R_{PS} = \left[1 - (1 - 0,9)^3\right]^3 = (1 - 0,1^3)^3 = (1 - 0,001)^3 = 0,999^3 = 0,997 \; (99,7\%)$

Structuri cu votare majoritară



$$R_{2/3} = R_V \cdot \left[R_A R_B R_C + (1-R_A) R_B R_C + R_A (1-R_B) R_C + R_A R_B (1-R_C) \right]$$

de obicei: $R_V \gg R_A, R_B, R_C$

$$R_V \simeq 1$$

$R_A = R_B = R_C = R$ și $R_V = 1 \Rightarrow R_{2/3} = R^3 + 3R^2(1-R) = R^3 + 3R^2 - 3R^3 \Rightarrow$

$$\Rightarrow R_{2/3} = 3R^2 - 2R^3$$

dacă $R = 99\%$ $\rightarrow$ $R_{2/3} = 3 \cdot 0{,}99^2 - 2 \cdot 0{,}99^3 = 3 \cdot 0{,}98 - 2 \cdot 0{,}97 =$

$= 0{,}9997\ldots$ $(99{,}97\%)$

dacă $R = 10\%$ $\Rightarrow$ $R_{2/3} = 3 \cdot 0{,}1^2 - 2 \cdot 0{,}1^3 = 0{,}028$ $(2{,}8\%)$

$$R_{3/5} = R^5 + 5(1-R)R^4 + 10(1-R)^2R^3 +$$

$$R_V = 1$$

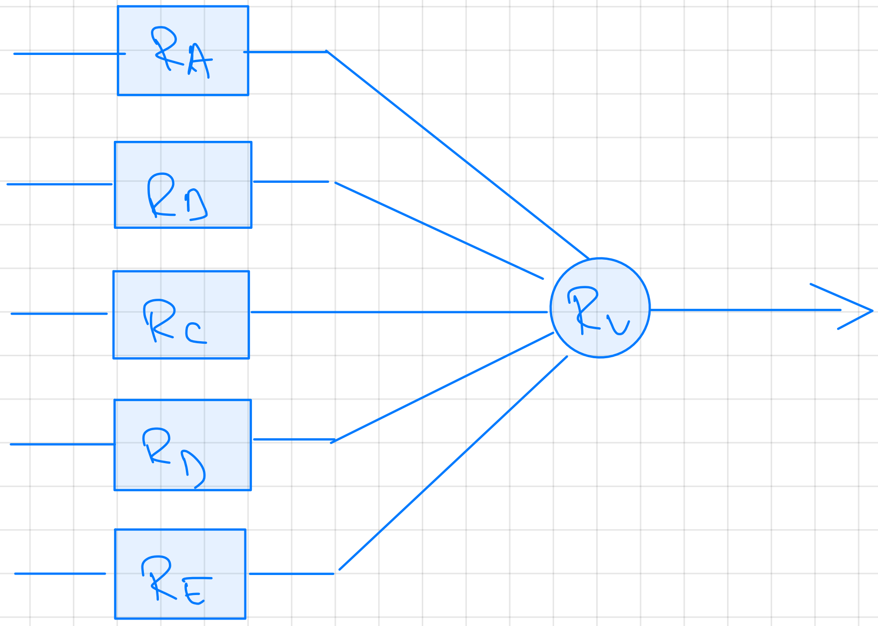
$$R_A = R_B = \dots R_E = R$$

$$R_{3/5} = R^5 + 5R^4 - 5R^5 + 10(1-2R+R^2)R^3 =$$

$$= 5R^4 - 4R^5 + 10R^3 - 20R^4 + 10R^5 =$$

$$= 6R^5 - 15R^4 + 10R^3$$

$$R_{3/5} > R \quad ?$$



Caz general $R_{n|2n-1}$

$$R_{n|2n-1} = \sum_{i=0}^{n-1} C_{2n-1}^{i} R^{2n-1-i}(1-R)^{i}$$

$$MTBF_{2|3} = \int_0^\infty R_{2|3}(t)\,dt$$

$$R_{2|3}(t) = 3R^2(t) - 2R^3(t) \qquad R(t) = e^{-\lambda t} \qquad MTBF = \frac{1}{\lambda}$$

$$R_{2|3}(t) = 3e^{-2\lambda t} - 2e^{-3\lambda t} \qquad MTBF_{2|3} = \int_0^\infty \left(3e^{-2\lambda t} - 2e^{-3\lambda t}\right)dt =$$

$$= \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{5}{6}\cdot\frac{1}{\lambda} = \frac{5}{6}\cdot MTBF < MTBF$$

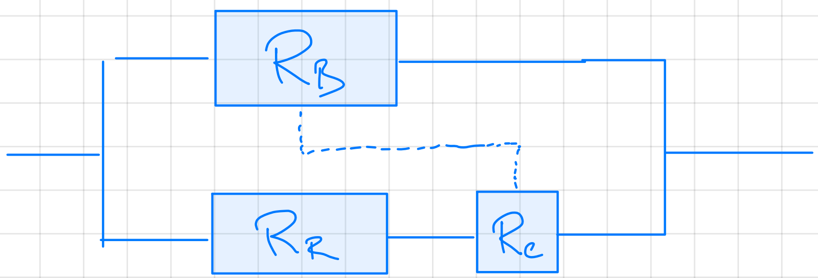
$$R_{3|5}(t) = 6e^{-5\lambda t} - 15e^{-4\lambda t} + 10e^{-3\lambda t}$$

$$MTBF_{3|5} = \int_0^\infty R_{3|5}(t)\,dt = \frac{6}{5\lambda} - 15\cdot\frac{1}{4\lambda} + 10\cdot\frac{1}{3\lambda} = (72 - 225 + 200)\frac{1}{60\lambda} =$$

$$= \frac{47}{60}\cdot\frac{1}{\lambda} = \frac{47}{60}\cdot MTBF = 0{,}78\ MTBF < MTBF$$

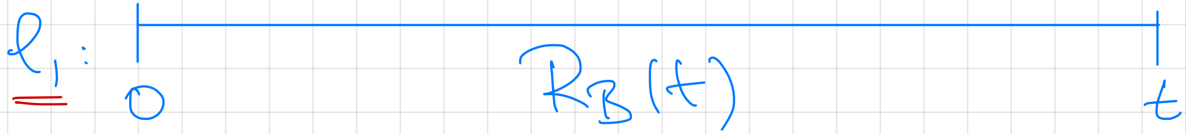
Structuri cu un singur element de rezervă

$$R_{TOTAL} = P(e_1) + P(e_2)$$

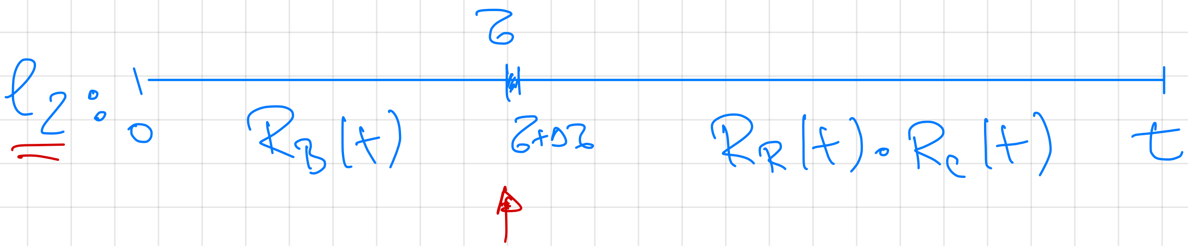


e_1: baza funcționează în $(0,t)$

$$P(e_1) = R_B(t)$$



e_2: baza funcționează $[0, \tau)$
și elem. rezervă comută în
stare de funcționare



$P_b \rightarrow$ prob. ca elem. de bază să se defecteze în $[\tau, \tau + d\tau]$

$$P_b = f_b(\tau)\, d\tau, \quad \text{unde } f_b(\tau) = \frac{dF_b(\tau)}{d\tau} = -\frac{dR_B(\tau)}{d\tau}$$

$P_r \rightarrow$ prob ca elem de rezervă să funcționeze în (τ, t)

$$P_r(t) = R_c(t-\tau) \cdot R_R(t-\tau) \;\Big|\; \text{pp. că } R_c \simeq 1 \Rightarrow P_r(t) = R_R(t-\tau)$$

τ - se produce oricând in $[0, t)$

$$P(e_2) = R_{e_{2 \leftarrow 1}} \cdot \int_0^t p_b(\tau) p_r(\tau) d\tau = \int_0^t -\frac{dR_B(\tau)}{d\tau} \cdot R_R(t-\tau) d\tau$$

$$P(e_2) = -\int_0^t \frac{dR_B(\tau)}{d\tau} R_R(t-\tau) d\tau$$

$$R_{TOTAL} = R_B(t) - \int_0^t \frac{dR_B(\tau)}{d\tau} R_R(t-\tau) d\tau$$

$$R_B(t) = e^{-\lambda_B t} \qquad R_R(t) = e^{-\lambda_R t} \qquad \left(e^{ax}\right)' = a e^{ax}$$

$$R_{TOTAL}(t) = e^{-\lambda_B t} - \int_0^t (-\lambda_B) e^{-\lambda_B \tau} e^{-\lambda_R (t-\tau)} d\tau =$$

$$= e^{-\lambda_B t} + \lambda_B \int_0^t e^{-\lambda_B \tau} \cdot e^{-\lambda_R t} \cdot e^{\lambda_R \tau} d\tau = e^{-\lambda_B t} + \lambda_B e^{-\lambda_R t} \int_0^t e^{-(\lambda_B - \lambda_R)\tau} d\tau =$$

$$= e^{-\lambda_B t} + \lambda_B e^{-\lambda_R t} \frac{1}{\lambda_R - \lambda_B} e^{(\lambda_R - \lambda_B)\tau} \Big|_0^t = e^{-\lambda_B t} + \frac{\lambda_B}{\lambda_R - \lambda_B} e^{-\lambda_R t} \cdot \left(e^{(\lambda_R - \lambda_B)t} - 1\right)$$

$$\Rightarrow R_{TOTAL}(t) = \frac{\lambda_R}{\lambda_R - \lambda_B} R_B(t) - \frac{\lambda_B}{\lambda_R - \lambda_B} R_R(t) > R_B(t) \, ?$$

$$MTBF_{TOTAL} = ? = \int_0^\infty R_{TOTAL}(t)\,dt = \int_0^\infty \left(\frac{\lambda_R}{\lambda_R-\lambda_B} e^{-\lambda_B t} - \frac{\lambda_B}{\lambda_R-\lambda_B} e^{-\lambda_R t} \right) dt$$

$$= \frac{\lambda_R}{\lambda_R-\lambda_B} \cdot \int_0^\infty e^{-\lambda_B t}\,dt - \frac{\lambda_B}{\lambda_R-\lambda_B} \int_0^\infty e^{-\lambda_R t}\,dt = \frac{\cancel{\lambda_R}\,\lambda_R}{\lambda_B(\lambda_R-\lambda_B)} - \frac{\cancel{\lambda_B}\,\lambda_B}{\lambda_R(\lambda_R-\lambda_B)} =$$

$$= \frac{\lambda_R^2 - \lambda_B^2}{\lambda_R\lambda_B(\lambda_R-\lambda_B)} = \frac{\lambda_R+\lambda_B}{\lambda_R\lambda_B} = \frac{1}{\lambda_B} + \frac{1}{\lambda_R} = MTBF_B + MTBF_R$$

Dacă $R_B(t) = R_R(t) = e^{-\lambda t}$

$$R_{TOTAL}(t) = e^{-\lambda t} - \int_0^t (-\lambda) e^{-\lambda z} e^{-\lambda(t-z)}\,dz = e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t dz =$$

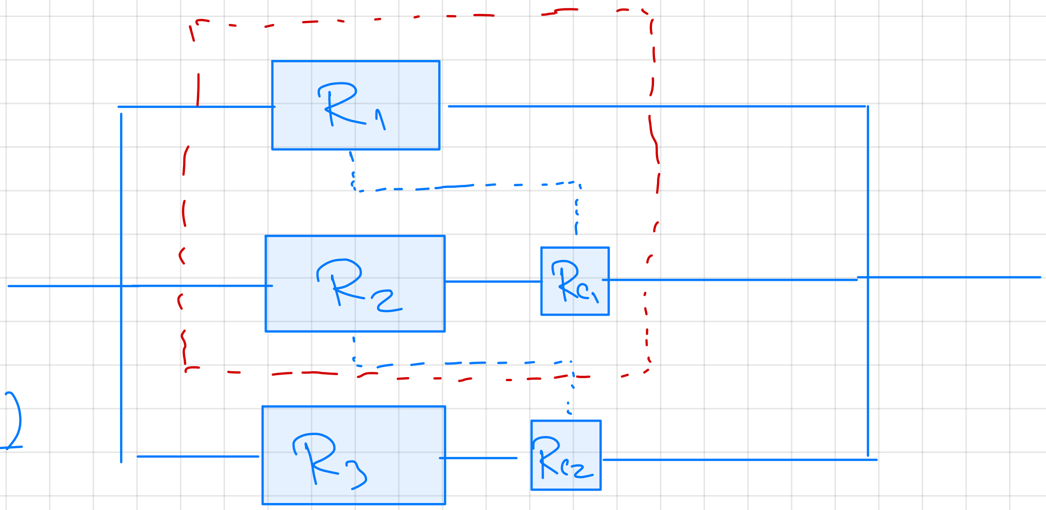
$$= e^{-\lambda t} + \lambda e^{-\lambda t}\, t = e^{-\lambda t}(1+\lambda t)$$

$$MTBF_{TOTAL} = 2 \cdot MTBF = \frac{2}{\lambda}$$

$$R_{TOTAL}(t) > R(t) \ ? \Rightarrow \cancel{e^{-\lambda t}}(1+\lambda t) > \cancel{e^{-\lambda t}} \Rightarrow 1+\lambda t > 1 \ (\forall)\ t \geq 0$$

$$\lambda t > 0$$

Structură cu două elemente de rezervă



$$R_{12}(t) = R_1(t) + \int_0^t f_1(\tau) R_2(t-\tau)\, d\tau$$

$$R_{123}(t) = R_{12}(t) + \int_0^t f_{12}(\tau) R_3(t-\tau)\, d\tau$$

$$f_{12}(\tau) = -\frac{dR_{12}(\tau)}{d\tau} \qquad f_1(\tau) = -\frac{dR_1(\tau)}{d\tau}$$

$$R_1(t) = R_2(t) = R_3(t) = e^{-\lambda t}$$

$$R_{12}(t) = e^{-\lambda t}(1+\lambda t), \quad f_{12}(\tau) = -\frac{dR_{12}(\tau)}{d\tau} = -\frac{d\, e^{-\lambda\tau}(1+\lambda\tau)}{d\tau} = \lambda^2 \tau e^{-\lambda\tau}$$

$$R_{123}(t) = e^{-\lambda t}(1+\lambda t) + \int_0^t (+\lambda^2 \tau e^{-\lambda\tau}) \cdot e^{-\lambda(t-\tau)}\, d\tau =$$

$$= e^{-\lambda t}(1+\lambda t) + \lambda^2 e^{-\lambda t}\int_0^t \tau\, d\tau = e^{-\lambda t}(1+\lambda t) + \frac{\lambda^2 t^2}{2} \cdot e^{-\lambda t} =$$

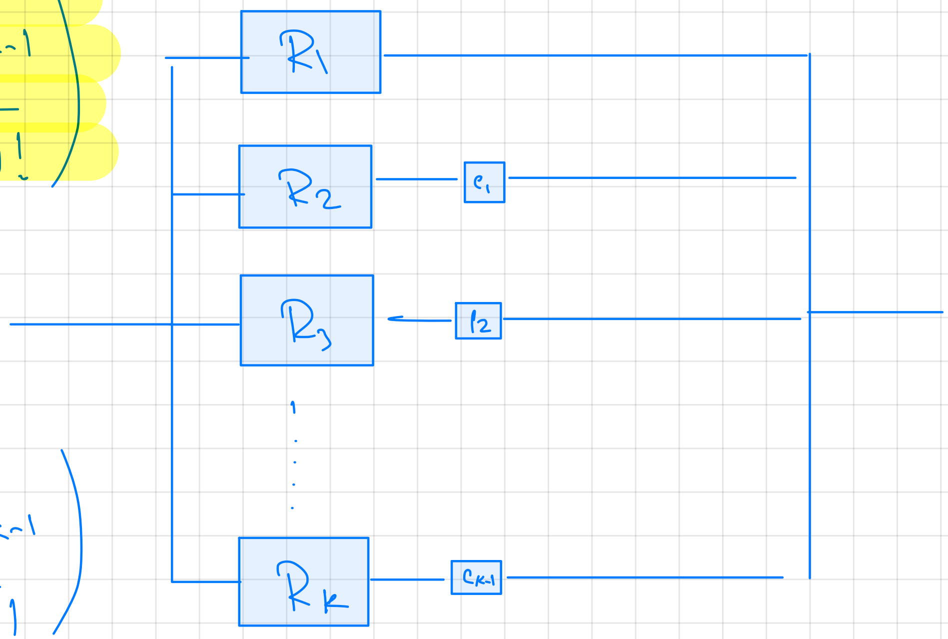
$$= e^{-\lambda t}\left(1 + \lambda t + \frac{\lambda^2 t^2}{2}\right)$$

$$MTBF_{123} = \int_0^{\infty} R_{123}(t)\,dt = 3 \cdot \frac{1}{\lambda} = 3 \cdot MTBF$$

Avem $(k-1)$ elemente de rezervă identice!

$$R_1(t) = R_2(t) = \dots R_k(t) = R(t) = e^{-\lambda t}$$

$$R_{TOTAL}(t) = e^{-\lambda t} \cdot \left(1 + \lambda t + \frac{\lambda^2 t^2}{2} + \frac{\lambda^3 t^3}{6} + \dots \frac{\lambda^{k-1} t^{k-1}}{(k-1)!}\right)$$



Dar dacă $k \to \infty$ (avem f. multe elem. de back-up)

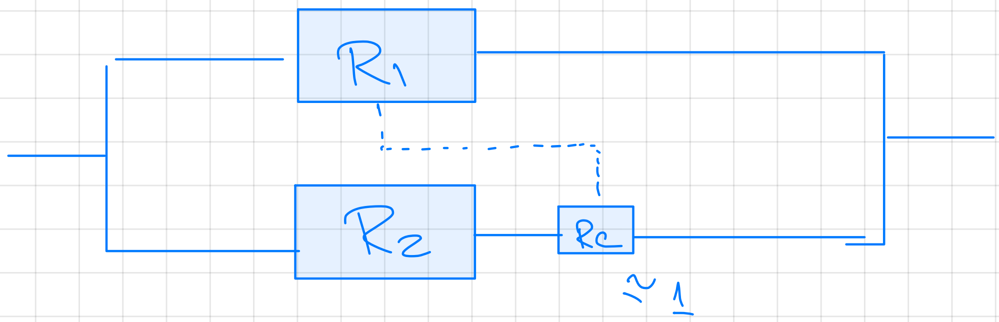
$$R_{TOTAL}(t) = \lim_{k \to \infty} e^{-\lambda t}\left(1 + \lambda t + \frac{\lambda^2 t^2}{2} + \dots \frac{\lambda^{k-1} t^{k-1}}{(k-1)!}\right)$$

$$= e^{-\lambda t} \lim_{k \to \infty} \sum_{i=0}^{k-1} \frac{(\lambda t)^i}{i!} = e^{-\lambda t} \cdot e^{\lambda t} = 1$$

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \frac{x^n}{n!} + \dots = e^x$$

Structură cu element de rezervă activ

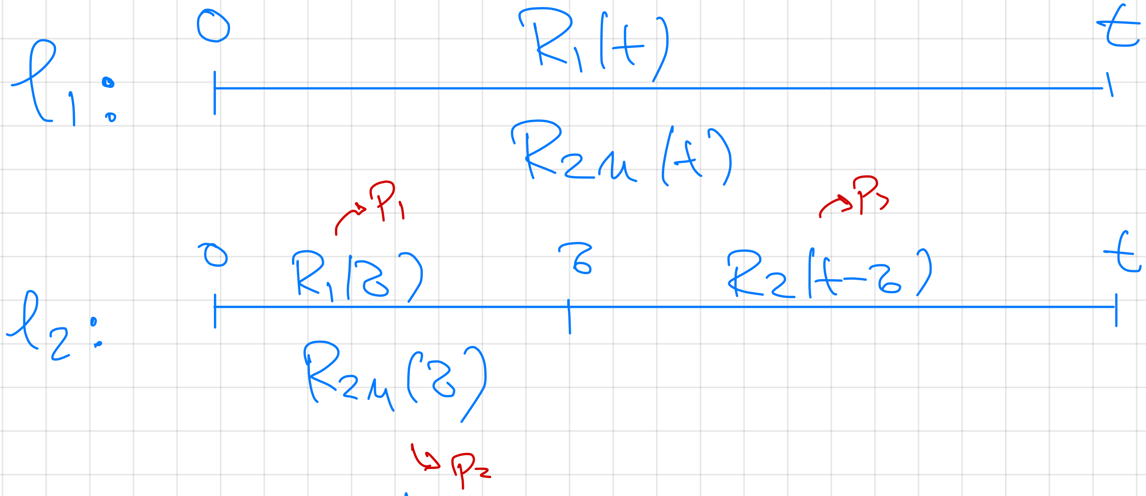
$$R_{TOTAL} = P(e_1) + P(e_2)$$



$$P(e_1) = R_1(t)$$

$$P(e_2) = \int_0^t p_1 \cdot p_2 \cdot p_3 \, d\tau =$$

$$= \int_0^t f_1(\tau) \cdot R_{2m}(\tau) \cdot R_2(t-\tau)\, d\tau$$



$$f_1(\tau) = -\frac{dR_1(\tau)}{d\tau}$$

$$R_1(t) = e^{-\lambda_1 t} \qquad R_2(t) = e^{-\lambda_2 t} \qquad R_{2m}(t) = e^{-\lambda_{2m} t}$$

$$R_{TOTAL}(t) = e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 + \lambda_{2m} - \lambda_2}\left[e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_{2m})t}\right]$$

dacă modulele sunt identice $\Rightarrow R_1 = R_2 = e^{-\lambda t}$ $\quad R_{2m} = e^{-\lambda_m t}$

$$R_{TOTAL}(t) = e^{-\lambda t}\left(1 + \frac{\lambda}{\lambda_m}\left(1 - e^{-\lambda_m t}\right)\right)$$

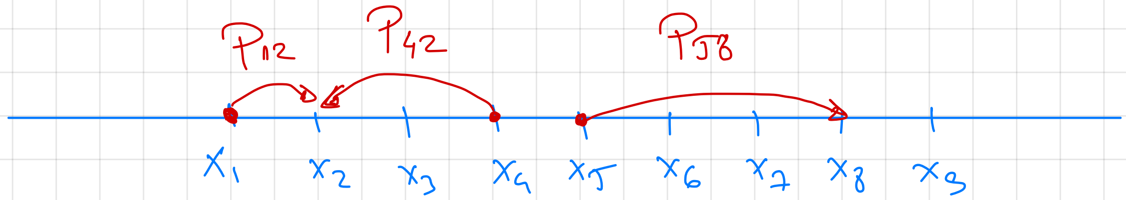
$$MTBF = \frac{1}{\lambda} + \frac{1}{\lambda + \lambda_m}$$

Modele Markov

două variabile:
- Starea sistemului: $(x_1, x_2 \dots x_n)$
- timpul de observare: $(t_1, t_2 \dots t_n)$

Lanțuri Markov

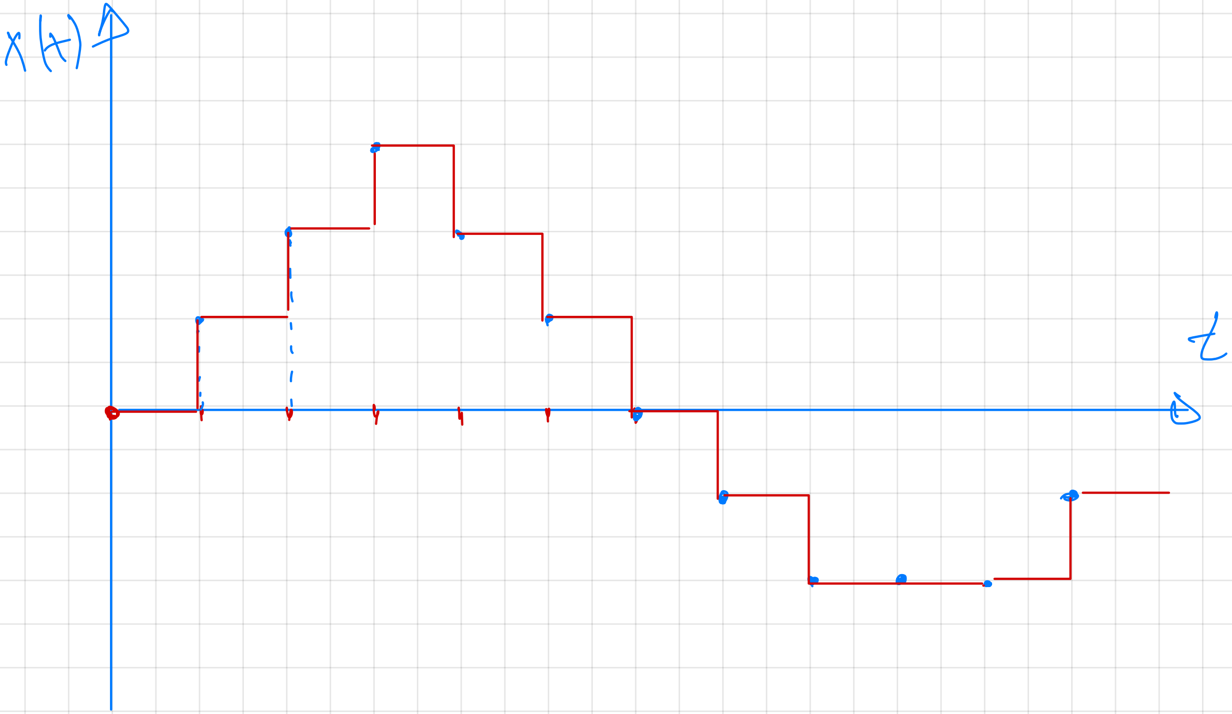
$X(1), X(2) \dots X(n) \dots$



$$P_i(k) = P(X(k) = S_i)$$

$$\sum_{i=1}^{n} P_i(k) = 1$$

$$P_{ij} = P(X(k) = S_i,\ X(k+1) = S_j)$$



n stări → P(n×n) prob de tranziție din o stare în orice altă stare

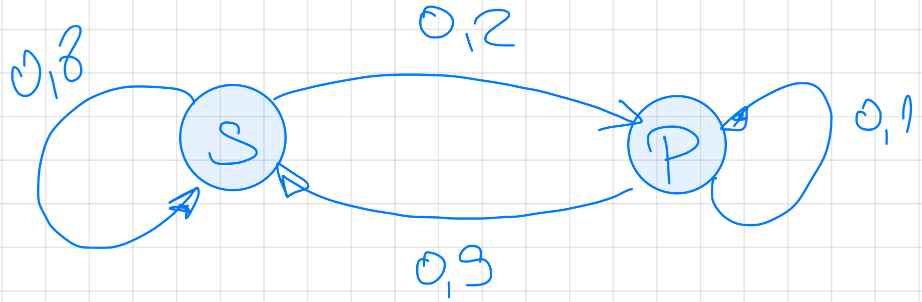
$$\begin{pmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & P_{n3} & \cdots & P_{nn} \end{pmatrix} = A$$

$$p_i(k) = \sum_{j=1}^{n} p_j(k-1) P_{ji}$$

$$\begin{pmatrix} p_1(t) & p_2(t) & \dots & p_n(t) \end{pmatrix} \cdot A = \begin{pmatrix} p_1(t+\Delta t) & p_2(t+\Delta t) & \dots & p_n(t+\Delta t) \end{pmatrix}$$

Ex: starea vremii

stări: < Soare (S), Ploaie (P)



	S	P
S	0,8	0,2
P	0,9	0,1

$$A = \begin{pmatrix} 0,8 & 0,2 \\ 0,9 & 0,1 \end{pmatrix}$$

$$\begin{array}{cc} S & P \\ \end{array}$$
$$\begin{pmatrix} 1 & 0 \end{pmatrix} \cdot A = \underset{\text{(azi)}}{\begin{pmatrix} 1 & 0 \end{pmatrix}} \begin{pmatrix} 0,8 & 0,2 \\ 0,9 & 0,1 \end{pmatrix} = \underset{\text{(mâine)}}{\begin{pmatrix} 0,8 & 0,2 \end{pmatrix}} \begin{pmatrix} 0,8 & 0,2 \\ 0,9 & 0,1 \end{pmatrix} = \underset{\text{(poimâine)}}{\begin{pmatrix} 0,82 & 0,18 \end{pmatrix}}$$

la steady state: $(q_1\ q_2)\, A = (q_1\ q_2)\, I_2 \Rightarrow$

$$\Rightarrow (q_1\ q_2)(A - I_2) = (0\ 0) \Rightarrow (q_1\ q_2)\begin{pmatrix} -0,2 & 0,2 \\ 0,9 & -0,9 \end{pmatrix} = (0\ 0) \Rightarrow -0,2q_1 + 0,9q_2 = 0$$

$$2q_1 - 9q_2 = 0$$

$$\begin{cases} 2q_1 - 9q_2 = 0 \\ q_1 + q_2 = 1 \end{cases} \Rightarrow q_1 = \frac{9}{11} \text{ și } q_2 = \frac{2}{11}$$

Procese Markov

$$S_i \xrightarrow{P_{ij}} S_j$$

$\lambda_{ij} \to$ densitate de probabilitate de tranziție

$$\lambda_{ij} = \lim_{\Delta t \to 0} \frac{P\{X(t)=S_i \mid X(t+\Delta t)=S_j\}}{\Delta t}$$

dacă $\Delta t \to 0 \Rightarrow \boxed{P_{ij} \simeq \lambda_{ij} \cdot \Delta t}$

$$P(t) = (P_1(t) \quad P_2(t) \ \dots \ P_n(t)) \qquad P(t) \cdot A = P(t+\Delta t)$$

$$A = \begin{pmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ - & - & - & - \\ P_{n1} & P_{n2} & \dots & P_{nn} \end{pmatrix} = \begin{pmatrix} \lambda_{11}\Delta t & \lambda_{12}\Delta t & \dots & \lambda_{1n}\Delta t \\ \lambda_{21}\Delta t & \lambda_{22}\Delta t & \dots & \lambda_{2n}\Delta t \\ - & - & - & - \\ \lambda_{n1}\Delta t & \lambda_{n2}\Delta t & \dots & \lambda_{nn}\Delta t \end{pmatrix} = \begin{pmatrix} 1-\sum\limits_{i=2}^{n}\lambda_{1i}\Delta t & \lambda_{12}\Delta t & \dots & \lambda_{1n}\Delta t \\ \lambda_{21}\Delta t & 1-\sum\limits_{\substack{i=1 \\ i\neq 2}}^{n}\lambda_{2i}\Delta t & \dots & \lambda_{2n}\Delta t \\ - & - & - & - \\ \lambda_{n1}\Delta t & \lambda_{n2}\Delta t & \dots & 1-\sum\limits_{i=1}^{n-1}\lambda_{ni}\Delta t \end{pmatrix}$$

$$P(t)\cdot A = \left(P_1(t)\;\; P_2(t)\; \dots\; P_n(t)\right)\cdot \begin{pmatrix} 1-\sum_{i=2}^{n}\lambda_{1i}\Delta t & \lambda_{12}\Delta t & \dots & \lambda_{1n}\Delta t \\ \lambda_{21}\Delta t & 1-\sum_{\substack{i=1\\ i\neq 2}}^{n}\lambda_{2i}\Delta t & \dots & \lambda_{2n}\Delta t \\ - & - & - & - \\ \lambda_{n1}\Delta t & \lambda_{n2}\Delta t & \dots & 1-\sum_{i=1}^{n-1}\lambda_{ni}\Delta t \end{pmatrix}$$

$$P_1(t+\Delta t) = P_1(t)\left(1-\sum_{i=2}^{n}\lambda_{1i}\Delta t\right) + P_2(t)\lambda_{21}\Delta t + \dots P_n(t)\lambda_{n1}\Delta t$$

$$P_1(t+\Delta t) - P_1(t) = -P_1(t)\sum_{i=2}^{n}\lambda_{1i}\Delta t + P_2(t)\lambda_{21}\Delta t + \dots P_n(t)\lambda_{n1}\Delta t \;/ :\Delta t$$

$$\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t} = -P_1(t)\sum_{i=2}^{n}\lambda_{1i} + P_2(t)\lambda_{21} + \dots P_n(t)\lambda_{n1}$$

$$\lim_{\Delta t\to 0}\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t} = -P_1(t)\sum_{i=2}^{n}\lambda_{1i} + P_2(t)\lambda_{21} + \dots P_n(t)\lambda_{n1}$$

$$\frac{dP_1(t)}{dt} = \sum_{i=2}^{n} P_i(t)\lambda_{i1} - P_1(t)\sum_{i=2}^{n}\lambda_{1i}$$

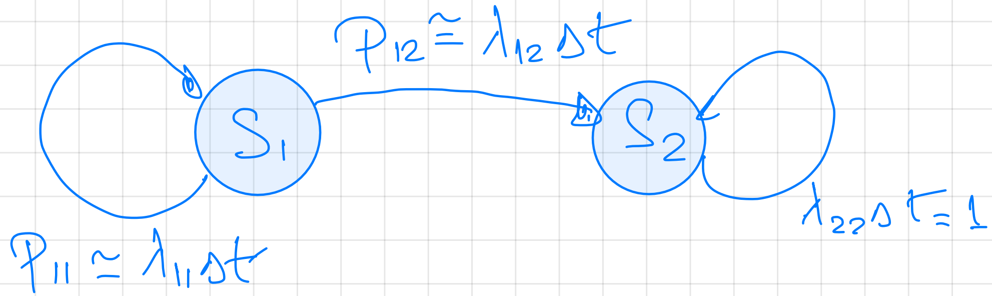
$$\frac{dP_j(t)}{dt} = \sum_{\substack{i=1\\ i\neq j}}^{n} P_i(t)\lambda_{ij} - P_j(t)\sum_{\substack{i=1\\ i\neq j}}^{n}\lambda_{ji}, \ (\forall)\ j=\overline{1,n}$$

Sistem de ecuații Chapman – Kolmogorov (C-K)

Formă matriceală

$[P(t)]\cdot A^{*} = [P'(t)]$, unde $A^{*} = \begin{pmatrix} -\sum_{i=2}^{n}\lambda_{1i} & \lambda_{12} & \lambda_{13} & \dots & \lambda_{1n} \\ \lambda_{21} & -\sum_{\substack{i=1\\ i\neq 2}}^{n}\lambda_{2i} & \lambda_{23} & \dots & \lambda_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \lambda_{n1} & \lambda_{n2} & \lambda_{n3} & \dots & -\sum_{i=1}^{n-1}\lambda_{ni} \end{pmatrix}$

Sistem cu două stări



$S_1 \to$ funcționare

$S_2 \to$ defect

$$p_{11}(t) + p_{12}(t) = 1 \Rightarrow \lambda_{11}\Delta t + \lambda_{12}\Delta t = 1 \Rightarrow \lambda_{11}\Delta t = 1 - \lambda_{12}\Delta t$$

$$[P(t)] \cdot A = [P(t+\Delta t)] \Rightarrow \begin{pmatrix} P_1(t) & P_2(t) \end{pmatrix} \cdot \begin{pmatrix} \lambda_{11}\Delta t & \lambda_{12}\Delta t \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} P_1(t+\Delta t) & P_2(t+\Delta t) \end{pmatrix}$$

$$[P(t)] \cdot A^* = [P'(t)] \Leftrightarrow \begin{pmatrix} P_1(t) & P_2(t) \end{pmatrix} \cdot \begin{pmatrix} -\lambda_{12} & \lambda_{12} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} P_1'(t) & P_2'(t) \end{pmatrix}$$

$$\begin{cases} P_1'(t) = -P_1(t)\,\lambda_{12} \\ P_2'(t) = P_1(t)\,\lambda_{12} \end{cases}$$

$$P_1'(t) = -\lambda_{12} P_1(t) \iff \frac{d P_1(t)}{dt} = -\lambda_{12} P_1(t) \Rightarrow$$

$$\Rightarrow \int \frac{1}{P_1(t)}\, dP_1(t) = \int -\lambda_{12}\, dt \Rightarrow \ln(P_1(t)) = -\lambda_{12} t + C$$

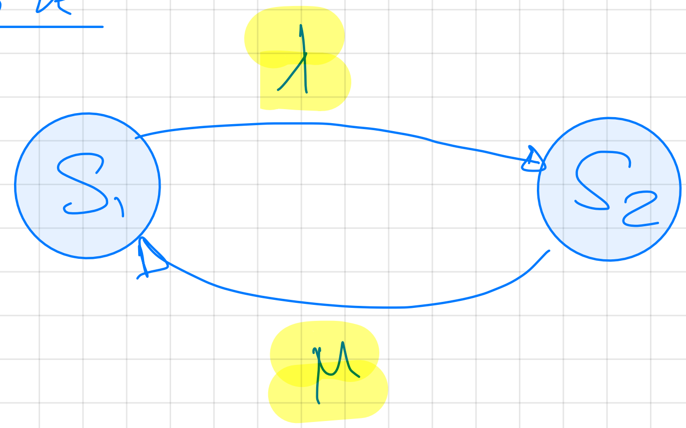
$$P_1(t) = e^{-\lambda_{12} t}$$

$$P_2(t) = 1 - P_1(t) = 1 - e^{-\lambda_{12} t}$$

Sistem cu două stări + reparare

S_1 – funcționare

S_2 – defect



$$A^* = \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix}$$

$$[P(t)] \cdot A^* = [P'(t)] \Rightarrow \begin{pmatrix} P_1(t) & P_2(t) \end{pmatrix} \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix} = \begin{pmatrix} P_1'(t) & P_2'(t) \end{pmatrix}$$

$$\begin{cases} \dfrac{dP_1(t)}{dt} = -\lambda P_1(t) + \mu P_2(t) \Rightarrow \dfrac{dP_1(t)}{dt} = -\lambda P_1(t) + \mu\left(1 - P_1(t)\right) \Rightarrow \\[2ex] \dfrac{dP_2(t)}{dt} = \lambda P_1(t) - \mu P_2(t) \\[2ex] P_1(t) + P_2(t) = 1 \end{cases}$$

$$\frac{dP_1(t)}{dt} + (\lambda+\mu) P_1(t) = \mu \longrightarrow P_1(t) = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda} e^{-(\lambda+\mu)t}$$

$$\lim_{t\to\infty} P_1(t) = \frac{\mu}{\mu+\lambda}^{(\cdot\mu\lambda)} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}+\frac{1}{\mu}} = \frac{MTBF}{MTBF+MTTR} = A$$

disponibilità dei sistemi

$$\underbrace{P_1'(t)}_{y'(t)} + \underbrace{(\lambda+\mu)}_{p(t)} \underbrace{P_1(t)}_{y(t)} = \underbrace{\mu}_{g(t)}$$

$$\frac{dy(t)}{dt} + p(t)\,y(t) = g(t) \rightarrow$$ ec. differenziale lineare di gr. 1

$$\mu(t) \rightarrow \mu(t)\,p(t) = \mu'(t) \iff \frac{\mu'(t)}{\mu(t)} = p(t) \Big| \int \Rightarrow \int \frac{1}{\mu(t)}\,\mu'(t) = \int p(t)\,dt$$

$$\ln(\mu(t)) = \int p(t)\,dt + k \Rightarrow \mu(t) = e^{\int p(t)\,dt + k} = e^{k} \cdot e^{\int p(t)\,dt} \Rightarrow$$

$$\mu(t) = k \cdot e^{\int p(t)\,dt}$$

$$\mu(t)\,y'(t) + \mu(t)\,p(t)\,y(t) = \mu(t)\,g(t) \iff$$

$$\mu(t)\,y'(t) + \mu'(t)\,y(t) = \mu(t)\,g(t) \iff \left(\mu(t)\,y(t)\right)' = \mu(t)\,g(t)$$

$$\int \left(\mu(t)\,y(t)\right)' dt = \int \mu(t)\,g(t)\,dt$$

$$\mu(t)\,y(t) + C = \int \mu(t)\,g(t)\,dt \Rightarrow y(t) = \frac{\int \mu(t)\,g(t)\,dt + C}{\mu(t)}$$

$$y(t) = \frac{\int K e^{\int p(t)dt} \cdot g(t)\,dt + C}{K \cdot e^{\int p(t)dt}} = \frac{\int e^{\int p(t)dt} g(t)dt + K}{e^{\int p(t)dt}}$$

$$y'(t) + p(t)\,y(t) = g(t) \implies$$

$$y(t) = \frac{\int e^{\int p(t)dt} g(t)\,dt + K}{e^{\int p(t)dt}}$$

$$\underbrace{P_1'(t)}_{y'(t)} + \underbrace{(\lambda+\mu)}_{p(t)}\underbrace{P_1(t)}_{y(t)} = \underbrace{\mu}_{g(t)}$$

$$P_1(t) = \frac{\int e^{\int(\lambda+\mu)dt}\,\mu\,dt + K}{e^{\int(\lambda+\mu)dt}} =$$

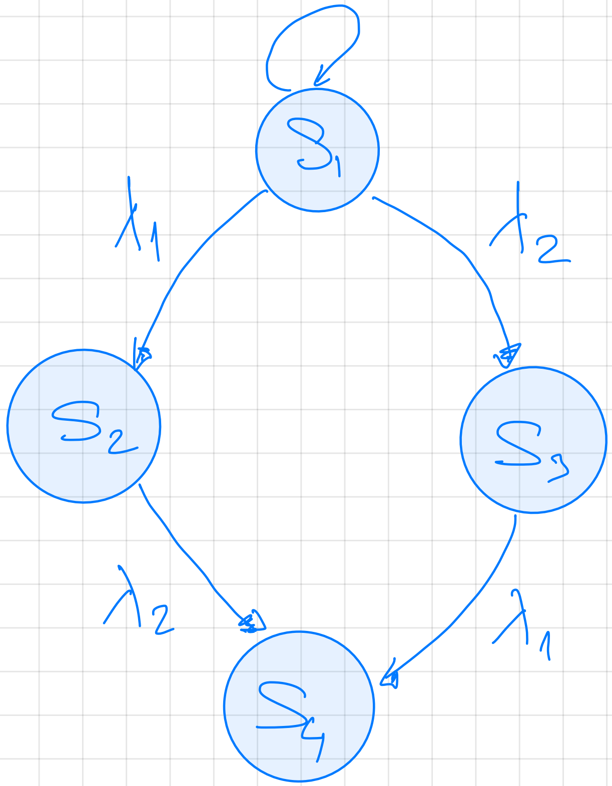
$$= \frac{\mu\int e^{(\lambda+\mu)t} + K}{e^{(\lambda+\mu)t}} = \frac{\mu\,\frac{1}{\lambda+\mu}\cdot e^{(\lambda+\mu)t} + K}{e^{(\lambda+\mu)t}}$$

$$P_1(0) = 1 \Rightarrow \frac{\mu}{\lambda+\mu} + K = 1 \Rightarrow K = 1 - \frac{\mu}{\lambda+\mu} = \frac{\lambda}{\lambda+\mu}$$

$$P_1(t) = \frac{\frac{\mu}{\lambda+\mu} \cdot e^{(\lambda+\mu)t} + \frac{\lambda}{\lambda+\mu}}{e^{(\lambda+\mu)t}} = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

Sistem cu două componente

Stare	Comp 1.	Comp. 2
S_1	Op.	Op.
S_2	F	Op.
S_3	Op.	F
S_4	F	F



$$A^* = \begin{pmatrix} -\lambda_1-\lambda_2 & \lambda_1 & \lambda_2 & 0 \\ 0 & -\lambda_2 & 0 & \lambda_2 \\ 0 & 0 & -\lambda_1 & \lambda_1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\left[P_1(t) \quad P_2(t) \quad P_3(t) \quad P_4(t)\right] \cdot A^* = \left[P_1'(t) \quad P_2'(t) \quad P_3'(t) \quad P_4'(t)\right]$$

$$P_1(t)(-\lambda_1-\lambda_2) = P_1'(t) = \frac{dP_1(t)}{dt} \Rightarrow \frac{1}{P_1(t)} dP_1(t) = -(\lambda_1+\lambda_2)dt \;\Big|\int \Rightarrow$$

$$\int \frac{1}{P_1(t)}\, dP_1(t) = -\int (\lambda_1+\lambda_2)\, dt \Rightarrow \ln(P_1(t)) = -(\lambda_1+\lambda_2)t \Rightarrow$$

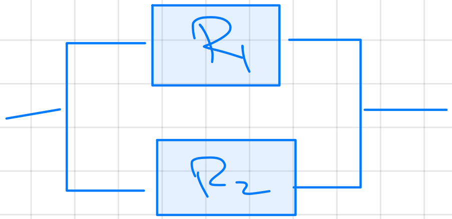
$$\Rightarrow P_1(t) = e^{-(\lambda_1+\lambda_2)t} = e^{-\lambda_1 t}\cdot e^{-\lambda_2 t} = R_1(t)\cdot R_2(t)$$

$$\lambda_1 P_1(t) - \lambda_2 P_2(t) = \frac{dP_2(t)}{dt} \Rightarrow \lambda_1 e^{-(\lambda_1+\lambda_2)t} - \lambda_2 P_2(t) = P_2'(t) \Rightarrow$$

$$\Rightarrow \dots \Rightarrow P_2(t) = e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

$$\lambda_2 P_1(t) - \lambda_1 P_3(t) = P_3'(t) \Rightarrow \dots\ P_3(t) = e^{-\lambda_1 t} - e^{-(\lambda_1+\lambda_2)t}$$

$$P_4(t) = 1 - P_1(t) - P_2(t) - P_3(t)$$



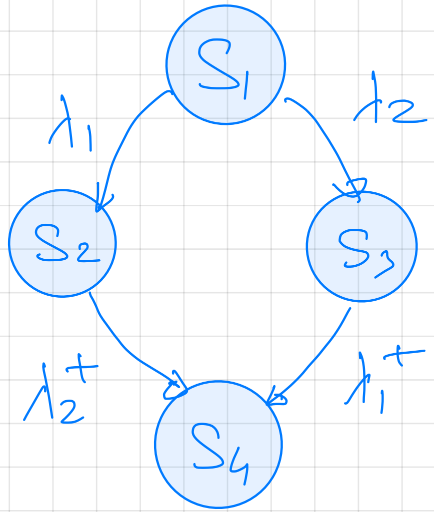
Serie $= P_1(t) = e^{-(\lambda_1+\lambda_2)t}$

Parallel $= P_1(t) + P_2(t) + P_3(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t} = R_1(t) + R_2(t) - R_1(t)R_2(t)$

$$= 1 - (1 - R_1(t))(1 - R_2(t))$$

Load sharing system

$$A^{+} = \begin{pmatrix} -\lambda_1-\lambda_2 & \lambda_1 & \lambda_2 & 0 \\ 0 & -\lambda_2^{+} & 0 & \lambda_2^{+} \\ 0 & 0 & -\lambda_1^{+} & \lambda_1^{+} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



$$P_1(t)(-\lambda_1-\lambda_2) = P_1'(t) \Rightarrow P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$

$$\lambda_1 P_1(t) - \lambda_2^{+} P_2(t) = P_2'(t) \Rightarrow \ldots \Rightarrow P_2(t) = \frac{\lambda_1}{\lambda_1+\lambda_2-\lambda_2^{+}}\left(e^{-\lambda_2^{+}t} - e^{-(\lambda_1+\lambda_2)t}\right)$$

$$\lambda_2 P_1(t) - \lambda_1^{+} P_3(t) = P_3'(t) \Rightarrow \ldots \Rightarrow P_3(t) = \frac{\lambda_1}{\lambda_1+\lambda_2-\lambda_1^{+}}\left(e^{-\lambda_1^{+}t} - e^{-(\lambda_1+\lambda_2)t}\right)$$

$$P_4(t) = 1 - P_1(t) - P_2(t) - P_3(t)$$

$$R(t) = P_1(t) + P_2(t) + P_3(t)$$

$$MTBF = \int_0^{\infty} R(t)\,dt = \ldots$$

$\lambda_1 = 10/an$ $\quad \lambda_1^{+} = 20/an$

$\lambda_2 = 5/an$ $\quad \lambda_2^{+} = 30/an$

Sistem cu 3 componente cu standby

State	Comp 1	Comp 2	Comp 3
S_1	Op.	Stby.	Stby.
S_2	F	Op.	Stby.
S_3	F	F	Op.
S_4	F	F	F

$$\begin{tikzcd} S_1 \arrow[d, bend left, "\lambda"] \\ S_2 \arrow[d, bend left, "\lambda"] \\ S_3 \arrow[d, bend left, "\lambda"] \\ S_4 \end{tikzcd}$$

$$A^* = \begin{pmatrix} -\lambda & \lambda & 0 & 0 \\ 0 & -\lambda & \lambda & 0 \\ 0 & 0 & -\lambda & \lambda \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow [P(t)] \cdot A^* = [P'(t)]$$

$$P_1(t) \cdot (-\lambda) = \frac{dP_1(t)}{dt} \Rightarrow P_1(t) = e^{-\lambda t}$$

$$\lambda P_1(t) - \lambda P_2(t) = P_2'(t) \Rightarrow \lambda e^{-\lambda t} - \lambda P_2(t) = P_2'(t) \Rightarrow$$

$$\Rightarrow P_2'(t) + \lambda P_2(t) = \lambda e^{-\lambda t}$$

$$P_2'(t) + \lambda P_2(t) = \lambda e^{-\lambda t} \qquad y'(t) + p(t)\,y(t) = g(t) \Rightarrow$$

$p(t) = \lambda$

$g(t) = \lambda e^{-\lambda t}$

$y(t) = P_2(t)$

$$y(t) = \frac{\int e^{\int p(t)dt} g(t)\,dt + K}{e^{\int p(t)dt}}$$

$$P_2(t) = \frac{\int e^{\int \lambda dt} \lambda e^{-\lambda t}\,dt + K}{e^{\int \lambda dt}} = \frac{\lambda \int \cancel{e^{\lambda t}}\,\cancel{e^{-\lambda t}}\,dt + K}{e^{\lambda t}} = \lambda e^{-\lambda t}\,t$$

$$P_2(t) = \lambda t\, e^{-\lambda t}$$

Fiabilitatea struct cu 1 elem. de back-up: $R(t) = P_1(t) + P_2(t)$

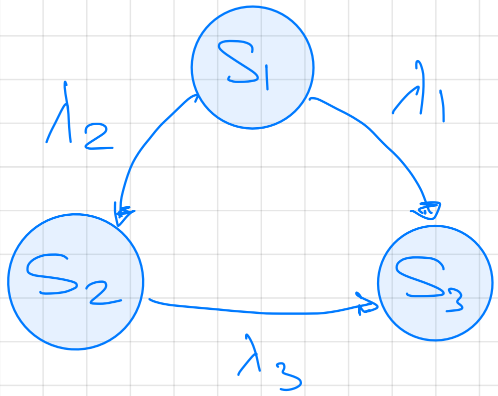
$$\lambda P_2(t) - \lambda P_3(t) = P_3'(t) \Rightarrow \dots \Rightarrow P_3(t) = \frac{(\lambda t)^2}{2} e^{-\lambda t}$$

Sistem cu funcționare degradată

S_1 : Complet funcțional

S_2 : Stare degradată de funcționare

S_3 : defect



$$A^* = \begin{pmatrix} -\lambda_1-\lambda_2 & \lambda_2 & \lambda_1 \\ 0 & -\lambda_3 & \lambda_3 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} P_1(t)(-\lambda_1-\lambda_2) = P_1'(t) \Rightarrow P_1(t) = e^{-(\lambda_1+\lambda_2)t} \\ \lambda_2 P_1(t) - \lambda_3 P_2(t) = P_2'(t) \\ \lambda_1 P_1(t) + \lambda_3 P_2(t) = P_3'(t) \end{cases}$$

$$\lambda_2 e^{-(\lambda_1+\lambda_2)t} - \lambda_3 P_2(t) = P_2'(t) \Rightarrow P_2'(t) + \lambda_3 P_2(t) = \lambda_2 e^{-(\lambda_1+\lambda_2)t}$$

$$P_2(t) = \frac{\lambda_2}{\lambda_1+\lambda_2-\lambda_3}\left(e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t}\right)$$

$\lambda_1 = 5/\text{an}$ $\qquad$ $MTBF = ?$

$\lambda_2 = 3/\text{an}$

$\lambda_3 = 2/\text{an}$

$$R(t) = P_1(t) + P_2(t) = e^{-(\lambda_1+\lambda_2)t} + \frac{\lambda_2}{\lambda_1+\lambda_2-\lambda_3}\left(e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t}\right)$$

$$MTBF = \int_0^{\infty} R(t)\,dt = \int_0^{\infty} \left(e^{-(\lambda_1+\lambda_2)t} + \frac{\lambda_2}{\lambda_1+\lambda_2-\lambda_3}\left(e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t}\right)\right) dt =$$

$$= \frac{1}{\lambda_1+\lambda_2} + \frac{\lambda_2}{\lambda_1+\lambda_2-\lambda_3}\left(\frac{1}{\lambda_3} - \frac{1}{\lambda_1+\lambda_2}\right) = \frac{1}{5+3} + \frac{3}{5+3-2}\left(\frac{1}{2} - \frac{1}{5+3}\right)$$

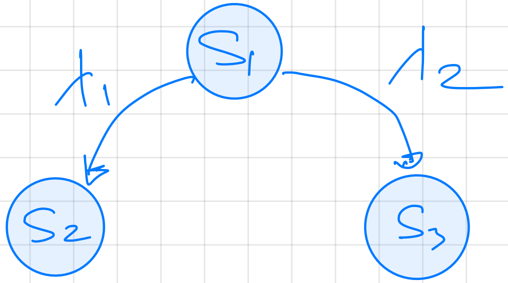
$$= \frac{1}{8} + \frac{3}{6}\left(\frac{1}{2} - \frac{1}{8}\right) = \frac{1}{8} + \frac{1}{2}\cdot\frac{3}{8} - - -$$

Sisteme three - state

S_1 : functionare

S_2 : fail - open

S_3 : fail - short



$$A^* = \begin{pmatrix} -\lambda_1-\lambda_2 & \lambda_1 & \lambda_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow$$

$$P_1(t)(-\lambda_1-\lambda_2) = P_1'(t) \Rightarrow P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$

$$P_1(t)\lambda_1 = P_2'(t) \Rightarrow P_2'(t) = \lambda_1 e^{-(\lambda_1+\lambda_2)t}$$

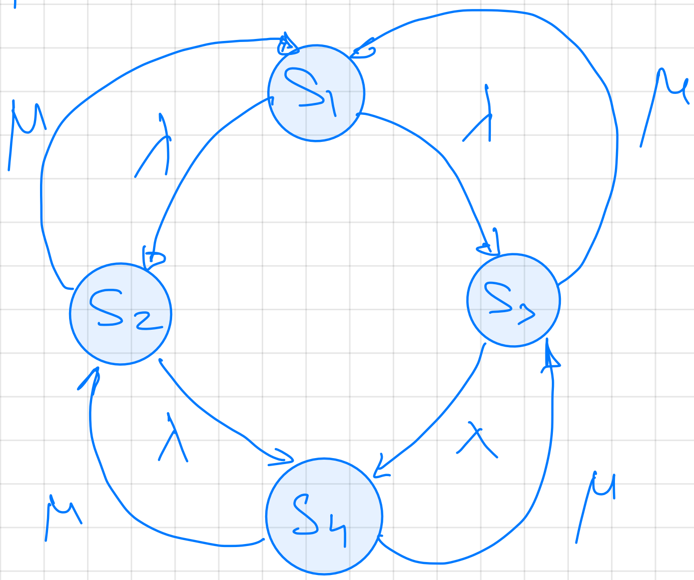
$$P_1(t)\lambda_2 = P_3'(t) \Rightarrow P_3'(t) = \lambda_2 e^{-(\lambda_1+\lambda_2)t}$$

$$R(t) = P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$

$$MTBF = \int_0^\infty R(t)\,dt = \int_0^\infty e^{-(\lambda_1+\lambda_2)t}\,dt = \frac{1}{\lambda_1+\lambda_2}$$

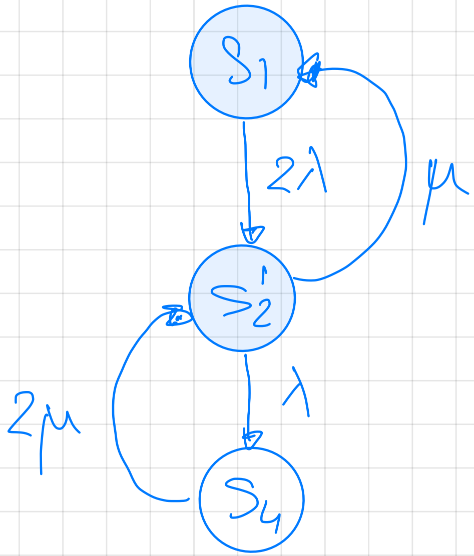
Sistem cu 2 componente și 2 depanatori

Stare	Comp 1.	Comp. 2	
S_1	Op.	Op.	
S_2	Rep.	Op.	S_2'
S_3	Op.	Rep.	
S_4	Rep.	Rep.	



$$A^* = \begin{pmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -\lambda-\mu & \lambda \\ 0 & 2\mu & -2\mu \end{pmatrix} \Rightarrow \begin{cases} -2\lambda P_1(t) + \mu P_2(t) = P_1'(t) \\ 2\lambda P_1(t) - (\lambda+\mu) P_2 + 2\mu P_3 = P_2' \\ \lambda P_2 - 2\mu P_3 = P_3' \end{cases}$$

$$P_1 + P_2 + P_3 = 1$$



Sisteme cu 2 componente și un singur depanator

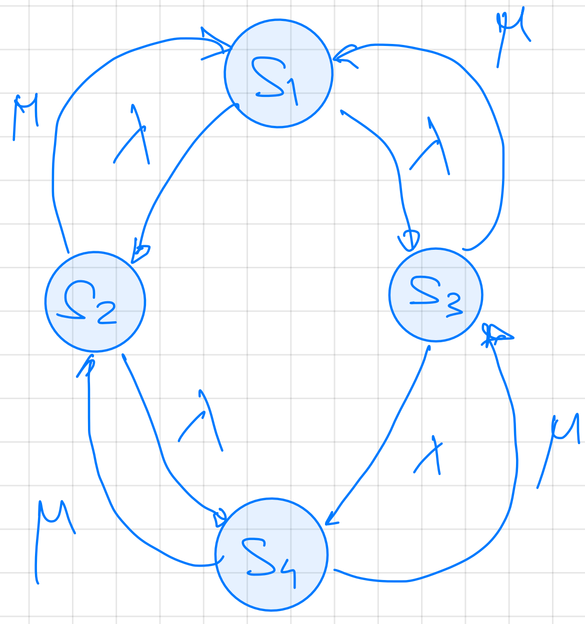
S_1 : ambele up

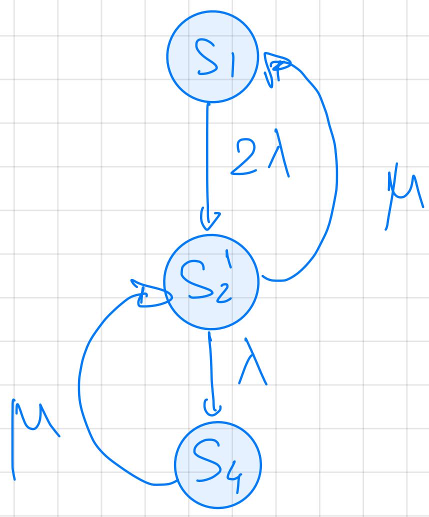
S_2 : C_1 în rep. și C_2 funcționala

S_3 : C_1 OK și C_2 rep.

S_4 : C_1 și C_2 în rep.

unificăm S_2 și S_3 = S_2'





Dacă sistemele nu sunt identice: