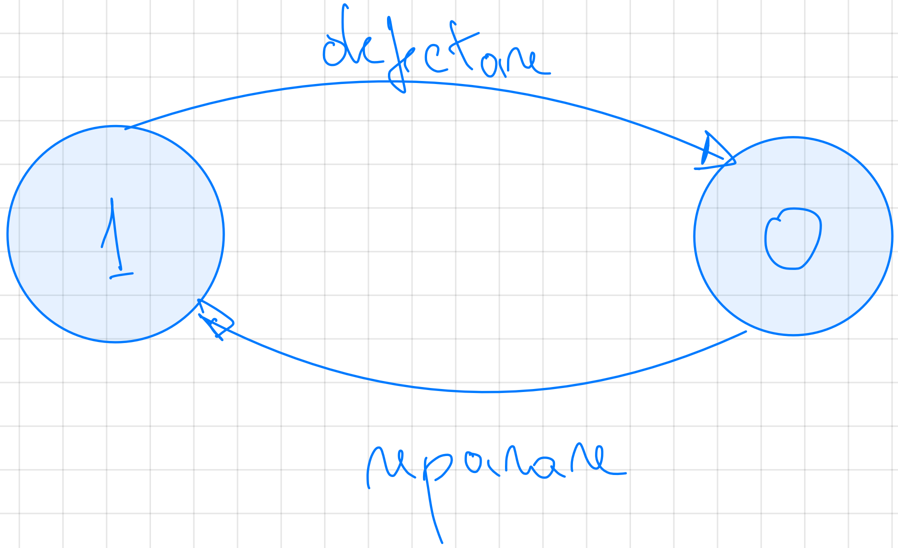
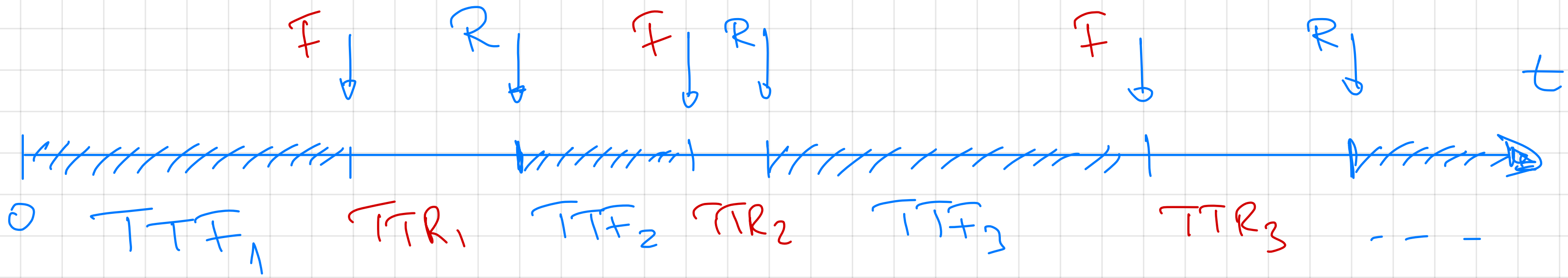
Fiabilitate și toleranță la defecte

$R(t)$ - fiabilitatea

$$R(t) = P(T > t \mid OK @ t=0)$$





$$MTBF = \sum_i \frac{TTF_i}{n}$$

$$MTR = \sum_i \frac{TTR_i}{n}$$

Disponibilitate – $A(t)$

$$A = \frac{\sum_i TTF_i}{\sum_i TTF_i + \sum_i TTR_i} = \frac{MTBF}{MTBF + MTR}$$

Availability (%)	Downtime / year	Downtime / month	Downtime / week
90% ("one nine")	36,5 days	72 h	16,8 h
99% ("2 nines")	3,65 days	7,2 h	1,68 h
99,9% ("3 nines")	8,76 h	43,2 min	10,1 min
99,99% ("4 nines")	52,56 min	4,32 min	1,01 min
99,999%	5,25 min	25,9 s	6,05 s
99,9999%	31,5 s	2,59 s	0,605 s

Probability theory 101

$$0 \leq P(A) \leq 1$$

$$P(\bar{A}) = 1 - P(A)$$

$P(A|B)$ – Prob A cond de B

$$P(A \cdot B) = P(A|B) \cdot P(B)$$

Dacă A şi B sunt independente

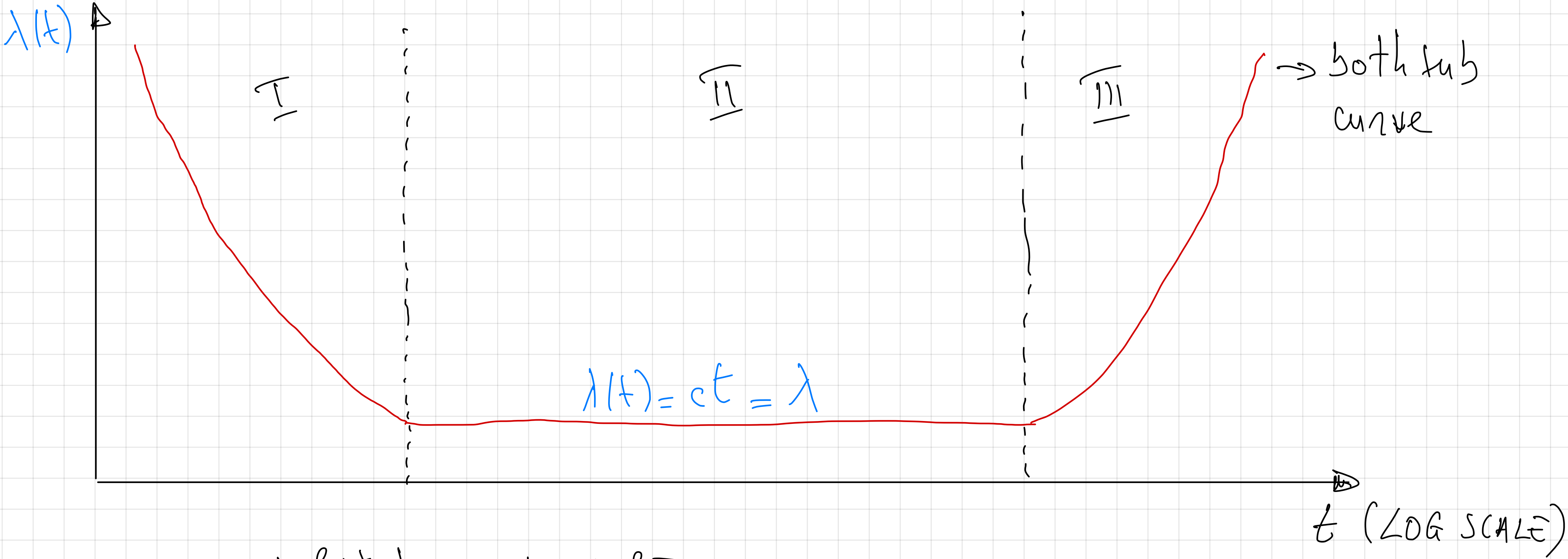
$$P(A \cdot B) = P(A) \cdot P(B)$$

$$P(A+B) = P(A) + P(B) - P(A \cdot B)$$

Dacă A şi B sunt mutual exclusive $\Rightarrow P(A \cdot B) = P(B \cdot A) = 0$

$$P(A+B) = P(A) + P(B)$$

Failure Rate → $\lambda(t)$ – intensitatea defecțiunilor

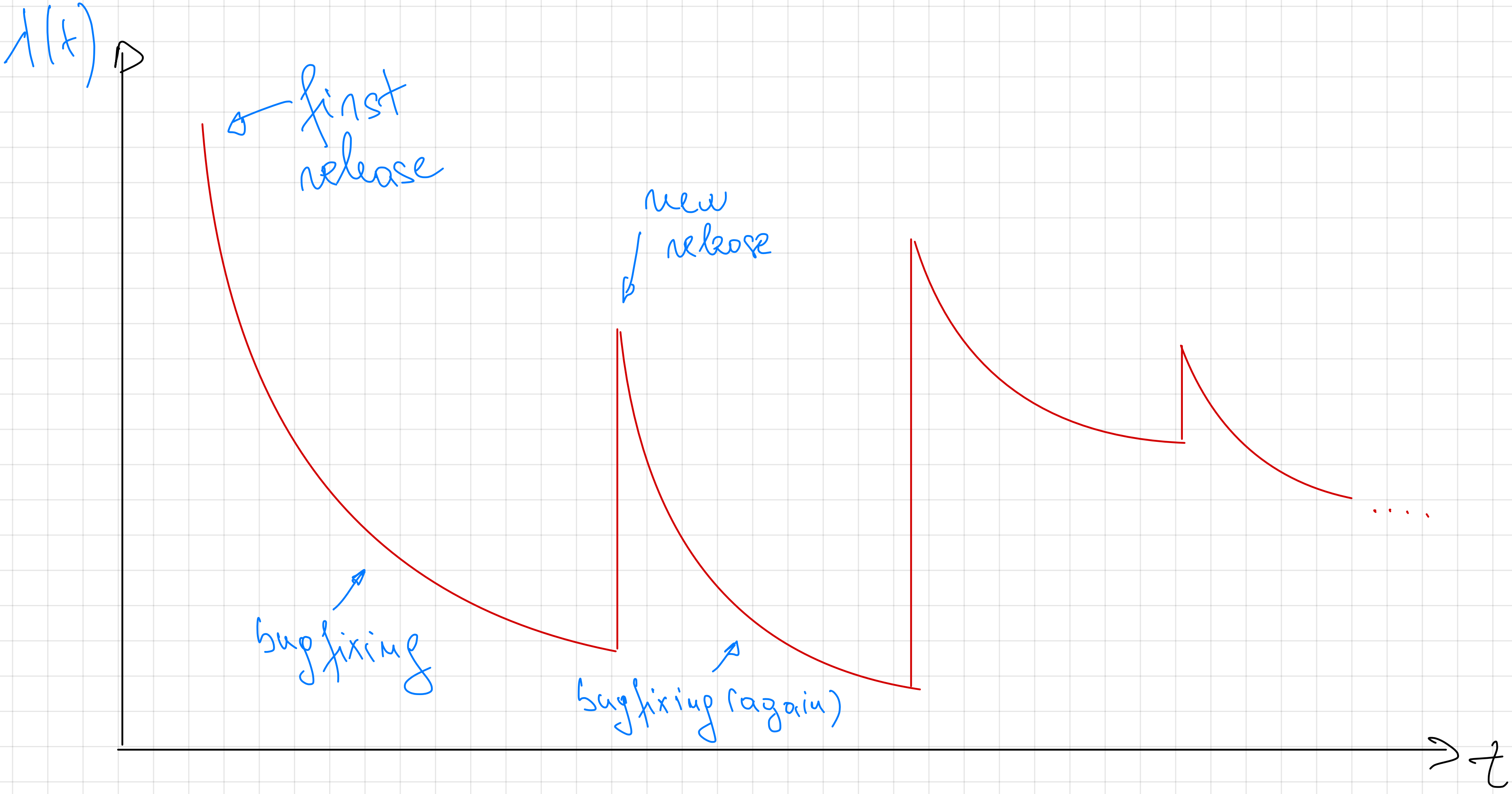


I – mortalitate infantilă

II – viața utilă

III – îmbătrânire

Failure rate - software



Câteva definiții noi:

$f(t)$ – funcție densitate probabilitate (pdf)

$F(t)$ – funcție cumulativă distribuție probabilitate

$$f(t) = \frac{dF(t)}{dt} \qquad F(t) = \int_0^t f(\tau)\,d\tau$$

$$R(t) = 1 - F(t)$$

$$\lambda(t) = \frac{f(t)}{1-F(t)} \Rightarrow \lambda(t) = \frac{f(t)}{R(t)}$$

$$f(t) = \frac{dF(t)}{dt} = \frac{d(1-R(t))}{dt} = -\frac{dR(t)}{dt}$$

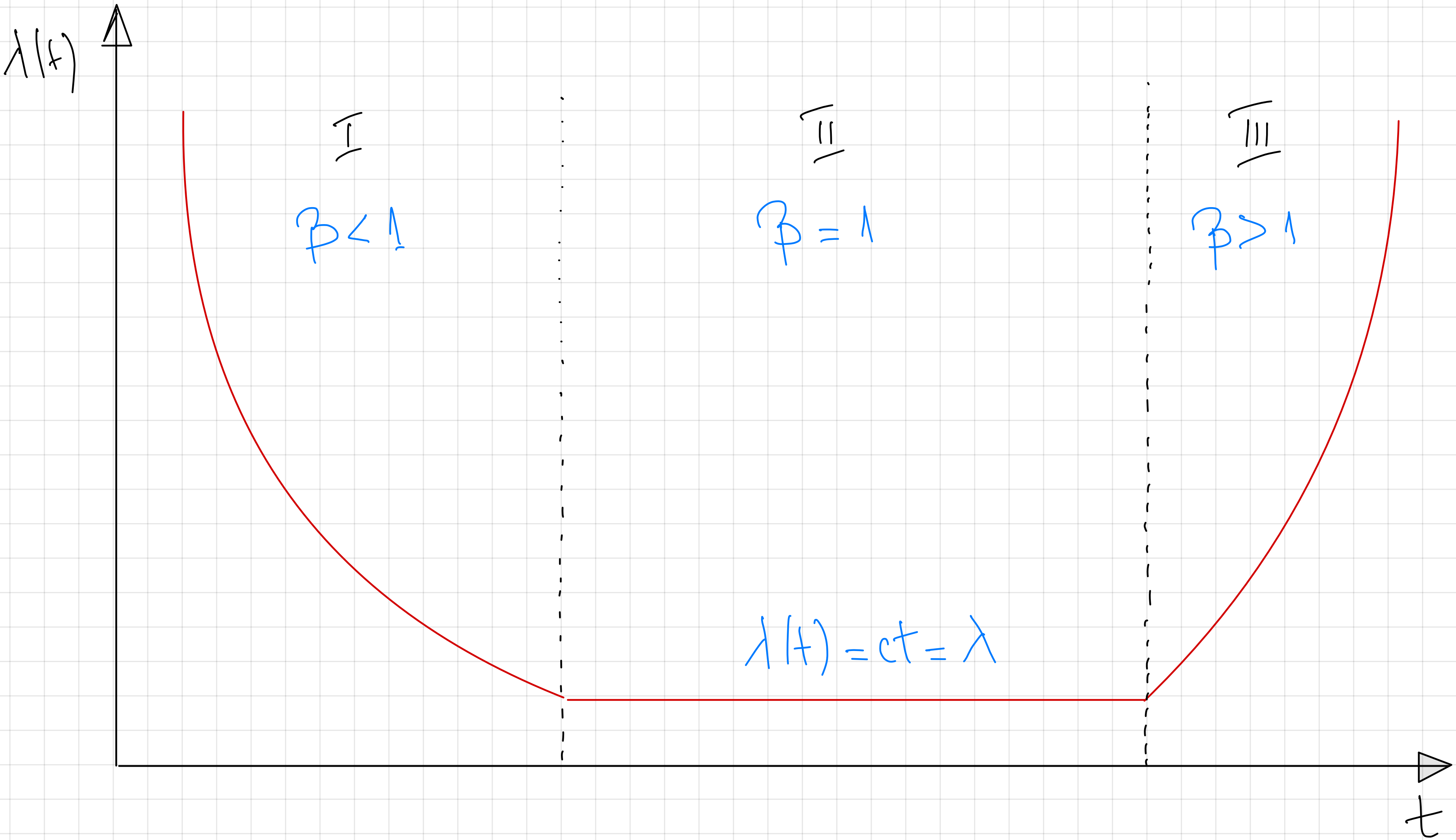
$$\lambda(t) = \frac{-\frac{dR(t)}{dt}}{R(t)} = -\frac{1}{R(t)}\frac{dR(t)}{dt}$$

Aproximare matematică a $\lambda(t)$ - folosim distribuția Weibull

$$f(t) = \lambda \beta t^{\beta-1} e^{-\lambda t^{\beta}}$$

$$\lambda(t) = \lambda \beta t^{\beta-1}$$

- dacă $\beta < 1 \Rightarrow \lambda(t) \searrow$ (zona I pe grafic)
- dacă $\beta = 1 \Rightarrow \lambda(t) = ct$ (zona II pe grafic)
- dacă $\beta > 1 \Rightarrow \lambda(t) \nearrow$ (zona III pe grafic)



1. Cazul în care $\lambda(t) = ct = \lambda$ (perioada de viață utilă a produsului)

$$\lambda(t) = \lambda = -\frac{1}{R(t)} \frac{dR(t)}{dt} \iff \lambda\, dt = -\frac{1}{R(t)}\, dR(t) \Rightarrow$$

$$\Rightarrow \int \lambda\, dt = -\int \frac{1}{R(t)}\, dR(t) \Rightarrow \lambda t + C_1 = -\ln R(t) + C_2 \Rightarrow$$

$$\Rightarrow \ln R(t) = -\lambda t + C \Rightarrow R(t) = e^{-\lambda t + C} = K \cdot e^{-\lambda t}$$

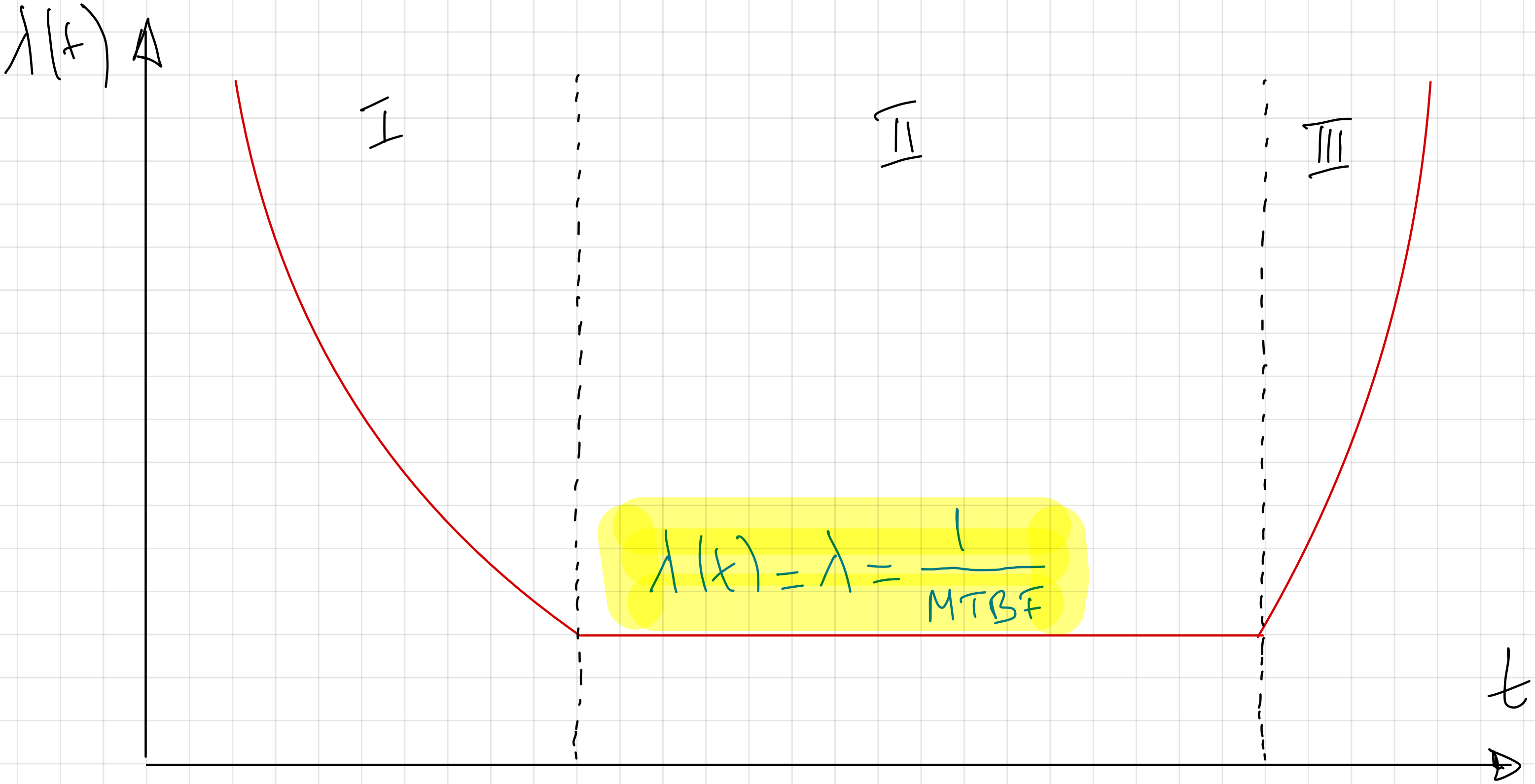
Dacă $t = 0 \Rightarrow R(0) = K$ – putem să presupunem că sistemul pornește la momentul zero de timp în stare de funcționare, deci $R(0) = 1$ (fiabilitate 100%) – alegem $K = 1 \Rightarrow$

$$\boxed{R(t) = e^{-\lambda t}}$$

$$MTBF = \int_0^\infty R(t)\,dt = \int_0^\infty e^{-\lambda t}\,dt = -\frac{1}{\lambda} e^{-\lambda t}\Big|_0^\infty = -\frac{1}{\lambda}\left(e^{-\lambda\infty} - e^{-\lambda \cdot 0}\right) =$$

$$= -\frac{1}{\lambda}(0 - 1) = \frac{1}{\lambda}$$

Deci, pt $\lambda(t) = ct = \lambda$, $MTBF = \frac{1}{\lambda}$



2. Dacă $\lambda(t) \neq ct$ (pt. zonele mortalitate infantilă și îmbătrânire)

$$f(t) = \lambda\beta t^{\beta-1} e^{-\lambda t^{\beta}}$$

$$F(t) = \int_0^t f(z)\,dz = \int_0^t \lambda\beta z^{\beta-1} e^{-\lambda z^{\beta}}\,dz = \dots = 1 - e^{-\lambda t^{\beta}}$$

dar $R(t) = 1 - F(t) \Rightarrow R(t) = e^{-\lambda t^{\beta}}$

$$MTBF = \int_0^\infty R(t)\,dt = \int_0^\infty e^{-\lambda t^{\beta}}\,dt = \frac{\Gamma(\beta^{-1})}{\beta \lambda^{\beta-1}},$$

unde $\Gamma(x)$ este funcția gamma, definită prin: $\Gamma(x) = \int_0^\infty e^{-y} y^{x-1}\,dy$ și este extensia pt. numere reale a funcției factoriale $(n!)$

$$\Gamma(x) = (-1+x)\,\Gamma(-1+x)$$

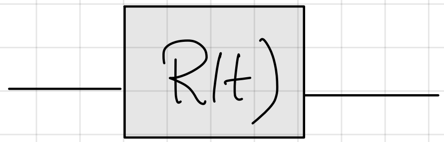
$$\Gamma(x) = \frac{\Gamma(x+1)}{x}$$

$$\Gamma(0) = \Gamma(1) = 1$$

Estimarea fiabilității

Folosim diagrame pentru a modela și estima fiabilitatea

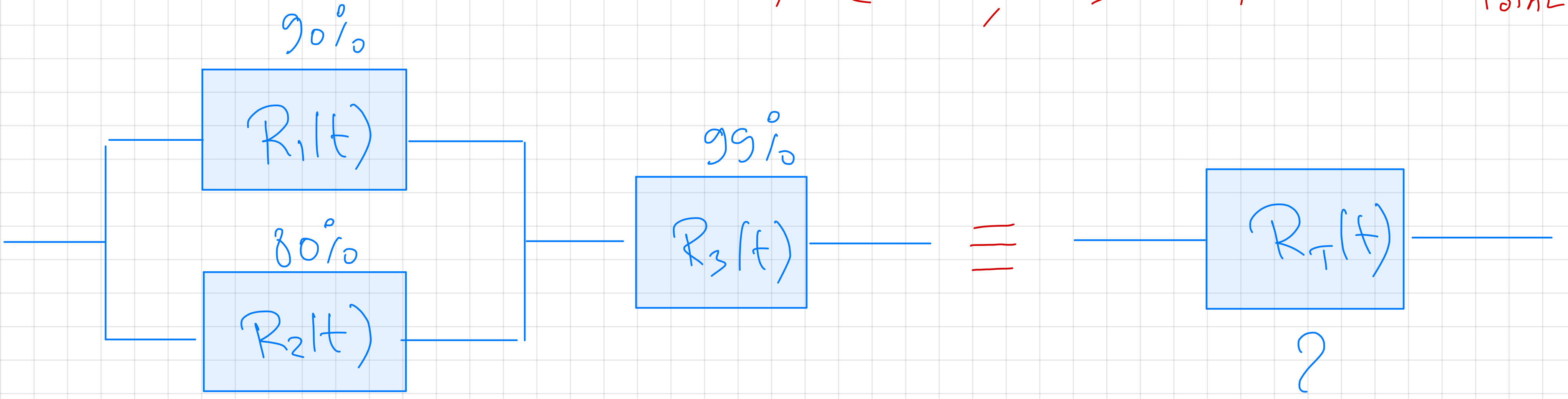
Modularizare: orice sistem, oricât de complex, poate fi modelat fiabilistic ca un modul de fiabilitate dată $R(t)$:



→ Poate fi orice dispozitiv (procesor, calculator, telefon etc) sau orice secvență de cod (bibliotecă, funcție, clasă etc.)

De ex.: Două procesoare ce accesează aceeași memorie.
Primul procesor are o fiabilitate $R_1(t)$, al doilea $R_2(t)$ și memoria $R_3(t)$

Dacă $R_1 = 90\%$, $R_2 = 80\%$ și $R_3 = 99\%$, cât este R_{TOTAL}?



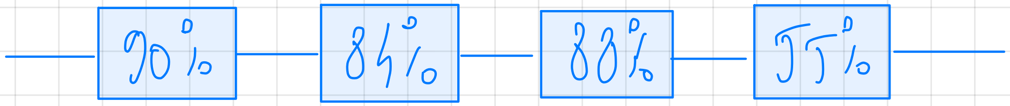
Structuri serie

– n module in serie

$$R_s(t) = R_1(t)\cdot R_2(t)\cdot \ldots \cdot R_n(t) = \prod_{i=1}^{n} R_i(t)$$

$$R_s(t) = \prod_{i=1}^{n} R_i(t)$$

De ex.:



$$R_s = 0{,}9 \cdot 0{,}84 \cdot 0{,}88 \cdot 0{,}55 = 0{,}3659 \simeq 0{,}37 = 37\%$$

Dacă $R_i(t) = e^{-\lambda_i t}$

$$R_s(t) = \prod_{i=1}^{n} e^{-\lambda_i t} = e^{-\sum_{i=1}^{n} \lambda_i t}$$

$$R_s(t) = e^{-\sum_{i=1}^{n} \lambda_i t}$$

$$\lambda_s = \sum_{i=1}^{n} \lambda_i$$

Definim $Q_i(t) \rightarrow$ inversul fiabilității

$$Q_i(t) = 1 - R_i(t)$$

$$R_s(t) = \prod_{i=1}^{n} R_i(t)$$

$$Q_s(t) = 1 - R_s(t) = 1 - \prod_{i=1}^{n} (1 - Q_i(t))^n$$

R_i este mare $\geq 90\%$ $\Rightarrow$ $Q_i \leq 10\%$ $\qquad Q_i Q_j \simeq 0$

$$Q_s(t) = 1 - \Big(1 - \sum_{i=1}^{n} Q_i + \sum_{\substack{i=1 \\ j=1 \\ i \neq j}}^{n} \underbrace{Q_i Q_j}_{\simeq 0} \ldots\Big) \simeq 1 - \Big(1 - \sum_{i=1}^{n} Q_i\Big) \Rightarrow$$

$$\boxed{Q_s(t) = \sum_{i=1}^{n} Q_i(t)}$$

$$MTBF_i = \int_0^\infty R_i(t)\,dt = \int_0^\infty e^{-\lambda_i t}\,dt = \frac{1}{\lambda_i}$$

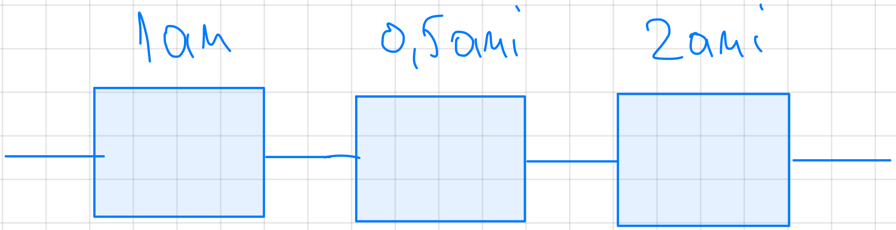
$$MTBF_S = \int_0^\infty R_S(t)\,dt = \int_0^\infty e^{-\sum_{i=1}^{n}\lambda_i t}\,dt = \int_0^\infty e^{-\lambda_S t}\,dt =$$

$$= -\frac{1}{\lambda_S} e^{-\lambda_S t}\Big|_0^\infty = -\frac{1}{\lambda_S}(0-1) = \frac{1}{\lambda_S}$$

$$MTBF_S = \frac{1}{\lambda_S}$$

Dar $\lambda_i = \frac{1}{MTBF_i} \Rightarrow$

$$MTBF_S = \frac{1}{\lambda_1+\lambda_2+\dots\lambda_n} = \frac{1}{\frac{1}{MTBF_1}+\frac{1}{MTBF_2}+\dots\frac{1}{MTBF_n}} = \frac{1}{\sum_{i=1}^{n}\frac{1}{MTBF_i}}$$



$$MTBF_S = \frac{1}{\frac{1}{1}+\frac{1}{0,5}+\frac{1}{2}} = \frac{1}{3+\frac{1}{2}} = \frac{1}{\frac{7}{2}} = \frac{2}{7}\ \text{ani}$$

$$MTBF_S = 0,285\ \text{ani}$$

Dacă avem n module identice în serie, atunci:

$$R_1(t) = R_2(t) = \dots R_n(t) = R(t) = e^{-\lambda t}$$

$$R_S(t) = \prod_{i=1}^{n} R(t) = \prod_{i=1}^{n} e^{-\lambda t} = e^{-n\lambda t}$$

$$R_S(t) = e^{-n\lambda t} = R^n(t)$$

$$\lambda_S(t) = \sum_{i=1}^{n} \lambda_i(t) = \sum_{i=1}^{n} \lambda = n\lambda$$

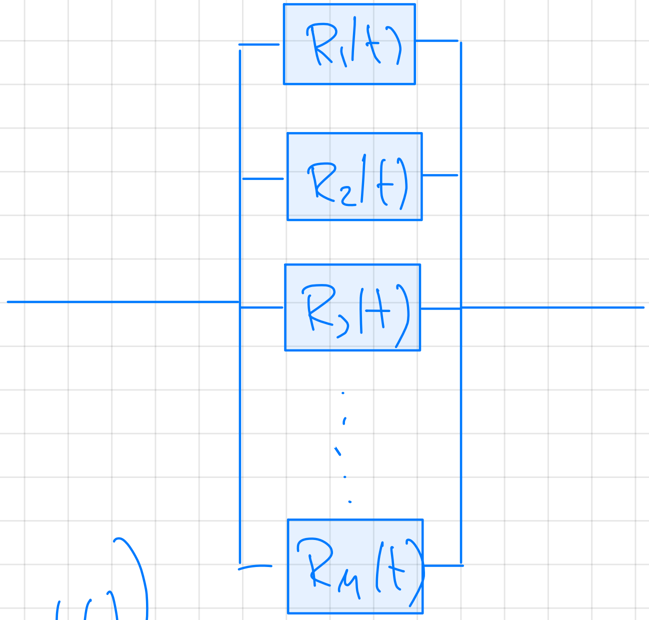
$$\lambda_S = n\lambda$$

$$MTBF_S = \frac{1}{\lambda_S} = \frac{1}{n\lambda} = \frac{MTBF}{n}$$

$$MTBF_S = \frac{MTBF}{n}$$

În general, pt structura serie, R total scade, λ total crește și MTBF total scade.

Structura paralel



$$R_p(t) = ? = 1 - Q_p(t)$$

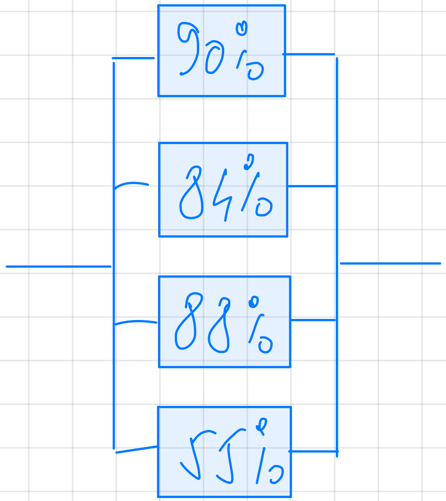
$$Q_p(t) = Q_1(t) \cdot Q_2(t) \cdots Q_n(t) =$$

$$= (1 - R_1(t))(1 - R_2(t)) \cdots (1 - R_n(t))$$

$$R_p(t) = 1 - (1 - R_1(t))(1 - R_2(t)) \cdots (1 - R_n(t))$$

$$R_p(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

De ex: patru module in paralel



$$R_p = 1 - (1 - 0,9)(1 - 0,84)(1 - 0,88)(1 - 0,55) =$$

$$= 1 - 0,1 \cdot 0,16 \cdot 0,12 \cdot 0,45 = 0,9991$$

$$R_p = 99,91\%$$

$$R_i(t) = e^{-\lambda_i t} \Rightarrow R_p(t) = 1 - \prod_{i=1}^{n} \left(1 - e^{-\lambda_i t}\right)$$

$$R_p(t) = 1 - \left(1 - e^{-\lambda_1 t}\right)\left(1 - e^{-\lambda_2 t}\right) \dots \left(1 - e^{-\lambda_n t}\right) \stackrel{?}{=} e^{-\lambda_p t}$$

Presupunem că $R_1(t) = R_2(t) = \dots = R_n(t) = e^{-\lambda t}$

$$R_p(t) = 1 - \left(1 - e^{-\lambda t}\right)^n$$

$$\lambda_p(t) = \frac{f_p(t)}{R_p(t)} = \frac{-\dfrac{dR_p(t)}{dt}}{R_p(t)}$$

$$\frac{dR_p(t)}{dt} = -n\left(1 - e^{-\lambda t}\right)^{n-1}\left(1 - e^{-\lambda t}\right)' = -n\lambda e^{-\lambda t}\left(1 - e^{-\lambda t}\right)^{n-1}$$

$$\lambda_p(t) = n\lambda \frac{e^{-\lambda t}\left(1 - e^{-\lambda t}\right)^{n-1}}{1 - \left(1 - e^{-\lambda t}\right)^n}$$ – nu este constantă!

$$\lambda_{P,\,\text{steady state}} = \lim_{t\to\infty} \lambda_P(t) = \lim_{t\to\infty} n\lambda \frac{e^{-\lambda t}\left(1-e^{-\lambda t}\right)^{n-1}}{1-\left(1-e^{-\lambda t}\right)^{n}} =$$

$e^{-\lambda t} = x$, dacă $t \to \infty$, atunci $x \to 0$

$$= \lim_{x\to 0} n\lambda \frac{x(1-x)^{n-1}}{1-(1-x)^n} \overset{\text{l'Hospital}}{=} n\lambda \lim_{x\to 0} \frac{\left(x(1-x)^{n-1}\right)'}{\left(1-(1-x)^n\right)'} =$$

$$= n\lambda \lim_{x\to 0} \frac{(1-x)^{n-1} - x(n-1)(1-x)^{n-2}}{+n(1-x)^{n-1}} = n\lambda \lim_{x\to 0} \frac{1-x-x(n-1)}{n(1-x)}$$

$$= n\lambda \cdot \frac{1}{n} = \lambda$$

$\boxed{\lambda_P = \lambda}$, la steady-state

$$MTBF_P = \int_0^\infty R_p(t)\,dt = \int_0^\infty \left[1 - \prod_{i=1}^{n}\left(1 - R_i(t)\right)\right] dt$$

Dacă modulele sunt identice $R(t) = e^{-\lambda t}$

$$MTBF_P = \int_0^\infty \left(1 - \left(1 - e^{-\lambda t}\right)^n\right) dt$$

$$\left(1 - e^{-\lambda t}\right)^n = 1 - n e^{-\lambda t} + C_n^2 e^{-2\lambda t} - C_n^3 e^{-3\lambda t} + \dots + (-1)^n e^{-n\lambda t}$$

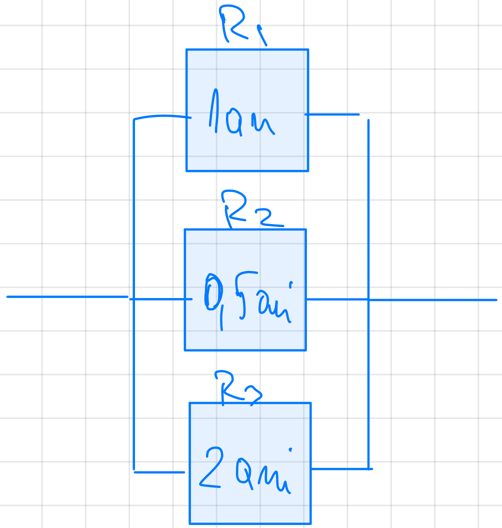
$$MTBF_P = \int_0^\infty \left(n e^{-\lambda t} - \frac{n(n-1)}{2} e^{-2\lambda t} + \dots + (-1)^{n+1} e^{-n\lambda t}\right) dt =$$

$$= n \frac{1}{\lambda} - \frac{n(n-1)}{2} \frac{1}{2\lambda} + \dots + (-1)^{n+1} \frac{1}{n\lambda} = \frac{1}{\lambda}\left(n - \frac{n(n-1)}{4} + \dots (-1)^{n+1} \frac{1}{n}\right)$$

$$= \frac{1}{\lambda} \sum_{i=1}^{n} C_n^i \frac{1}{i} (-1)^{i+1}$$

În general, R total crește, λ total rămâne constant și MTBF total crește

De ex.: trei module în paralel, $MTBF_1 = 1$ an, $MTBF_2 = 0,5$ ani, $MTBF_3 = 2$ ani



$$MTBF_p = \int_0^\infty R_p(t)\,dt = \int_0^\infty \left(1-(1-R_1)(1-R_2)(1-R_3)\right)dt$$

$$(1-R_1)(1-R_2)(1-R_3) = (1-R_2-R_1+R_1R_2)(1-R_3) =$$

$$= 1 - R_1 - R_2 + R_1R_2 - R_3 + R_2R_3 + R_1R_3 - R_1R_2R_3$$

$$\int_0^\infty (R_1+R_2+R_3 - R_1R_2 - R_2R_3 - R_1R_3 + R_1R_2R_3)\,dt$$

$$\int_0^\infty \left(e^{-\lambda_1 t} + e^{-\lambda_2 t} + e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t} - e^{-(\lambda_2+\lambda_3)t} - e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_1+\lambda_2+\lambda_3)t}\right)dt$$

$$= \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} - \frac{1}{\lambda_1+\lambda_2} - \frac{1}{\lambda_2+\lambda_3} - \frac{1}{\lambda_1+\lambda_3} + \frac{1}{\lambda_1+\lambda_2+\lambda_3} =$$

$$= MTBF_1 + MTBF_2 + MTBF_3 - \frac{1}{\frac{1}{MTBF_1}+\frac{1}{MTBF_2}} + \ldots = 1 + 0,5 + 2 - \frac{1}{1+2} - \frac{1}{2+\frac{1}{2}} - \frac{1}{1+\frac{1}{2}}$$

$$+ \frac{1}{1+2+\frac{1}{2}} = 3,5 - \frac{1}{3} - \frac{2}{5} - \frac{2}{3} + \frac{2}{7} = 2,385 \text{ ani}$$

Concluzii:

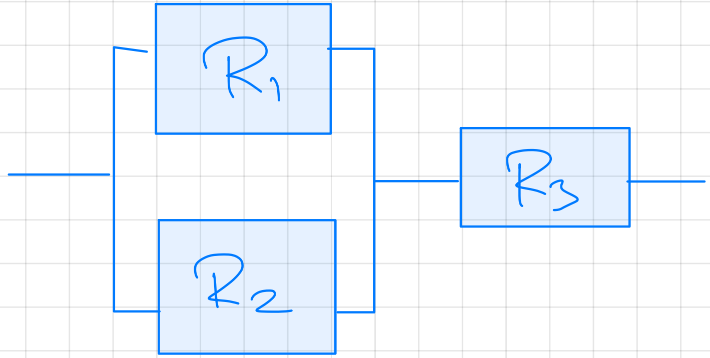
	n module identice în Serie	n module identice în Paralel
R_{TOTAL}	$R_s(t) = R^n(t) = e^{-n\lambda t}$	$R_p(t) = 1 - (1 - R(t))^n = 1 - (1 - e^{-\lambda t})^n$
λ_{TOTAL}	$\lambda_s = n\lambda$	$\lambda_p(t) = n\lambda \dfrac{e^{-\lambda t}(1 - e^{-\lambda t})^{n-1}}{1 - (1 - e^{-\lambda t})^n}$, $\lambda_{p,\text{stabile}} = \lambda$
$MTBF_{TOTAL}$	$MTBF_s = \dfrac{MTBF}{n}$	$MTBF_p = MTBF \cdot \sum_{i=1}^{n} (-1)^{i+1} \cdot C_n^i \dfrac{1}{i}$

$$R_{TOTAL} = (R_1 \| R_2) \cdot R_3 =$$

$$= [1-(1-R_1)(1-R_2)]\, R_3 =$$

$$= [1-(1-R_2-R_1+R_1R_2)]\, R_3 =$$

$$= (R_1+R_2-R_1R_2)R_3 = R_1R_3+R_2R_3-R_1R_2R_3$$

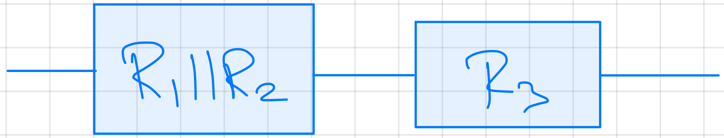


$R_1(t)=e^{-\lambda_1 t}$

$R_2(t)=e^{-\lambda_2 t}$

$R_3(t)=e^{-\lambda_3 t}$

$R_{TOTAL} = ?$



După un timp t $R_1(t)=80\%$,

$R_2(t)=90\%$ şi $R_3(t)=55\%$

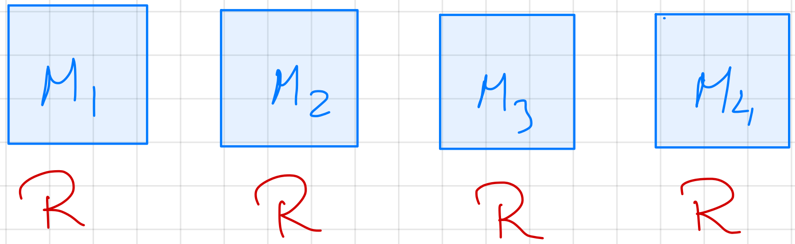
$$R_{TOTAL} = 0{,}8\cdot 0{,}55 + 0{,}9\cdot 0{,}55 - 0{,}8\cdot 0{,}9\cdot 0{,}55 = \ldots$$

$MTBF_1 = 2$ ani $\quad MTBF_2 = 3$ ani şi $MTBF_3 = 5$ ani

$$MTBF_{TOTAL} = \int_0^\infty R_{TOTAL}(t)\,dt = \int_0^\infty \left(e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_2+\lambda_3)t} - e^{-(\lambda_1+\lambda_2+\lambda_3)t}\right)dt =$$

$$= \frac{1}{\lambda_1+\lambda_3} + \frac{1}{\lambda_2+\lambda_3} - \frac{1}{\lambda_1+\lambda_2+\lambda_3} = \frac{1}{\frac{1}{2}+\frac{1}{5}} + \frac{1}{\frac{1}{3}+\frac{1}{5}} - \frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{5}} = \ldots$$

Fiabilitatea structurilor r din n



Ex: avion cu 4 motoare, poate tolera maxim 2 motoare defecte (2 din 4)

Fiecare motor are aceeași fiabilitate R(t):

$$R_{2/4} = R^4 + 4 \cdot R^3 \cdot (1-R) + 6R^2(1-R)^2$$

Cazul general:

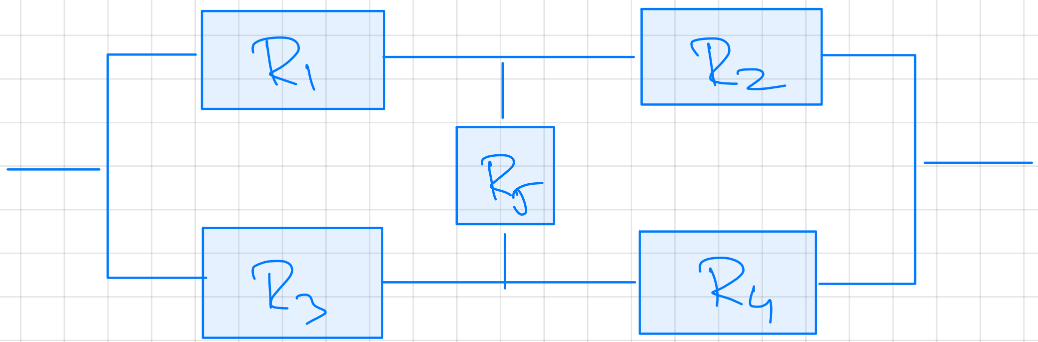
$$R_{r|n} = \sum_{i=0}^{n-r} C_n^{n-i} R^{n-i} (1-R)^i$$

dacă $r = n \rightarrow R_{n|n} = R^n$ – fiabilitatea structurii serie

$r = 1 \rightarrow R_{1|n} = 1-(1-R)^n$ – fiabilitatea structurii paralel

Fiabilitatea structurilor nedecompozabile

$R_{TOTAL} = ?$



Cazul 1: R_5 funcționează ($R_5 = 1$)

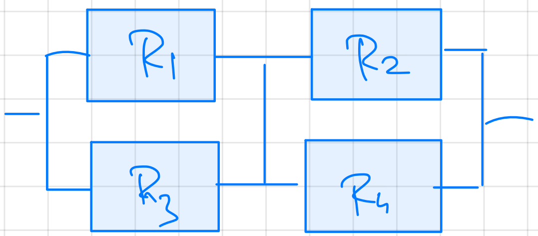
$$R_{c_1}(S \mid R_5) = \ldots$$

Cazul 2: R_5 defect ($R_5 = 0$)

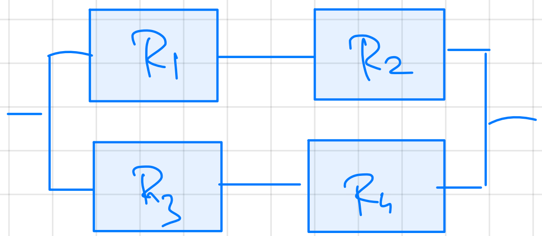
$$R_{c_2}(S \mid \overline{R_5}) = \ldots$$

$$R_{TOTAL} = R_{c_1}(S \mid R_5) \cdot R_5 + R_{c_2}(S \mid \overline{R_5}) \cdot (1 - R_5)$$

$$R_{c_1}(S \mid R_5) = (R_1 \parallel R_3) \cdot (R_2 \parallel R_4) = (R_1 + R_3 - R_1 R_3)(R_2 + R_4 - R_4 R_2)$$



$$R_{c_2}(S \mid \overline{R_5}) = (R_1 R_2) \parallel R_3 R_4 = R_1 R_2 + R_3 R_4 - R_1 R_2 R_3 R_4$$



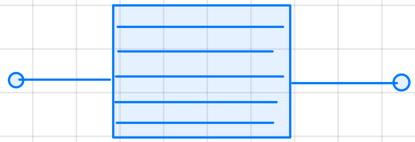
$R_{TOTAL} = \ldots$

Fiabilitatea structurilor serie-paralel și paralel-serie

Să considerăm un modul format dintr-o celulă fotovoltaică.

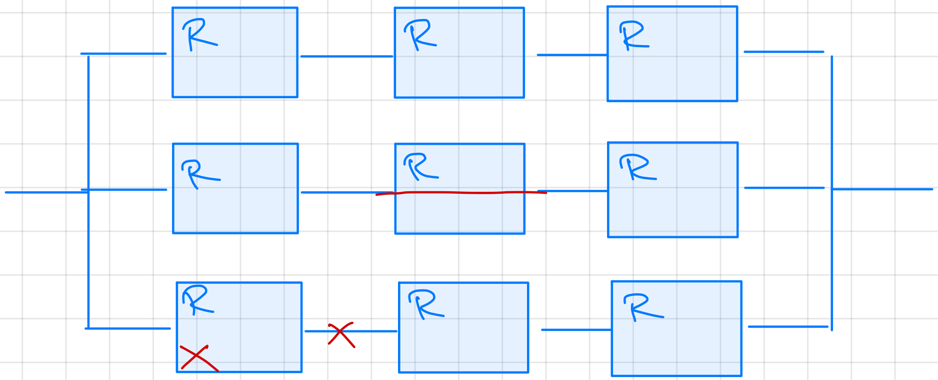
Moduri de defectare:

- întrerupere (prevalent)
- scurtcircuit (relativ rar)

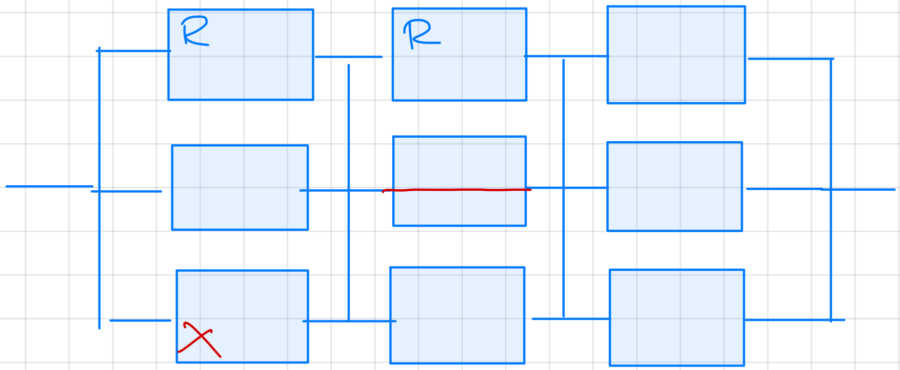


Cum construiesc un panou fotovoltaic cu 9 celule care să fie fiabil?

Serie-Paralel



Paralel - Serie



Scurtcircuit: pierdem 1 celulă din 9

întrerupere: pierdem 3 celule din 9

Scurtcircuit: pierdem 3 celule din 9

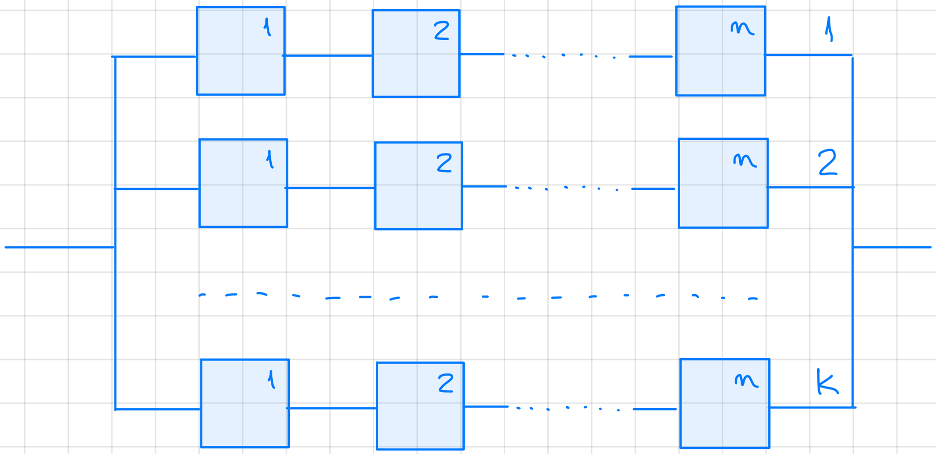
întrerupere: pierdem 1 celulă din 9

$$R_{SP} = 1 - (1 - R^3)^3$$

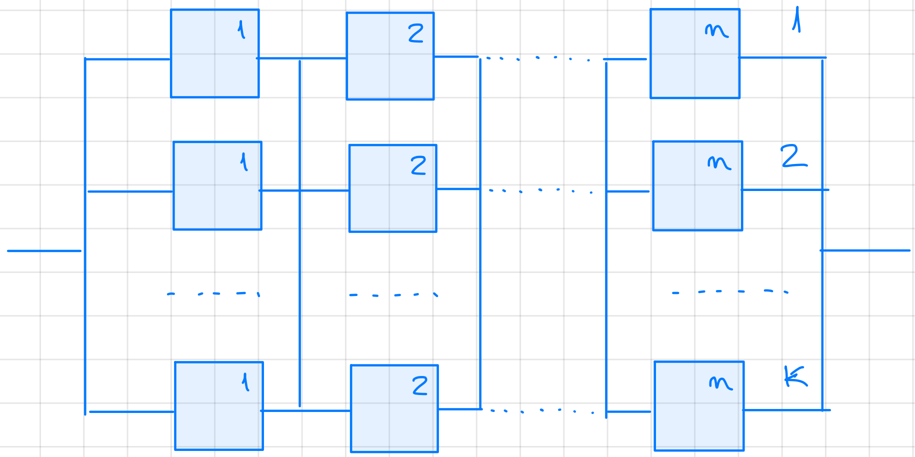
$$R_{PS} = \left(1 - (1-R)^3\right)^3$$

Cazul general:

Serie-Paralel



Paralel - Serie

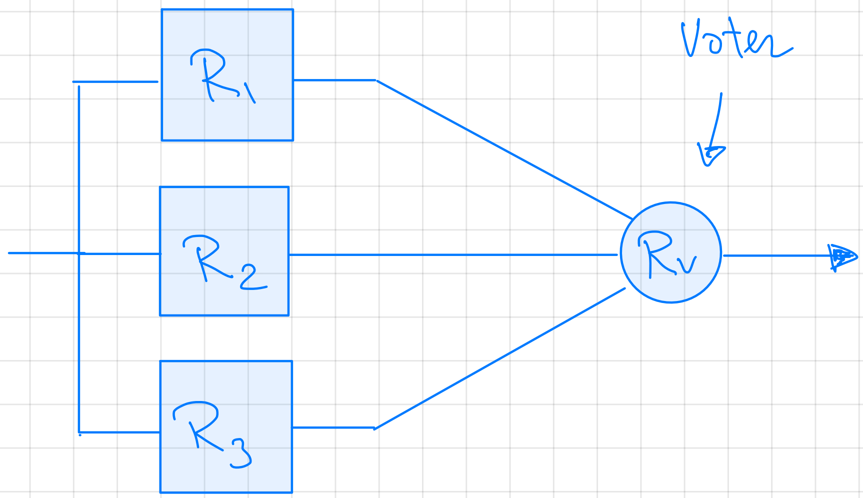


Presupunem că toate modulele sunt identice și au fiabilitatea $R(t)$

$$R_{SP} = 1 - (1 - R^n)^k$$

$$R_{PS} = \left[1 - (1 - R)^k\right]^n$$

Structuri cu votare majoritară



$$R_{2/3} = R_V \cdot \left(R_1R_2R_3 + R_1R_2(1-R_3) + R_1R_3(1-R_2) + (1-R_1)R_2R_3\right)$$

de obicei: $R_V >> R_1, R_2$ sau R_3

$$R_V \simeq 1$$

$$R_1 = R_2 = R_3 = R$$

$$R_{2/3} = R^3 + 3R^2(1-R) = R^3 + 3R^2 - 3R^3 = 3R^2 - 2R^3$$

dacă $R = 99\% \rightarrow R_{2/3} = 3 \cdot 0{,}99^2 - 2 \cdot 0{,}99^3 = 3 \cdot 0{,}98 - 2 \cdot 0{,}97 = 0{,}9997...$

$R_{2/3} = 99{,}97\% > R$

dacă $R = 10\% \rightarrow R_{2/3} = 3 \cdot 0{,}1^2 - 2 \cdot 0{,}1^3 = 3 \cdot 0{,}01 - 2 \cdot 0{,}001 = 0{,}028 = 2{,}8\%$

$R_{2/3} = 2{,}8\% < R$

$R_{2/3} > R$? $\Rightarrow 3R^2 - 2R^3 > R \Rightarrow 2R^3 - 3R^2 + R < 0$

$R(2R^2 - 3R + 1) < 0$, dar $R \in [0,1] \Rightarrow 2R^2 - 3R + 1 < 0 \Rightarrow$

$(R-1)(R-\frac{1}{2}) < 0 \Rightarrow$

$R \in (1/2, 1)$, deci:

$R_{2/3} > R$ doar dacă $R > 50\%$

R				$\frac{1}{2}$			1		
$R-1$	$-$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$
$R-\frac{1}{2}$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$(R-1)(R-\frac{1}{2})$	$+$	$+$	$+$	0	$-$	$-$	0	$+$	$+$

$$MTBF_{2/3} = \int_0^\infty R_{2/3}(t)\,dt = \int_0^\infty \left(3R^2(t) - 2R^3(t)\right)dt = 3\int_0^\infty e^{-2\lambda t}dt - 2\int_0^\infty e^{-3\lambda t}dt$$

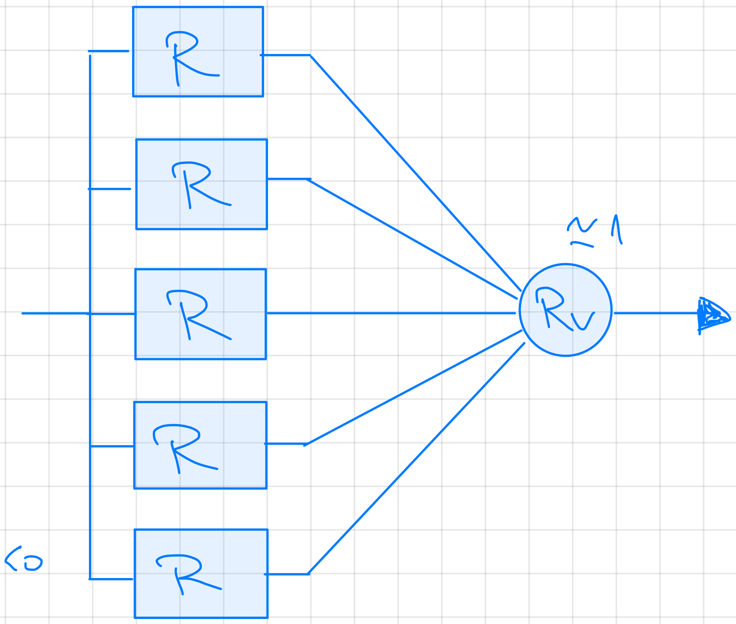
$R(t) = e^{-\lambda t}$

$$= \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{9-4}{6\lambda} = \frac{5}{6}\cdot\frac{1}{\lambda} = \frac{5}{6}\cdot MTBF$$

$MTBF_{2/3} = \frac{5}{6} MTBF$

$MTBF_{2/3} < MTBF$

$$R_{3/5} = R^5 + 5(1-R)R^4 + 10(1-R)^2 R^3$$



$$R_{3/5} = R^5 + 5R^4 - 5R^5 + 10R^3 - 20R^4 + 10R^5 =$$
$$= 6R^5 - 15R^4 + 10R^3$$

$R_{3/5} > R$? $\Rightarrow 6R^5 - 15R^4 + 10R^3 > R \Rightarrow$

$$\Rightarrow -6R^5 + 15R^4 - 10R^3 + R < 0 \Rightarrow R(1 - 10R^2 + 15R^3 - 6R^4) < 0$$

dar $R \geq 0 \Rightarrow 1 - 10R^2 + 15R^3 - 6R^4 < 0 \Rightarrow$

$(R-1)\left(R-\frac{1}{2}\right)\left(R^2 - R - \frac{1}{3}\right) > 0 \Rightarrow$

- $R \in \left(-\infty, \frac{1}{6}(3-\sqrt{21})\right)$ – negativ ×
- $R \in \left(\frac{1}{6}(3+\sqrt{21}), \infty\right) \rightarrow$ pozitiv > 1 ×
- $R \in \left(\frac{1}{2}, 1\right)$ ✓

Deci $R_{3/5} > R$ doar dacă $R > 50\%$

$$MTBF_{3/5} = \int_0^\infty R_{3/5}(t)\,dt = \int_0^\infty \left(6R^5(t) - 15R^4(t) + 10R^3(t)\right)dt = 6\int_0^\infty e^{-5\lambda t}dt - 15\int_0^\infty e^{-4\lambda t}dt +$$

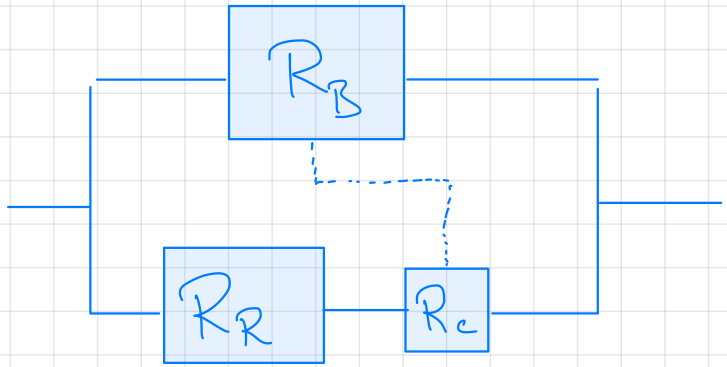
$$+ 10\int_0^\infty e^{-3\lambda t}dt = {}^{12/}\frac{6}{5\lambda} - {}^{15/}\frac{15}{4\lambda} + {}^{20/}\frac{10}{3\lambda} = \frac{72-225+200}{60\lambda} = \frac{47}{60}\cdot\frac{1}{\lambda} = 0{,}783\ MTBF < MTBF$$

Caz general: Votare majoritară n din 2n-1

$$R_{n|2n-1} = \sum_{i=0}^{n-1} C_{2n-1}^{i} R^{2n-1-i} (1-R)^{i}$$

Structuri cu un element de rezervă

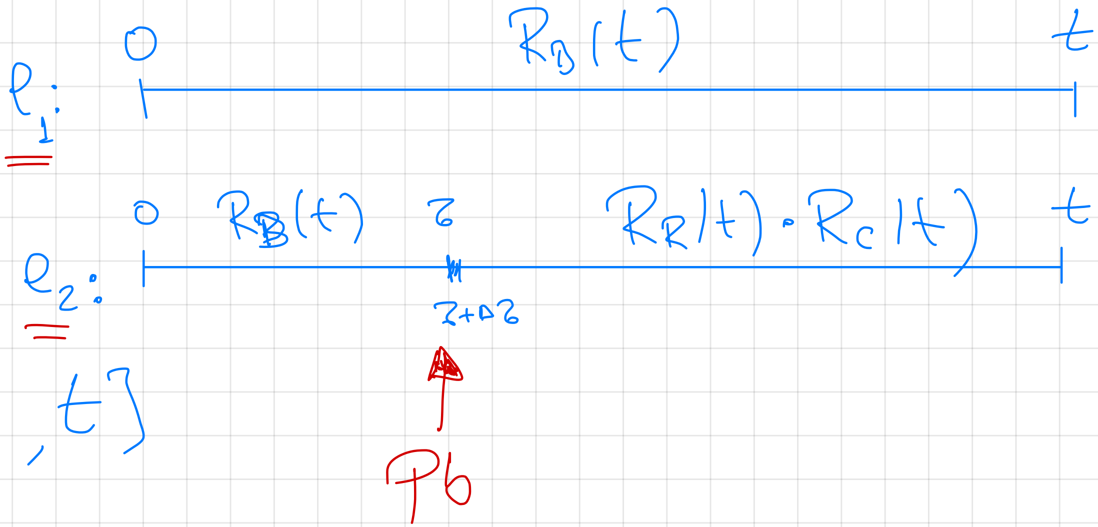
$$R_{TOTAL} = P(e_1) + P(e_2)$$



$P(e_1)$ – baza funcționează pe toată durata misiunii

$$P(e_1) = R_B(t)$$

$P(e_2)$ – baza funcționează în intervalul $(0, \tau)$ și back-up-ul comută în funcționare în $[\tau+\Delta\tau, t]$



$p_b \rightarrow$ prob ca elem. de bază să se defecteze în $[\tau, \tau+\Delta\tau]$

$p_b(\tau) = f_b(\tau)\, d\tau$, unde $f_b(\tau)$ este funcția densitate prob.

$$f_b(\tau) = \frac{dF_b(\tau)}{d\tau} = -\frac{dR_B(\tau)}{d\tau}$$

$P_r \to$ prob. ca elementul de rezervă să funcționeze în $[\tau+d\tau, t]$

$P_r(t) = R_R(t-\tau) \cdot R_C(t)$ | $R_C \simeq 1 \Rightarrow P_r(t) = R_R(t-\tau)$

$$P(e_2) = \underbrace{R_C}_{\simeq 1} \cdot \int_0^t p_b(\tau) \cdot p_r(\tau)\, d\tau = \int_0^t -\frac{dR_B(\tau)}{d\tau} \cdot R_R(t-\tau)\, d\tau$$

$$R_{TOTAL} = R_B(t) + R_C \cdot \int_0^t -\frac{dR_B(\tau)}{d\tau} \cdot R_R(t-\tau)\, d\tau$$

$e^{x\,\prime} = e^x$

$e^{ax\,\prime} = a\, e^{ax}$

$R_B(t) = e^{-\lambda_B t}$ $\qquad R_R(t) = e^{-\lambda_R t}$

$$R_{TOTAL} = e^{-\lambda_B t} + \int_0^t -(-\lambda_B)\, e^{-\lambda_B \tau} \cdot e^{-\lambda_R (t-\tau)}\, d\tau =$$

$$e^{-\lambda_B t} + \lambda_B e^{-\lambda_R t} \int_0^t e^{-(\lambda_B - \lambda_R)\tau}\, d\tau = e^{-\lambda_B t} + \lambda_B e^{-\lambda_R t} \frac{1}{\lambda_R - \lambda_B} \cdot e^{-(\lambda_B - \lambda_R)t} \Big|_0^t$$

$$= e^{-\lambda_B t} + \frac{\lambda_B}{\lambda_R - \lambda_B} e^{-\lambda_R t} \left(e^{(\lambda_R - \lambda_B)t} - 1 \right) = e^{-\lambda_B t} + \frac{\lambda_B}{\lambda_R - \lambda_B} e^{-\lambda_B t} - \frac{\lambda_B}{\lambda_R - \lambda_B} \cdot e^{-\lambda_R t}$$

$$= \frac{\lambda_R}{\lambda_R - \lambda_B} e^{-\lambda_B t} - \frac{\lambda_B}{\lambda_R - \lambda_B} e^{\lambda_R t} = \frac{\lambda_R}{\lambda_R - \lambda_B} R_B(t) - \frac{\lambda_B}{\lambda_R - \lambda_B} R_R(t)$$

$$MTBF_{TOTAL} = ? = \int_0^\infty R_{TOTAL}(t)\,dt = \int_0^\infty \left(\frac{\lambda_R}{\lambda_R-\lambda_B} e^{-\lambda_B t} - \frac{\lambda_B}{\lambda_R-\lambda_B} e^{-\lambda_R t} \right) dt =$$

$$= \frac{\lambda_R}{\lambda_R-\lambda_B} \cdot \frac{1}{\lambda_B} - \frac{\lambda_B}{\lambda_R-\lambda_B} \cdot \frac{1}{\lambda_R} = \frac{\lambda_R^2 - \lambda_B^2}{\lambda_R \lambda_B \cancel{(\lambda_R-\lambda_B)}} = \frac{\lambda_R+\lambda_B}{\lambda_R \cdot \lambda_B} = \frac{1}{\lambda_B} + \frac{1}{\lambda_R}$$

$$MTBF_{TOTAL} = \frac{1}{\lambda_B} + \frac{1}{\lambda_R} = MTBF_B + MTBF_R$$

Caz special: $R_B(t) = R_R(t) = R(t) = e^{-\lambda t}$

$$R_{TOTAL}(t) = e^{-\lambda t} + \int_0^t \lambda e^{-\lambda \tau} \cdot e^{-\lambda(t-\tau)}\, d\tau = e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t d\tau =$$

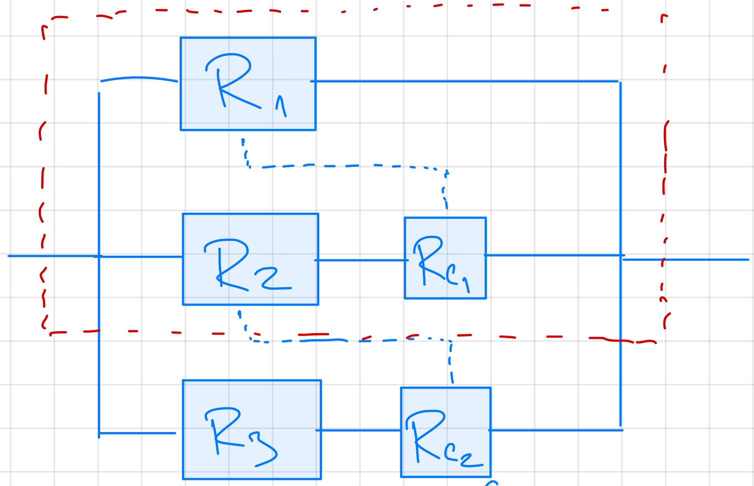
$$= e^{-\lambda t} + \lambda t e^{-\lambda t} = e^{-\lambda t}(1+\lambda t)$$

$$R_{TOTAL}(t) = e^{-\lambda t}(1+\lambda t) = R(t)(1+\lambda t) > R(t) \quad \forall\, t>0$$

$$MTBF_{TOTAL} = 2 \cdot MTBF = 2 \cdot \frac{1}{\lambda} > MTBF$$

Structură cu două elemente de rezervă

$$R_{12}(t) = R_1(t) + \int_0^t f_1(\tau) R_2(t-\tau)\,d\tau$$



$$R_{123}(t) = R_{12}(t) + \int_0^t f_{12}(\tau) R_3(t-\tau)\,d\tau$$

Dar $R_1(t) = R_2(t) = R_3(t) = e^{-\lambda t}$

$$f_1(\tau) = -\frac{dR_1(\tau)}{d\tau} = -\left(e^{-\lambda\tau}\right)' = \lambda e^{-\lambda\tau}$$

$$R_{12}(t) = e^{-\lambda t}(1+\lambda t)$$

$$f_{12}(\tau) = -\frac{dR_{12}(\tau)}{d\tau} = -\frac{d\left(e^{-\lambda\tau} + \lambda\tau e^{-\lambda\tau}\right)}{d\tau} = -\left(-\lambda e^{-\lambda\tau} + \lambda\left(e^{-\lambda\tau} - \lambda\tau e^{-\lambda\tau}\right)\right)$$

$$= \cancel{\lambda e^{-\lambda\tau}} - \cancel{\lambda e^{-\lambda\tau}} + \lambda^2 \tau e^{-\lambda\tau} = \lambda^2 \tau e^{-\lambda\tau}$$

$$R_{123}(t) = e^{-\lambda t}(1+\lambda t) + \int_0^t \lambda^2 \tau e^{-\lambda\tau} \cdot e^{-\lambda(t-\tau)}\,d\tau =$$

$$= e^{-\lambda t}(1+\lambda t) + \lambda^2 e^{-\lambda t} \cdot \int_0^t \tau\,d\tau = e^{-\lambda t}\left(1 + \lambda t + \frac{\lambda^2 t^2}{2}\right)$$

$$R_{TOTAL}(t) = e^{-\lambda t}\left(1 + \lambda t + \frac{(\lambda t)^2}{2}\right)$$

$$MTBF_{123} = \int_0^{\infty} R_{123}(t)\,dt = 3 \cdot \frac{1}{\lambda} = 3 \cdot MTBF$$

Generalizare: k-1 elemente de back-up

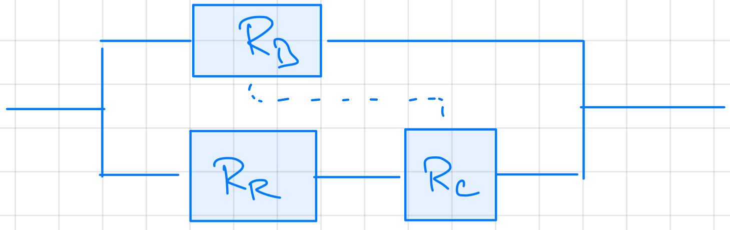
$$R_{TOTAL}(t) = e^{-\lambda t} \cdot \left(1 + \lambda t + \frac{(\lambda t)^2}{2} + \frac{(\lambda t)^3}{6} + \ldots + \frac{(\lambda t)^{k-1}}{(k-1)!} \right)$$

Dacă $k \to \infty$

$$R_{TOTAL}(t) = \lim_{k \to \infty} e^{-\lambda t} \sum_{i=0}^{k-1} \frac{(\lambda t)^i}{i!} = e^{-\lambda t} \lim_{k \to \infty} \sum_{i=0}^{k-1} \frac{(\lambda t)^i}{i!} = e^{-\lambda t} \cdot e^{\lambda t} \Rightarrow$$

$$R_{TOTAL}(t) = 1$$

Structură cu element de rezervă activ



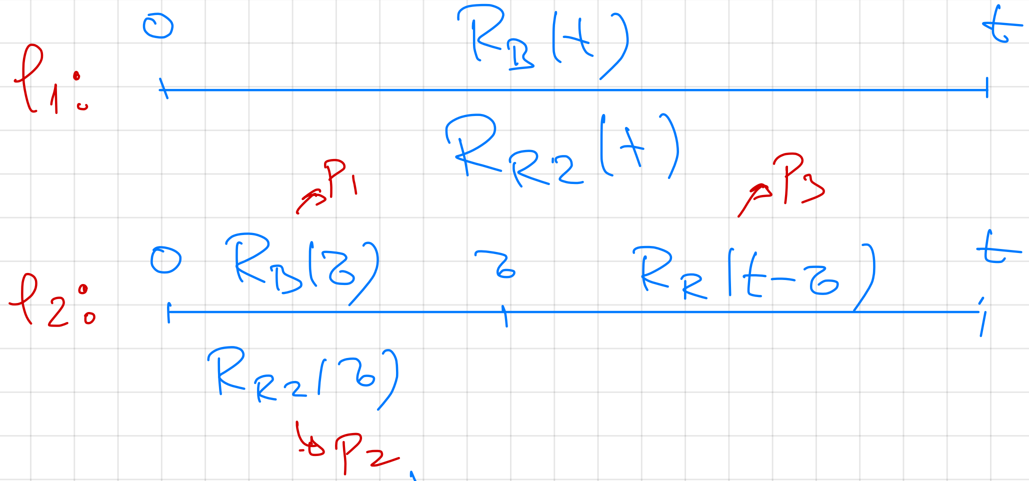
$R_C \cong 1$

$$R_{TOTAL}(t) = P(e_1) + P(e_2)$$

$$P(e_1) = R_B(t)$$

$$P(e_2) = \int_0^t p_1 \cdot p_2 \cdot p_3 \, d\tau =$$

$$= \int_0^t f_1(\tau) \cdot R_{R2}(\tau) \cdot R_R(t-\tau) \, d\tau$$



$$f_1(\tau) = -\frac{dR_B(\tau)}{d\tau}$$

$R_B(t) = e^{-\lambda_B t}$ $\quad R_R(t) = e^{-\lambda_R t}$ $\quad R_{R2}(t) = e^{-\lambda_{R2} t}$

$$R_{TOTAL}(t) = e^{-\lambda_B t} + \frac{\lambda_B}{\lambda_B + \lambda_{R2} - \lambda_R}\left(e^{-\lambda_R t} - e^{-(\lambda_B + \lambda_{R2})t}\right)$$

Dacă modulele sunt identice $\Rightarrow R_B(t) = R_R(t) = e^{-\lambda t}$; $R_{R2} = e^{-\lambda_{R2} t}$

$$R_{TOTAL}(t) = e^{-\lambda t}\left(1 + \frac{\lambda}{\lambda_{R2}}\left(1 - e^{-\lambda_{R2} t}\right)\right)$$

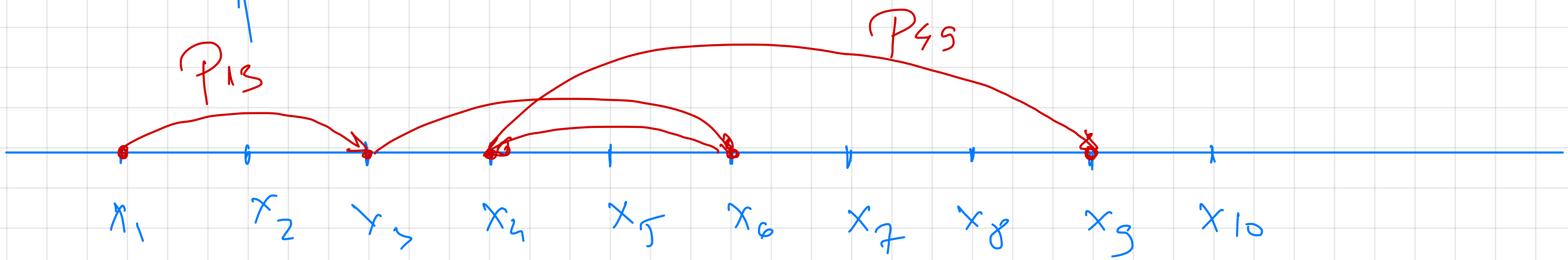
$$MTBF = \frac{1}{\lambda} + \frac{1}{\lambda + \lambda_{R2}}$$

Modele Markov

- Starea sistemului: $x_1, x_2, x_3 \dots x_n$
- timpul de observare: $t_1, t_2, t_3 \dots t_n$

Lanțuri Markov

- stările prin care trece sistemul - discrete
- timpul de observare - discret

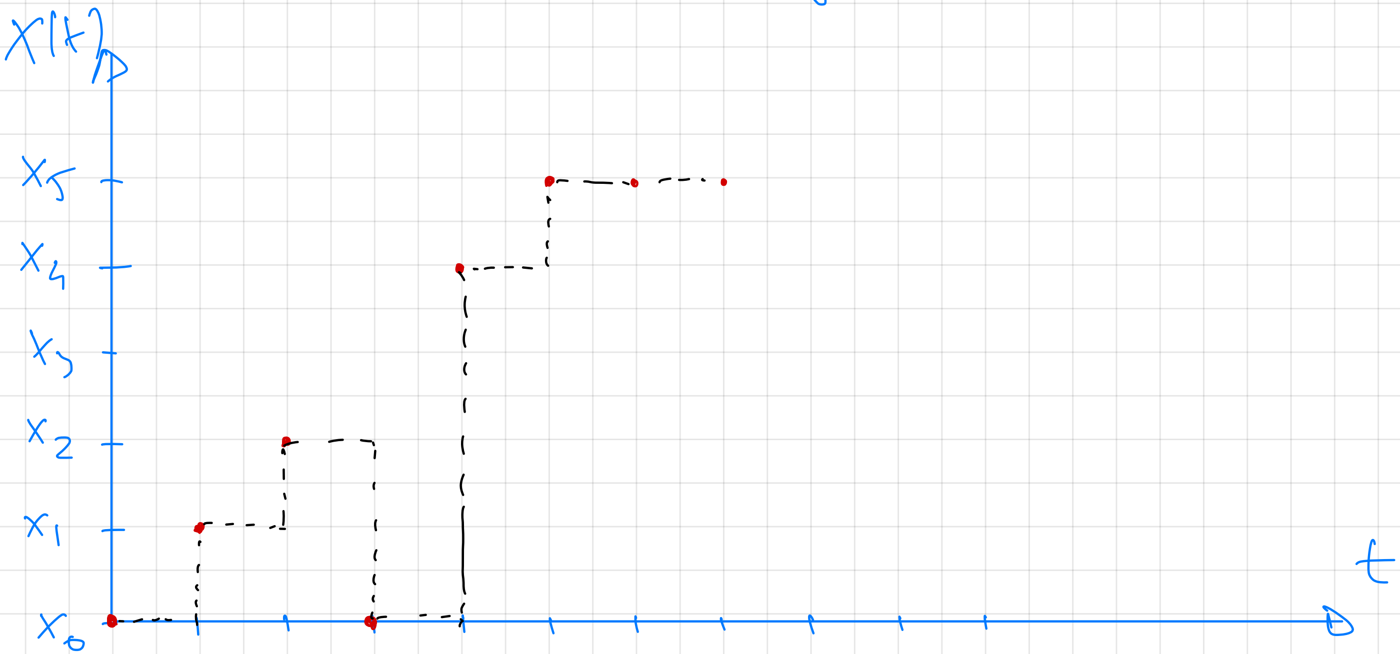


$x_1, x_2 \dots x_m$ - stările sistemului

$P_i(k) = P(S_i = x_k)$ - probabilitatea ca sistemul să fie în starea x_k

$$\sum_{i=1}^{n} P_i(k) = 1$$

$P_{ij} = P(X_k = S_i, X_{k+1} = S_j) \to$ prob. de tranziție din starea i în starea j.



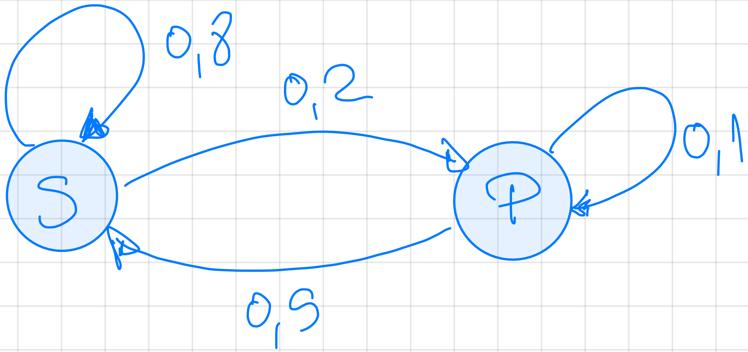
$$\begin{pmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ - & - & - & - \\ P_{n1} & P_{n2} & \dots & P_{nn} \end{pmatrix} = A$$

$$[P(t+\Delta t)] = [P(t)] \cdot A$$

vector de prob.

Ex: Starea vremii

Stările sistemului : Soare (S), Ploaie (P)



$P_S(t)$ - probabilitatea să fie soare

$P_P(t)$ - probabilitatea să fie ploaie

$$A = \begin{pmatrix} 0,8 & 0,2 \\ 0,5 & 0,1 \end{pmatrix}$$

$$\left[P_S(t+\Delta t) \quad P_P(t+\Delta t)\right] = \left[P_S(t) \quad P_P(t)\right] \cdot A$$

$$\underbrace{\begin{array}{cc} S & P \\ \end{array}}_{} $$

$$\underset{\text{azi}}{\begin{pmatrix} 1 & 0 \end{pmatrix}} \cdot \begin{pmatrix} 0,8 & 0,2 \\ 0,5 & 0,1 \end{pmatrix} = \underset{\text{mâine}}{\begin{pmatrix} 0,8 & 0,2 \end{pmatrix}} \begin{pmatrix} 0,8 & 0,2 \\ 0,5 & 0,1 \end{pmatrix} = \underset{\text{poimâine}}{\begin{pmatrix} 0,82 & 0,18 \end{pmatrix}} \ldots$$

Cum este vremea după o perioadă lungă de timp (steady state)?

→ Se stabilizează în jurul unei valori constante a vectorului?

$$(q_1\ \ q_2)\cdot A = (q_1\ \ q_2) \Rightarrow (q_1\ \ q_2)(A - I_2) = (0\ \ 0) \Rightarrow$$

$$(q_1\ \ q_2)\cdot\begin{pmatrix}-0,2 & 0,2\\ 0,9 & -0,9\end{pmatrix} = (0\ \ 0) \Rightarrow 0,2q_1 - 0,9\cdot q_2 = 0 \Rightarrow 2q_1 - 9q_2 = 0$$

$$\begin{cases} 2q_1 - 9q_2 = 0 \\ q_1 + q_2 = 1 \end{cases} \Rightarrow 2(1 - q_2) - 9q_2 = 0 \Rightarrow 2 - 11q_2 = 0 \Rightarrow q_2 = \frac{2}{11}$$

$$q_1 = \frac{9}{11}, \quad q_2 = \frac{2}{11}$$

81% prob să fie soare

19% prob să fie ploaie

Procese Markov

$$S_i \xrightarrow{P_{ij}} S_j$$

λ_{ij} – densitate de probabilitate de tranziție din starea i în starea j

$$\lambda_{ij} = \lim_{\Delta t \to 0} \frac{P\left(X(t)=S_i \mid X(t+\Delta t)=S_j\right)}{\Delta t}$$

dacă $\Delta t \to 0 \Rightarrow P_{ij} \simeq \lambda_{ij} \cdot \Delta t$

$$P(t) = \left(P_1(t) \quad P_2(t) \quad \dots \quad P_n(t) \right)$$

Se păstrează proprietatea:

$$P(t+\Delta t) = P(t) \cdot A$$

$$A = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ - & - & - & - \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{pmatrix} = \begin{pmatrix} \lambda_{11} \cdot \Delta t & \lambda_{12}\Delta t & \cdots & \lambda_{1n}\Delta t \\ \lambda_{21}\Delta t & \lambda_{22}\Delta t & \cdots & \lambda_{2n}\Delta t \\ - & - & - & - \\ \lambda_{n1}\Delta t & \lambda_{n2}\Delta t & \cdots & \lambda_{nn}\Delta t \end{pmatrix} = \begin{pmatrix} 1-\sum_{i=2}^{n}\lambda_{1i}\Delta t & \lambda_{12}\Delta t & \cdots & \lambda_{1n}\Delta t \\ \lambda_{21}\Delta t & 1-\sum_{\substack{i=1\\ i\neq 2}}^{n}\lambda_{2i}\Delta t & \cdots & \lambda_{2n}\Delta t \\ - & - & - & - \\ \lambda_{n1}\Delta t & \lambda_{n2}\Delta t & \cdots & 1-\sum_{i=1}^{n-1}\lambda_{ni}\Delta t \end{pmatrix}$$

$$P(t)\cdot A = (P_1(t)\ P_2(t)\ \cdots P_n(t)) \cdot \begin{pmatrix} 1-\sum_{i=2}^{n}\lambda_{1i}\Delta t & \lambda_{12}\Delta t & \cdots & \lambda_{1n}\Delta t \\ \lambda_{21}\Delta t & 1-\sum_{\substack{i=1\\ i\neq 2}}^{n}\lambda_{2i}\Delta t & \cdots & \lambda_{2n}\Delta t \\ - & - & - & - \\ \lambda_{n1}\Delta t & \lambda_{n2}\Delta t & \cdots & 1-\sum_{i=1}^{n-1}\lambda_{ni}\Delta t \end{pmatrix}$$

$$P_1(t+\Delta t) = P_1(t)\cdot\left(1-\sum_{i=2}^{n}\lambda_{1i}\Delta t\right) + P_2(t)\lambda_{21}\Delta t + P_3(t)\lambda_{31}\Delta t + \cdots P_n(t)\lambda_{n1}\Delta t$$

$$P_1(t+\Delta t) - P_1(t) = -P_1(t)\sum_{i=2}^{n}\lambda_{1i}\Delta t + P_2(t)\lambda_{21}\Delta t + \cdots P_n(t)\lambda_{n1}\Delta t$$

$$\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t} = -P_1(t)\sum_{i=2}^{n}\lambda_{1i} + P_2(t)\lambda_{21} + \dots P_n(t)\lambda_{n1}$$

$$\lim_{\Delta t \to 0} \frac{P_1(t+\Delta t)-P_1(t)}{\Delta t} = -P_1(t)\sum_{i=2}^{n}\lambda_{1i} + P_2(t)\lambda_{21} + \dots + P_n(t)\lambda_{n1}$$

$$\frac{dP_1(t)}{dt} = \sum_{i=2}^{n}\lambda_{i1}P_i(t) - P_1(t)\sum_{i=2}^{n}\lambda_{1i}$$

$$\frac{dP_j(t)}{dt} = \sum_{\substack{i=1\\ i\neq j}}^{n}\lambda_{ij}P_i(t) - P_j(t)\sum_{\substack{i=1\\ i\neq j}}^{n}\lambda_{ji}, \quad (\forall)\, j=\overline{1,n}$$

Sistem de ecuații Chapman - Kolmogorov (C-K)

Forma matriceală a sistemului C-K

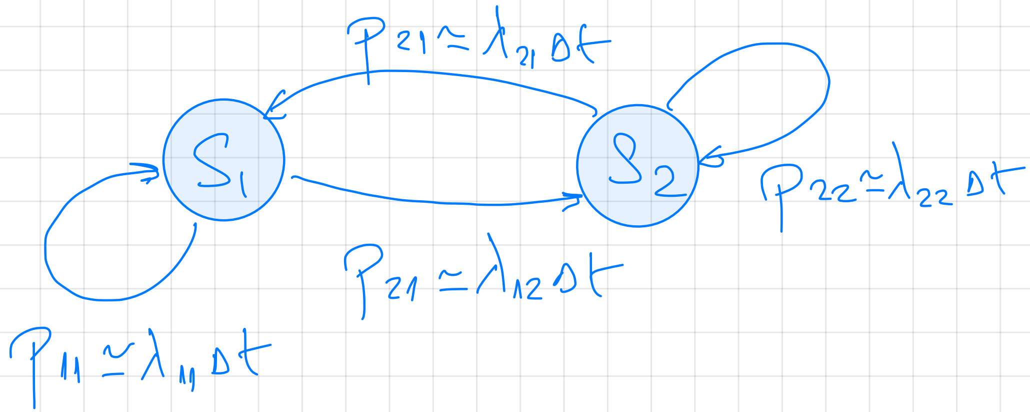
$$[P(t)] \cdot A^* = [P'(t)]$$

$$A^* = \begin{pmatrix} -\sum_{i=2}^{n} \lambda_{1i} & \lambda_{12} & \lambda_{13} & \dots & \lambda_{1n} \\ \lambda_{21} & -\sum_{\substack{i=1 \\ i\neq 2}}^{n} \lambda_{2i} & \lambda_{22} & \dots & \lambda_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \lambda_{n1} & \lambda_{n2} & \dots & & -\sum_{i=1}^{n-1} \lambda_{ni} \end{pmatrix}$$

Sistem cu două stări

S_1 – funcționare

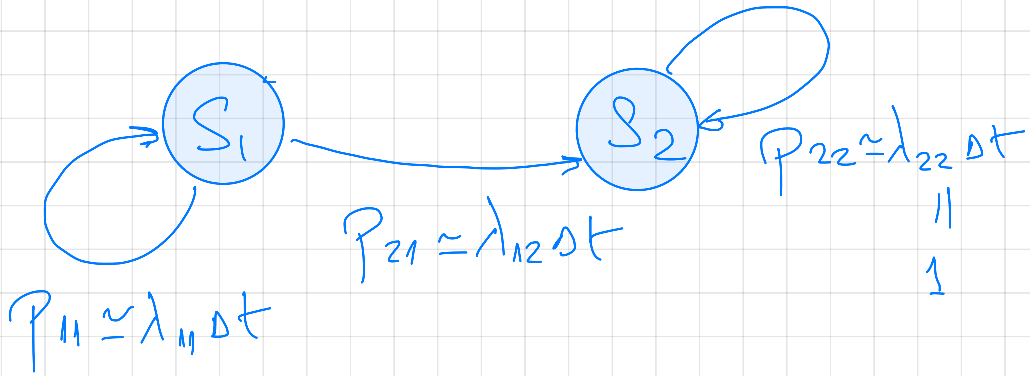
S_2 – defect



$$[P(t)] \cdot A = [P(t+\Delta t)] \Rightarrow [P_1(t) \;\; P_2(t)] \cdot \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} = [P_1(t+\Delta t) \;\; P_2(t+\Delta t)]$$

Caz simplificator: sistemul nu poate fi reparat $\Rightarrow P_{21}(t) = 0$

$$A = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} = \begin{pmatrix} \lambda_{11} \Delta t & \lambda_{12} \Delta t \\ 0 & 1 \end{pmatrix}$$



$$A^* = \begin{pmatrix} -\lambda_{12} & \lambda_{12} \\ 0 & 0 \end{pmatrix}$$

$$[P(t)] \, A^* = [P'(t)] \Rightarrow$$

$$[P_1(t) \;\; P_2(t)] \cdot \begin{pmatrix} -\lambda_{12} & \lambda_{12} \\ 0 & 0 \end{pmatrix} = \left[\frac{dP_1(t)}{dt} \;\; \frac{dP_2(t)}{dt} \right] \Rightarrow$$

$$\frac{dP_1(t)}{dt} = -\lambda_{12} P_1(t) \Rightarrow \frac{dP_1(t)}{P_1(t)} = -\lambda_{12}\, dt \Rightarrow \int \frac{1}{P_1(t)}\, dP_1(t) = -\lambda_{12} \int dt$$

$$\frac{dP_2(t)}{dt} = \lambda_{12} P_1(t)$$

$$\Rightarrow \ln(P_1(t)) = -\lambda_{12}(t + c) \Rightarrow P_1(t) = e^{-\lambda_{12}(t+c)} = K \cdot e^{-\lambda_{12} t}$$

$$P_1(t) = K \cdot e^{-\lambda_{12} t}$$

$$P_1(0) = K$$

Presupunem că sistemul a pornit din starea de funcționare:

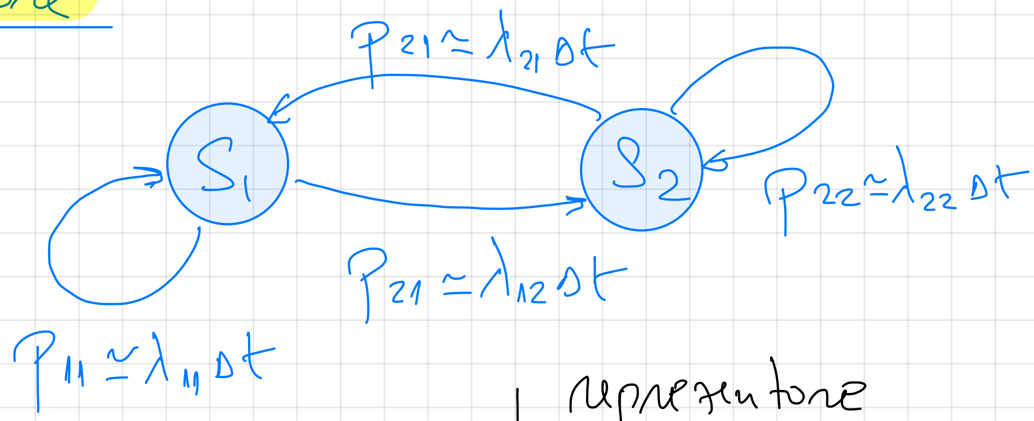
$$P_1(0) = 1 = K \Rightarrow P_1(t) = e^{-\lambda_{12} t}$$

$$P_2(t) = 1 - P_1(t) = 1 - e^{-\lambda_{12} t}$$

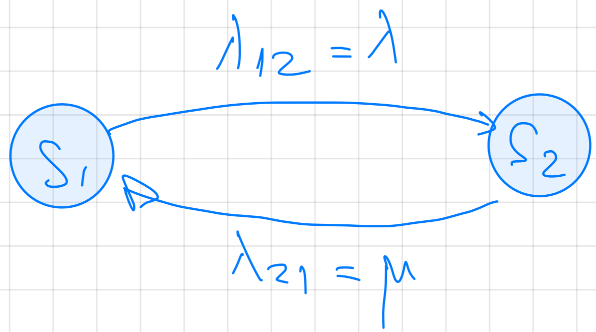
Sistem cu două stări + reparare

S_1 - funcționează

S_2 - defect



reprezentare simplificată



$$A^* = \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix}$$

$$\begin{bmatrix} P_1(t) & P_2(t) \end{bmatrix} \cdot \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix} = \begin{bmatrix} \frac{dP_1}{dt} & \frac{dP_2}{dt} \end{bmatrix}$$

$$\begin{cases} \frac{dP_1(t)}{dt} = -\lambda P_1(t) + \mu P_2(t) \\ \frac{dP_2(t)}{dt} = \lambda P_1(t) - \mu P_2(t) \end{cases}$$

$$P_1(t) + P_2(t) = 1$$

λ - intensitatea defectărilor

μ - intensitatea reparațiilor

$$\frac{dP_1(t)}{dt} = -\lambda P_1(t) + \mu(1 - P_1(t)) = -(\lambda+\mu) P_1(t) + \mu$$

Flashback:

$$\frac{dy(t)}{dt} + p(t)\, y(t) = g(t) \rightarrow$$ ec. diferențială liniară de gr. 1

Folosim o funcție ajutătoare $\mu(t)$ cu proprietatea $\mu'(t) = \mu(t) \cdot p(t)$

$$\mu(t) \rightarrow \mu(t) \cdot p(t) = \mu'(t) \Leftrightarrow \frac{\mu'(t)}{\mu(t)} = p(t) \Big| \int \Rightarrow \int \frac{1}{\mu(t)}\, d\mu(t) = \int p(t)\, dt \Rightarrow$$

$$\Rightarrow \ln(\mu(t)) = \int p(t)\, dt + k \Rightarrow \mu(t) = e^{\int p(t)\, dt + k} \Rightarrow$$

$$\Rightarrow \mu(t) = k \cdot e^{\int p(t)\, dt}$$

$$\frac{dy(t)}{dt} + p(t)\, y(t) = g(t) \Big| \cdot \mu(t) \Rightarrow$$

$$\mu(t)\, y'(t) + \mu(t)\, p(t)\, y(t) = \mu(t)\, g(t) \Leftrightarrow \mu(t)\, y'(t) + \mu'(t)\, y(t) = \mu(t)\, g(t)$$

$$\Rightarrow \left(\mu(t)\, y(t)\right)' = \mu(t)\, g(t) \Rightarrow \int \left(\mu(t)\, y(t)\right)' dt = \int \mu(t)\, g(t)\, dt \Rightarrow$$

$$\mu(t)\,y(t) + C = \int \mu(t)\,g(t)\,dt \Rightarrow y(t) = \frac{\int \mu(t)\,g(t)\,dt + C}{\mu(t)}$$

$$\Rightarrow y(t) = \frac{\int k e^{\int p(t)\,dt} \cdot g(t)\,dt + C}{k e^{\int p(t)\,dt}} = \frac{\int e^{\int p(t)\,dt} g(t)\,dt + K}{e^{\int p(t)\,dt}}$$

Prin urmare, soluția pentru ecuația diferențială:

$$\frac{dy}{dt}(t) + p(t)\,y(t) = g(t)$$

este:

$$y(t) = \frac{\int e^{\int p(t)\,dt} g(t)\,dt + K}{e^{\int p(t)\,dt}}$$

end of flashback

Revenind la sistemul cu două stări și reparare:

$$\frac{dP_1(t)}{dt} = -(\lambda+\mu)P_1(t) + \mu \Rightarrow$$

$$\frac{dP_1(t)}{dt} + (\lambda+\mu)P_1(t) = \mu \Rightarrow p(t) = \lambda+\mu \quad g(t) = \mu$$

$$P_1(t) = \frac{\int e^{\int(\lambda+\mu)dt} \cdot \mu dt + K}{e^{\int(\lambda+\mu)dt}} = \frac{\int e^{(\lambda+\mu)t} \cdot \mu dt + K}{e^{(\lambda+\mu)t}} = \frac{\frac{\mu}{\lambda+\mu} e^{(\lambda+\mu)t} + K}{e^{(\lambda+\mu)t}}$$

$$= \frac{\mu}{\lambda+\mu} + K e^{-(\lambda+\mu)t}$$

Presupunem că sistemul pornește în stare de funcționare:

$$P_1(0) = 1 \Rightarrow P_1(0) = \frac{\mu}{\lambda+\mu} + K = 1 \Rightarrow K = \frac{\lambda}{\lambda+\mu}$$

$$\Rightarrow P_1(t) = \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} + \frac{\mu}{\lambda+\mu}$$

$P_1(t) = R(t)$ – fiabilitatea sistemului

$P_1(0) = 1$; si $\lim_{t\to\infty} P_1(t) = \frac{\mu}{\lambda+\mu}$ → pentru că am introdus posibilitatea de reparare, fiabilitatea sistemului nu mai tinde spre zero.

Dar $\lambda = \frac{1}{MTBF}$ si $\mu = \frac{1}{MTR}$ (MTR = media timpului de reparare) ⇒

$$\Rightarrow \frac{\mu}{\lambda+\mu} = \frac{\frac{1}{\lambda}}{\frac{1}{\mu}+\frac{1}{\lambda}} = \frac{MTBF}{MTR+MTBF} = A$$ (disponibilitatea sistemului)