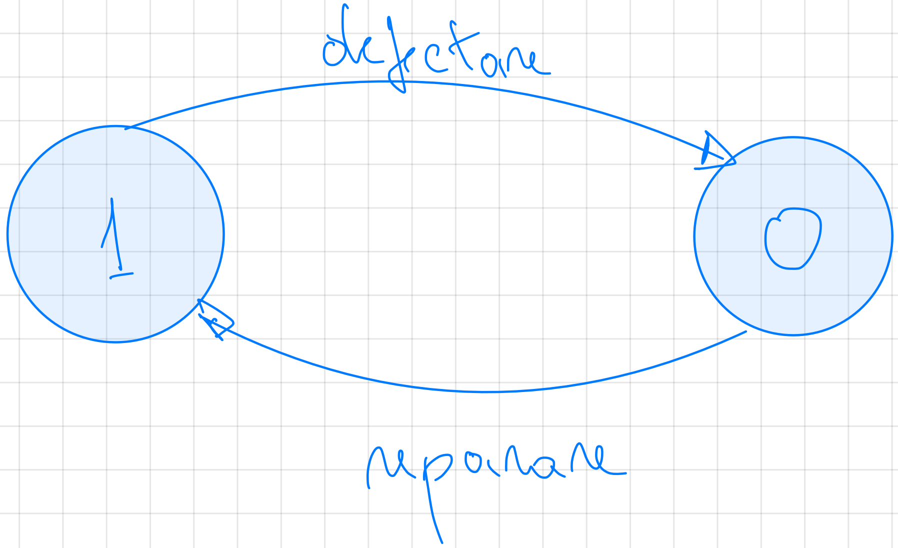
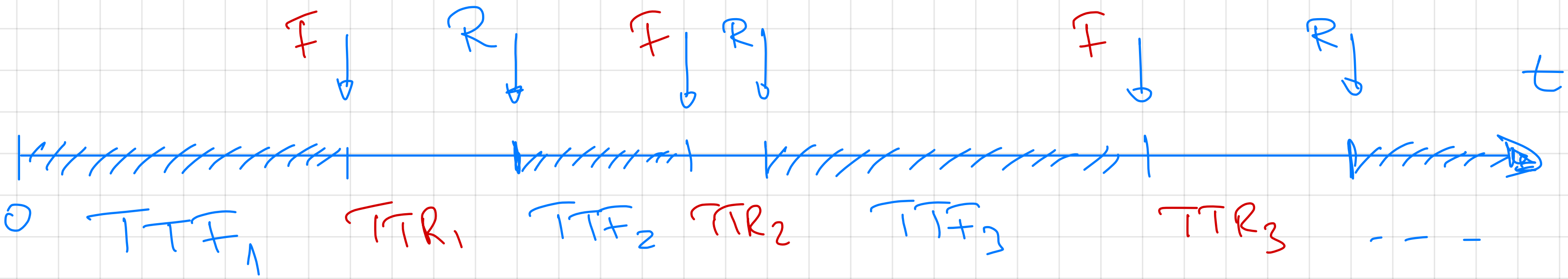
Fiabilitate și toleranță la defecte

$R(t)$ - fiabilitatea

$$R(t) = P(T > t \mid OK \,@\, t=0)$$





$$MTBF = \sum_i \frac{TTF_i}{n}$$

$$MTR = \sum_i \frac{TTR_i}{n}$$

Disponibilitate - $A(t)$

$$A = \frac{\sum_i TTF_i}{\sum_i TTF_i + \sum_i TTR_i} = \frac{MTBF}{MTBF + MTR}$$

Availability (%)	Downtime / year	Downtime / month	Downtime / week
90% ("one nine")	36,5 days	72 h	16,8 h
99% ("2 nines")	3,65 days	7,2 h	1,68 h
99,9% ("3 nines")	8,76 h	43,2 min	10,1 min
99,99% ("4 nines")	52,56 min	4,32 min	1,01 min
99,999%	5,25 min	25,9 s	6,05 s
99,9999%	31,5 s	2,59 s	0,605 s

Probability theory 101

$$0 \leq P(A) \leq 1$$

$$P(\bar{A}) = 1 - P(A)$$

$P(A|B)$ – Prob A cond de B

$$P(A \cdot B) = P(A|B) \cdot P(B)$$

Dacă A și B sunt independente

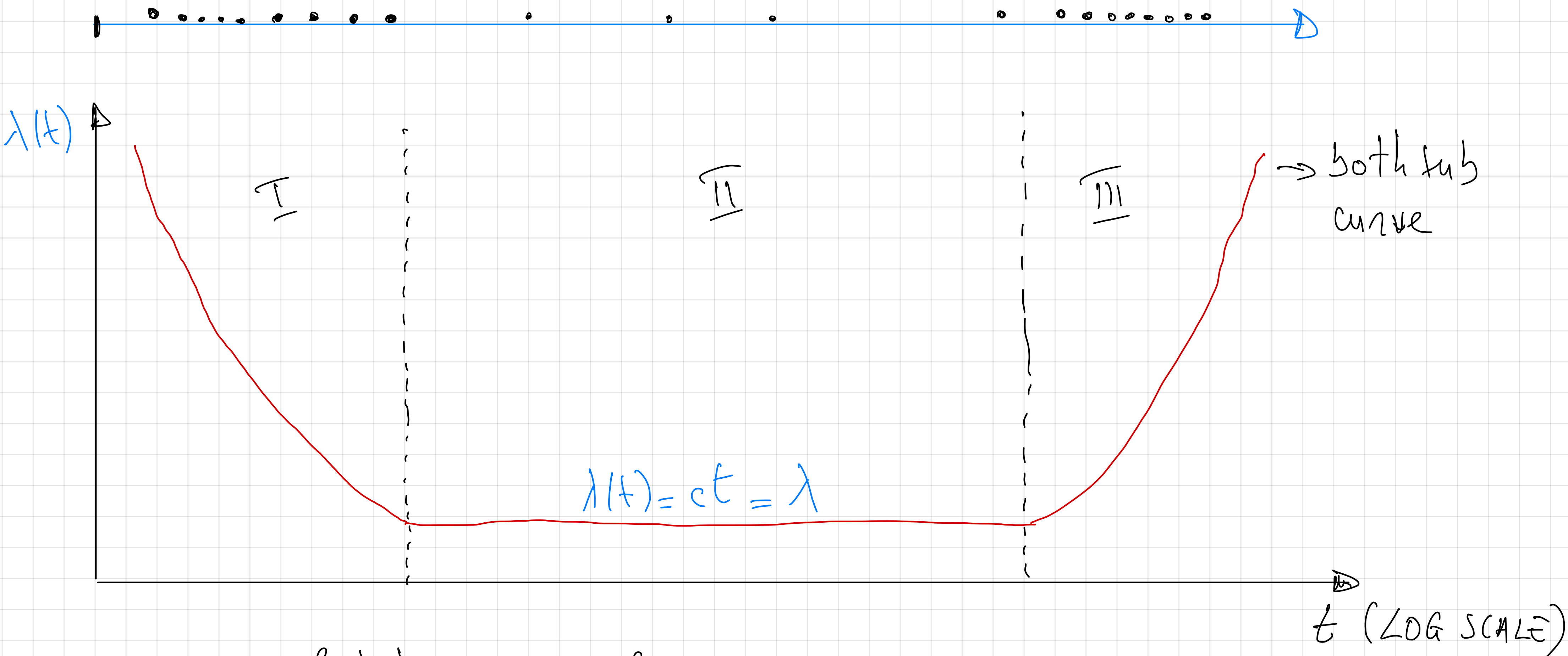
$$P(A \cdot B) = P(A) \cdot P(B)$$

$$P(A + B) = P(A) + P(B) - P(A \cdot B)$$

Dacă A și B sunt mutual exclusive $\Rightarrow P(A \cdot B) = P(B \cdot A) = 0$

$$P(A + B) = P(A) + P(B)$$

Failure Rate → $\lambda(t)$ – intensitatea defecțiunilor



I – mortalitate infantilă

II – viața utilă

III – îmbătrânire

Failure rate - software

Câteva definiții noi:

$f(t)$ – funcție densitate probabilitate (pdf)

$F(t)$ – funcție cumulativă distribuție probabilitate

$$\boxed{f(t) = \frac{dF(t)}{dt}} \qquad \boxed{F(t) = \int_0^t f(\tau)\,d\tau}$$

$$R(t) = 1 - F(t)$$

$$\lambda(t) = \frac{f(t)}{1-F(t)} \Rightarrow \boxed{\lambda(t) = \frac{f(t)}{R(t)}}$$

$$f(t) = \frac{dF(t)}{dt} = \frac{d(1-R(t))}{dt} = -\frac{dR(t)}{dt}$$

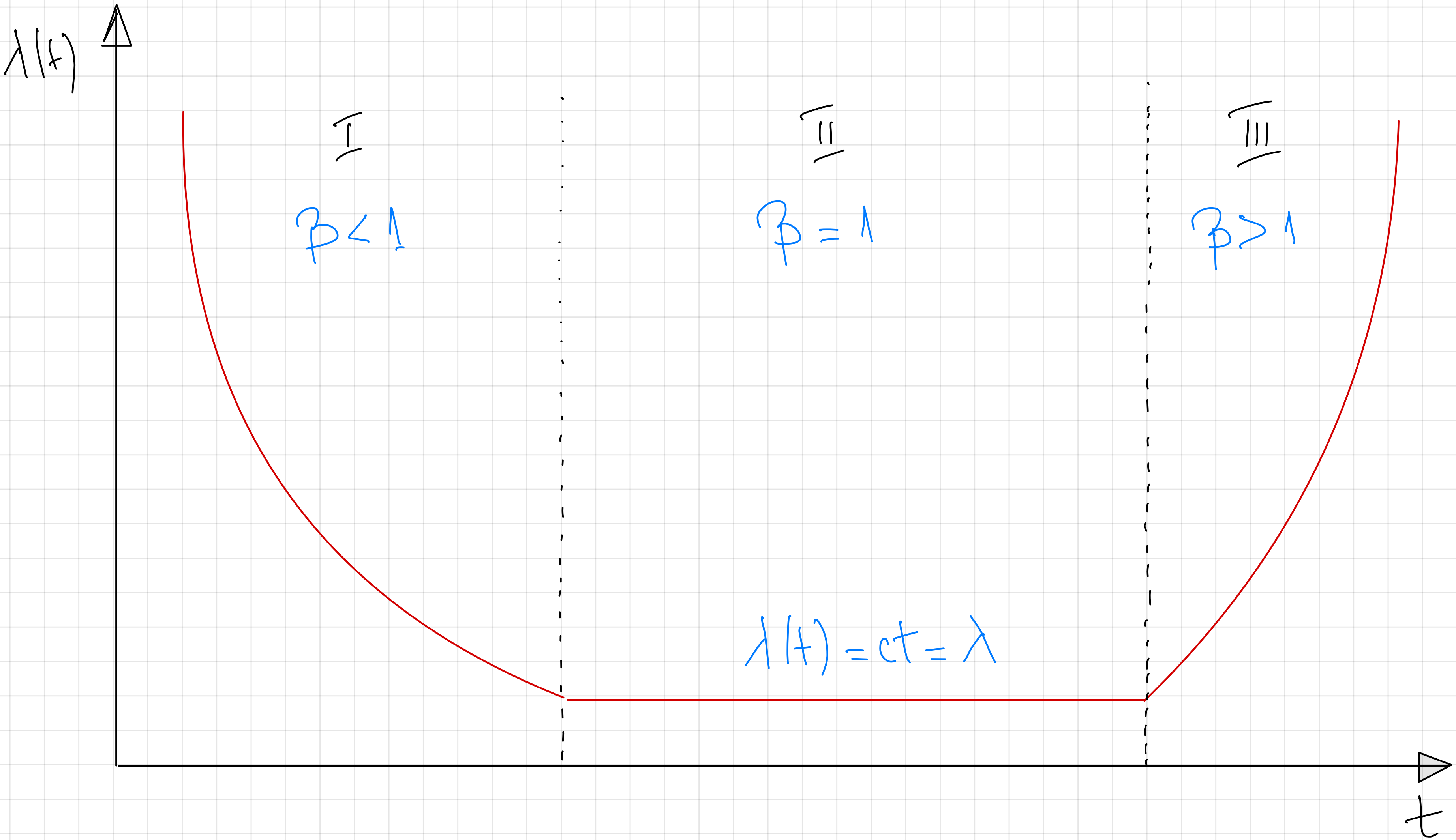
$$\lambda(t) = \frac{-\frac{dR(t)}{dt}}{R(t)} = -\frac{1}{R(t)}\frac{dR(t)}{dt}$$

Aproximare matematică a $\lambda(t)$ - folosim distribuția Weibull

$$f(t) = \lambda \beta t^{\beta-1} e^{-\lambda t^{\beta}}$$

$$\lambda(t) = \lambda \beta t^{\beta-1}$$

- dacă $\beta < 1 \Rightarrow \lambda(t) \searrow$ (zona I pe grafic)
- dacă $\beta = 1 \Rightarrow \lambda(t) = ct$ (zona II pe grafic)
- dacă $\beta > 1 \Rightarrow \lambda(t) \nearrow$ (zona III pe grafic)



1. Cazul în care $\lambda(t) = ct = \lambda$ (perioada de viață utilă a produsului)

$$\lambda(t) = \lambda = -\frac{1}{R(t)}\frac{dR(t)}{dt} \Leftrightarrow \lambda\, dt = -\frac{1}{R(t)}\, dR(t) \Rightarrow$$

$$\Rightarrow \int \lambda\, dt = -\int \frac{1}{R(t)}\, dR(t) \Rightarrow \lambda t + C_1 = -\ln R(t) + C_2 \Rightarrow$$

$$\Rightarrow \ln R(t) = -\lambda t + C \Rightarrow R(t) = e^{-\lambda t + C} = K \cdot e^{-\lambda t}$$

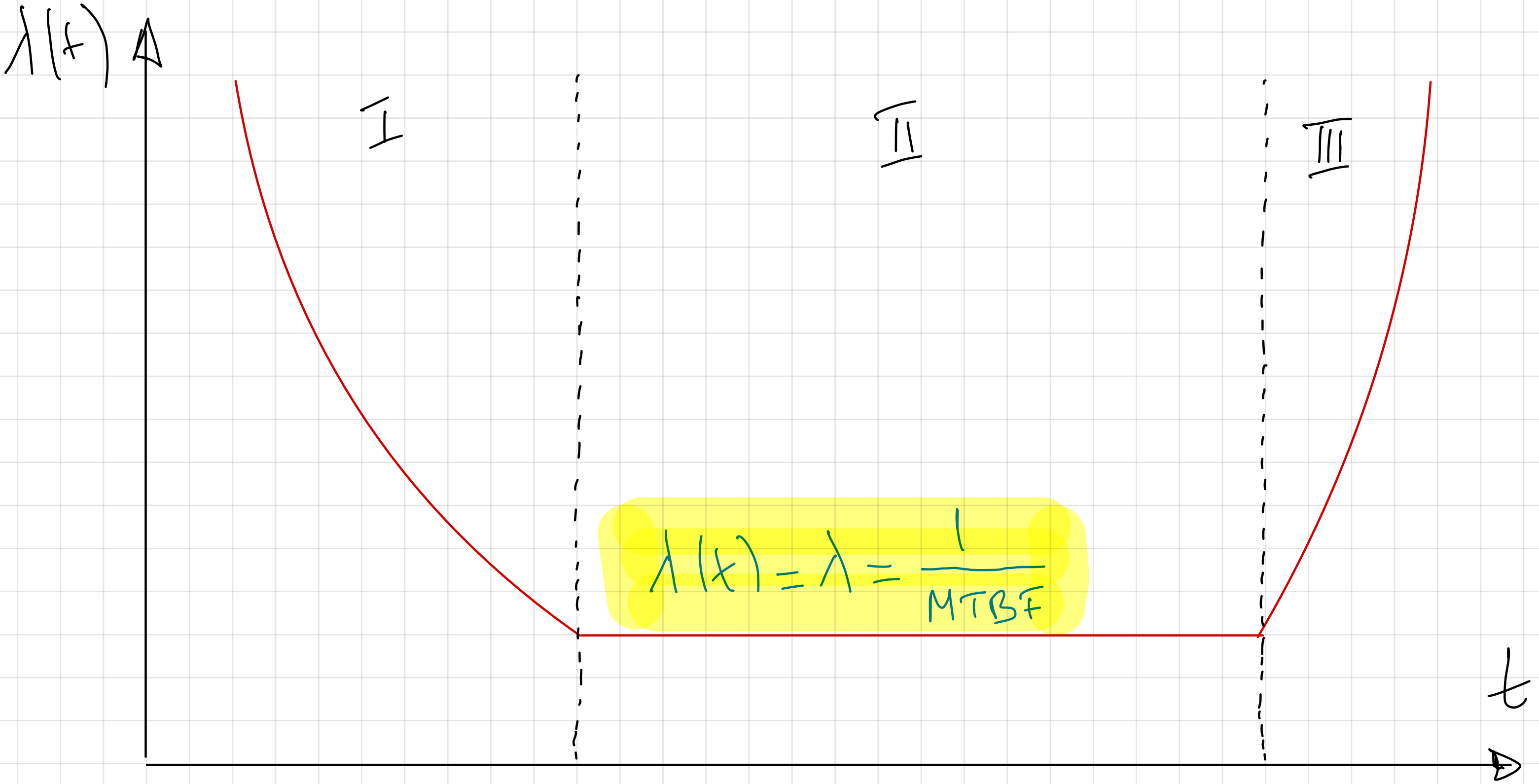
Dacă $t = 0 \Rightarrow R(0) = K$ – putem să presupunem că sistemul pornește la momentul zero de timp în stare de funcționare, deci $R(0) = 1$ (fiabilitate 100%) – alegem $K = 1 \Rightarrow$

$$R(t) = e^{-\lambda t}$$

$$MTBF = \int_0^\infty R(t)\,dt = \int_0^\infty e^{-\lambda t}\,dt = -\frac{1}{\lambda} e^{-\lambda t}\Big|_0^\infty = -\frac{1}{\lambda}\left(e^{-\lambda \infty} - e^{-\lambda \cdot 0}\right) =$$

$$= -\frac{1}{\lambda}(0 - 1) = \frac{1}{\lambda}$$

Deci, pt $\lambda(t) = ct = \lambda$, $MTBF = \frac{1}{\lambda}$



2. Dacă $\lambda(t) \neq ct$ (pt. zonele mortalitate infantilă și îmbătrânire)

$$f(t) = \lambda\beta t^{\beta-1} e^{-\lambda t^{\beta}}$$

$$F(t) = \int_0^t f(z)dz = \int_0^t \lambda\beta z^{\beta-1} e^{-\lambda z^{\beta}} dz = \ldots = 1 - e^{-\lambda t^{\beta}}$$

dar $R(t) = 1 - F(t) \Rightarrow R(t) = e^{-\lambda t^{\beta}}$

$$MTBF = \int_0^\infty R(t)dt = \int_0^\infty e^{-\lambda t^{\beta}} dt = \frac{\Gamma(\beta^{-1})}{\beta \lambda^{\beta^{-1}}}$$, unde $\Gamma(x)$ este funcția

gamma, definită prin: $\Gamma(x) = \int_0^\infty e^{-y} y^{x-1} dy$ și este extensia pt. numere reale a funcției factoriale $(n!)$

$$\Gamma(x) = (-1+x)\,\Gamma(-1+x)$$

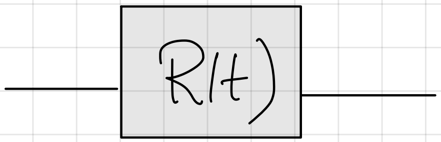
$$\Gamma(x) = \frac{\Gamma(x+1)}{x}$$

$$\Gamma(0) = \Gamma(1) = 1$$

Estimarea fiabilității

Folosim diagrame pentru a modela și estima fiabilitatea

Modularizare: orice sistem, oricât de complex, poate fi modelat fiabilistic ca un modul de fiabilitate dată $R(t)$:

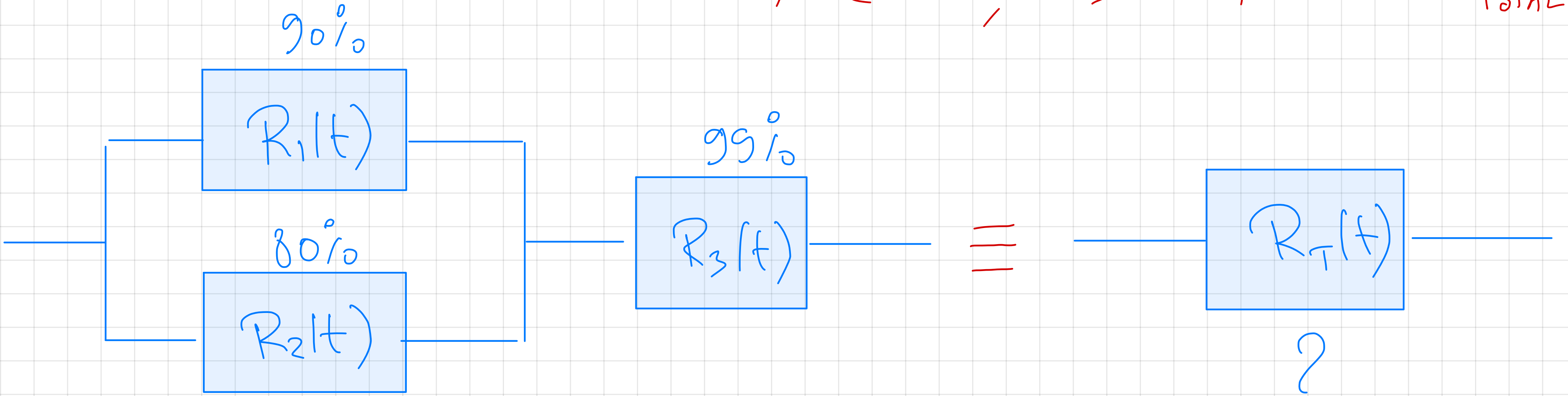


→ Poate fi orice dispozitiv (procesor, calculator, telefon etc) sau orice secvență de cod (bibliotecă, funcție, clasă etc.)

De ex.: Două procesoare ce accesează aceeași memorie.

Primul procesor are o fiabilitate $R_1(t)$, al doilea $R_2(t)$ și memoria $R_3(t)$

Dacă $R_1 = 90\%$, $R_2 = 80\%$ și $R_3 = 99\%$, cât este R_{TOTAL}?

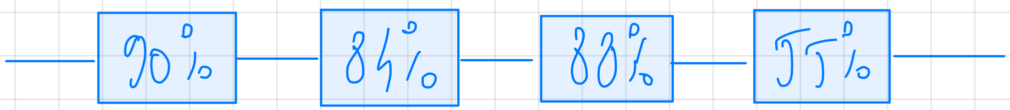


Structuri serie — n module în serie

$$R_s(t) = R_1(t)\cdot R_2(t)\cdot \ldots \cdot R_n(t) = \prod_{i=1}^{n} R_i(t)$$

$$R_s(t) = \prod_{i=1}^{n} R_i(t)$$

De ex.:



$$R_s = 0,9 \cdot 0,84 \cdot 0,88 \cdot 0,55 = 0,3659 \simeq 0,37 = 37\%$$

Dacă $R_i(t) = e^{-\lambda_i t}$

$$R_s(t) = \prod_{i=1}^{n} e^{-\lambda_i t} = e^{-\sum_{i=1}^{n} \lambda_i t}$$

$$R_s(t) = e^{-\sum_{i=1}^{n} \lambda_i t} \qquad \lambda_s = \sum_{i=1}^{n} \lambda_i$$

Definim $Q_i(t) \to$ inversul fiabilității

$$Q_i(t) = 1 - R_i(t)$$

$$R_s(t) = \prod_{i=1}^{n} R_i(t)$$

$$Q_s(t) = 1 - R_s(t) = 1 - \prod_{i=1}^{n} (1 - Q_i(t))^n$$

R_i este mare $> 90\% \Rightarrow Q_i < 10\%$ $\quad Q_i Q_j \simeq 0$

$$Q_s(t) = 1 - \left(1 - \sum_{i=1}^{n} Q_i + \underbrace{\sum_{\substack{i=1 \\ j=1 \\ i \neq j}}^{n} Q_i Q_j}_{\simeq 0} \dots \right) \simeq 1 - \left(1 - \sum_{i=1}^{n} Q_i\right) \Rightarrow$$

$$\boxed{Q_s(t) = \sum_{i=1}^{n} Q_i(t)}$$

$$MTBF_i = \int_0^\infty R_i(t)\,dt = \int_0^\infty e^{-\lambda_i t}\,dt = \frac{1}{\lambda_i}$$

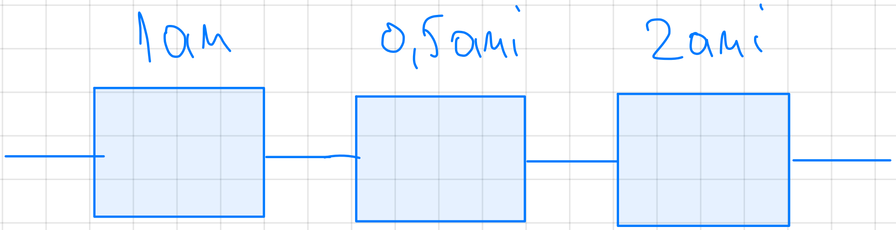
$$MTBF_S = \int_0^\infty R_S(t)\,dt = \int_0^\infty e^{-\sum_{i=1}^{n}\lambda_i t}\,dt = \int_0^\infty e^{-\lambda_s t}\,dt =$$

$$= -\frac{1}{\lambda_s} e^{-\lambda_s t}\Big|_0^\infty = -\frac{1}{\lambda_s}(0-1) = \frac{1}{\lambda_s}$$

$$\boxed{MTBF_S = \frac{1}{\lambda_S}}$$

Dar $\lambda_i = \frac{1}{MTBF_i} \Rightarrow$

$$MTBF_S = \frac{1}{\lambda_1+\lambda_2+\dots\lambda_n} = \frac{1}{\frac{1}{MTBF_1}+\frac{1}{MTBF_2}+\dots\frac{1}{MTBF_n}} = \frac{1}{\sum_{i=1}^{n}\frac{1}{MTBF_i}}$$



$$MTBF_S = \frac{1}{\frac{1}{1}+\frac{1}{0,5}+\frac{1}{2}} = \frac{1}{3+\frac{1}{2}} = \frac{1}{\frac{7}{2}} = \frac{2}{7}\text{ ani}$$

$$MTBF_S = 0,285 \text{ ani}$$

Dacă avem n module identice în serie, atunci:

$$R_1(t) = R_2(t) = \dots R_n(t) = R(t) = e^{-\lambda t}$$

$$R_S(t) = \prod_{i=1}^{n} R(t) = \prod_{i=1}^{n} e^{-\lambda t} = e^{-n\lambda t}$$

$$R_S(t) = e^{-n\lambda t} = R^n(t)$$

$$\lambda_S(t) = \sum_{i=1}^{n} \lambda_i(t) = \sum_{i=1}^{n} \lambda = n\lambda$$

$$\lambda_S = n\lambda$$

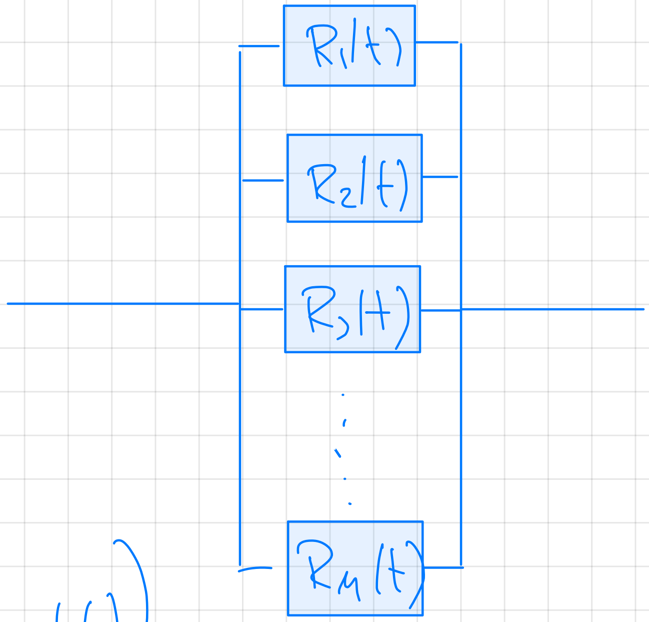
$$MTBF_S = \frac{1}{\lambda_S} = \frac{1}{n\lambda} = \frac{MTBF}{n}$$

$$MTBF_S = \frac{MTBF}{n}$$

În general, pt structura serie, R total scade, λ total crește și MTBF total scade.

Structura paralel

$R_p(t) = ? = 1 - Q_p(t)$



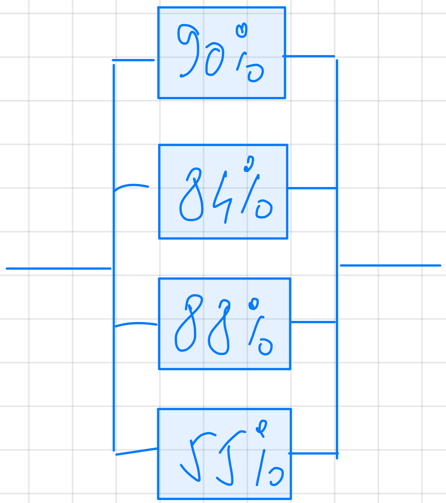
$Q_p(t) = Q_1(t) \cdot Q_2(t) \dots Q_n(t) =$

$= (1 - R_1(t))(1 - R_2(t)) \dots (1 - R_n(t))$

$R_p(t) = 1 - (1 - R_1(t))(1 - R_2(t)) \dots (1 - R_n(t))$

$$R_p(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

De ex: patru module in paralel



$R_p = 1 - (1 - 0,9)(1 - 0,84)(1 - 0,88)(1 - 0,55) =$

$= 1 - 0,1 \cdot 0,16 \cdot 0,12 \cdot 0,45 = 0,9991$

$R_p = 99,91\%$

$$R_i(t) = e^{-\lambda_i t} \Rightarrow R_p(t) = 1 - \prod_{i=1}^{n} \left(1 - e^{-\lambda_i t}\right)$$

$$R_p(t) = 1 - \left(1-e^{-\lambda_1 t}\right)\left(1-e^{-\lambda_2 t}\right) \dots \left(1-e^{-\lambda_n t}\right) \overset{?}{=} e^{-\lambda_p t}$$

Presupunem că $R_1(t) = R_2(t) = \dots = R_n(t) = e^{-\lambda t}$

$$\boxed{R_p(t) = 1 - \left(1-e^{-\lambda t}\right)^n}$$

$$\lambda_p(t) = \frac{f_p(t)}{R_p(t)} = \frac{-\frac{dR_p(t)}{dt}}{R_p(t)}$$

$$\frac{dR_p(t)}{dt} = -n\left(1-e^{-\lambda t}\right)^{n-1}\left(1-e^{-\lambda t}\right)' = -n\lambda e^{-\lambda t}\left(1-e^{-\lambda t}\right)^{n-1}$$

$$\boxed{\lambda_p(t) = n\lambda \frac{e^{-\lambda t}\left(1-e^{-\lambda t}\right)^{n-1}}{1-\left(1-e^{-\lambda t}\right)^n}}$$ – nu este constantă!

$$\lambda_{P,\text{steady state}} = \lim_{t\to\infty} \lambda_P(t) = \lim_{t\to\infty} n\lambda \frac{e^{-\lambda t}\left(1-e^{-\lambda t}\right)^{n-1}}{1-\left(1-e^{-\lambda t}\right)^{n}} =$$

$e^{-\lambda t} = x$, dacă $t \to \infty$, atunci $x \to 0$

$$= \lim_{x\to 0} n\lambda \frac{x(1-x)^{n-1}}{1-(1-x)^{n}} \overset{\text{l'Hospital}}{=} n\lambda \lim_{x\to 0} \frac{\left(x(1-x)^{n-1}\right)'}{\left(1-(1-x)^{n}\right)'} =$$

$$= n\lambda \lim_{x\to 0} \frac{(1-x)^{n-1} - x(n-1)(1-x)^{n-2}}{+n(1-x)^{n-1}} = n\lambda \lim_{x\to 0} \frac{1-x-x(n-1)}{n(1-x)}$$

$$= n\lambda \cdot \frac{1}{n} = \lambda$$

$\lambda_P = \lambda$, la steady-state

$$MTBF_p = \int_0^\infty R_p(t)\,dt = \int_0^\infty \left[1 - \prod_{i=1}^{n}(1 - R_i(t))\right] dt$$

Dacă modulele sunt identice $R(t) = e^{-\lambda t}$

$$MTBF_p = \int_0^\infty \left(1 - \left(1 - e^{-\lambda t}\right)^n\right) dt$$

$$\left(1 - e^{-\lambda t}\right)^n = 1 - n e^{-\lambda t} + C_n^2 e^{-2\lambda t} - C_n^3 e^{-3\lambda t} + \dots + (-1)^n e^{-n\lambda t}$$

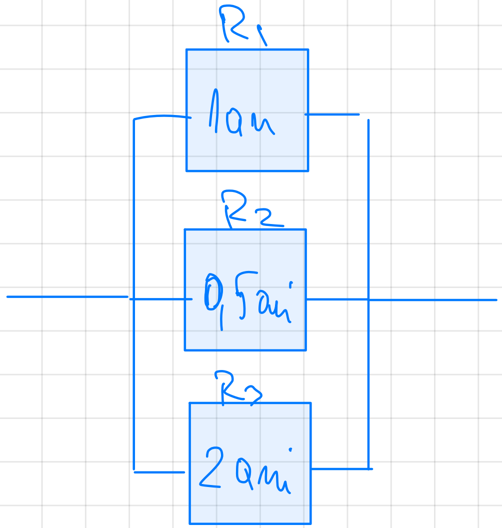
$$MTBF_p = \int_0^\infty \left(n e^{-\lambda t} - \frac{n(n-1)}{2} e^{-2\lambda t} + \dots + (-1)^{n+1} e^{-n\lambda t}\right) dt =$$

$$= n \frac{1}{\lambda} - \frac{n(n-1)}{2} \frac{1}{2\lambda} + \dots + (-1)^{n+1} \frac{1}{n\lambda} = \frac{1}{\lambda}\left(n - \frac{n(n-1)}{4} + \dots (-1)^{n+1} \frac{1}{n}\right)$$

$$= \frac{1}{\lambda} \sum_{i=1}^{n} C_n^i \frac{1}{i} (-1)^{i+1}$$

în general, R total crește, λ total rămâne constant și MTBF total crește

De ex.: trei module in paralel, $MTBF_1 = 1$ an, $MTBF_2 = 0,5$ ani, $MTBF_3 = 2$ ani



$$MTBF_P = \int_0^\infty R_P(t)\,dt = \int_0^\infty \left(1-(1-R_1)(1-R_2)(1-R_3)\right)dt$$

$$(1-R_1)(1-R_2)(1-R_3) = (1-R_2-R_1+R_1R_2)(1-R_3) =$$

$$= 1 - R_1 - R_2 + R_1R_2 - R_3 + R_2R_3 + R_1R_3 - R_1R_2R_3$$

$$\int_0^\infty \left(R_1+R_2+R_3 - R_1R_2 - R_2R_3 - R_1R_3 + R_1R_2R_3\right)dt$$

$$\int_0^\infty \left(e^{-\lambda_1 t} + e^{-\lambda_2 t} + e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t} - e^{-(\lambda_2+\lambda_3)t} - e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_1+\lambda_2+\lambda_3)t}\right)dt$$

$$= \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} - \frac{1}{\lambda_1+\lambda_2} - \frac{1}{\lambda_2+\lambda_3} - \frac{1}{\lambda_1+\lambda_3} + \frac{1}{\lambda_1+\lambda_2+\lambda_3} =$$

$$= MTBF_1 + MTBF_2 + MTBF_3 - \frac{1}{\frac{1}{MTBF_1}+\frac{1}{MTBF_2}} + \ldots = 1+0,5+2 - \frac{1}{1+2} - \frac{1}{2+\frac{1}{2}} - \frac{1}{1+\frac{1}{2}}$$

$$+ \frac{1}{1+2+\frac{1}{2}} = 3,5 - \frac{1}{3} - \frac{2}{5} - \frac{2}{3} + \frac{2}{7} = 2,385 \text{ ani}$$

Concluzii:

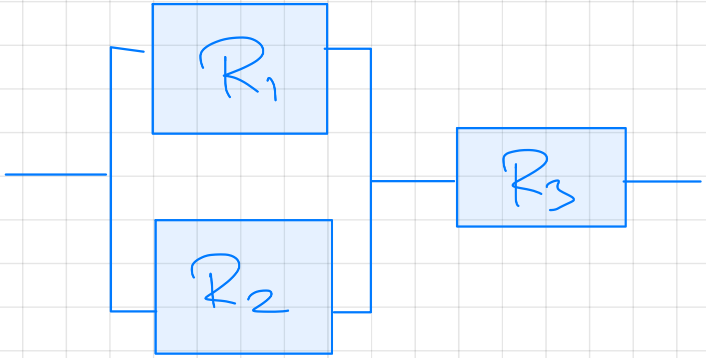
	n module identice în Serie	n module identice în Paralel
R_{TOTAL}	$R_s(t) = R^n(t) = e^{-n\lambda t}$	$R_p(t) = 1 - (1 - R(t))^n = 1 - (1 - e^{-\lambda t})^n$
λ_{TOTAL}	$\lambda_S = n\lambda$	$\lambda_p(t) = n\lambda \frac{e^{-\lambda t}(1 - e^{-\lambda t})^{n-1}}{1 - (1 - e^{-\lambda t})^n}$, $\lambda_{p,\text{stsbte}} = \lambda$
$MTBF_{TOTAL}$	$MTBF_S = \frac{MTBF}{n}$	$MTBF_p = MTBF \cdot \sum_{i=1}^{n} (-1)^{i+1} \cdot C_n^i \frac{1}{i}$

$$R_{TOTAL} = (R_1 \parallel R_2) \cdot R_3 =$$

$$= [1-(1-R_1)(1-R_2)]\, R_3 =$$

$$= [1-(1-R_2-R_1+R_1R_2)]\, R_3 =$$

$$= (R_1+R_2-R_1R_2)R_3 = R_1R_3+R_2R_3-R_1R_2R_3$$

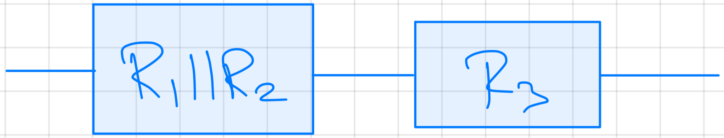


$R_1(t)=e^{-\lambda_1 t}$

$R_2(t)=e^{-\lambda_2 t}$

$R_3(t)=e^{-\lambda_3 t}$

$R_{TOTAL} = ?$



După un timp t $R_1(t)=80\%$, $R_2(t)=90\%$ și $R_3(t)=55\%$

$$R_{TOTAL} = 0{,}8 \cdot 0{,}55 + 0{,}9 \cdot 0{,}55 - 0{,}8 \cdot 0{,}9 \cdot 0{,}55 = \ldots$$

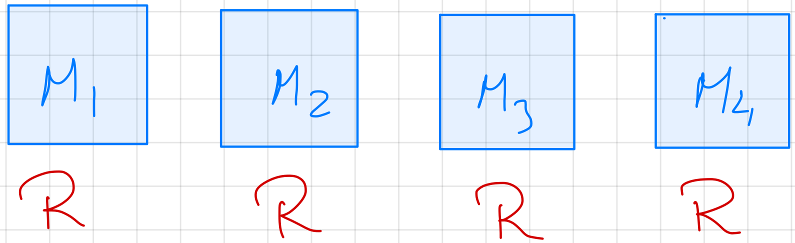
$MTBF_1 = 2$ ani $\quad MTBF_2 = 3$ ani și $MTBF_3 = 5$ ani

$$MTBF_{TOTAL} = \int_0^\infty R_{TOTAL}(t)\,dt = \int_0^\infty \left(e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_2+\lambda_3)t} - e^{-(\lambda_1+\lambda_2+\lambda_3)t}\right)dt =$$

$$= \frac{1}{\lambda_1+\lambda_3} + \frac{1}{\lambda_2+\lambda_3} - \frac{1}{\lambda_1+\lambda_2+\lambda_3} = \frac{1}{\frac{1}{2}+\frac{1}{5}} + \frac{1}{\frac{1}{3}+\frac{1}{5}} - \frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{5}} = \ldots$$

Fiabilitatea structurilor r din n

Ex: avion cu 4 motoare, poate tolera maxim 2 motoare defecte (2 din 4)



Fiecare motor are aceeași fiabilitate R(t):

$$R_{2/4} = R^4 + 4\cdot R^3\cdot(1-R) + 6R^2(1-R)^2$$

Cazul general:

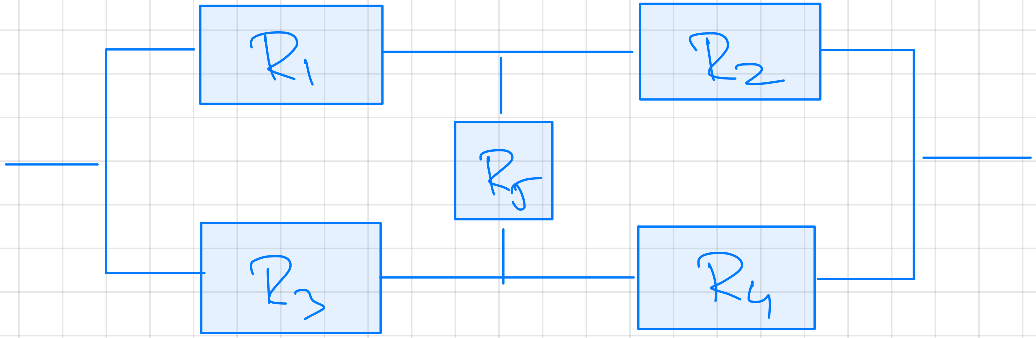
$$R_{r|n} = \sum_{i=0}^{n-r} C_n^{n-i} R^{n-i} (1-R)^i$$

dacă $r = n \rightarrow R_{n|n} = R^n$ – fiabilitatea structurii serie

$r = 1 \rightarrow R_{1|n} = 1-(1-R)^n$ – fiabilitatea structurii paralel

Fiabilitatea structurilor nedecompozabile

$R_{TOTAL} = ?$



Cazul 1: R_5 funcționează ($R_5 = 1$)

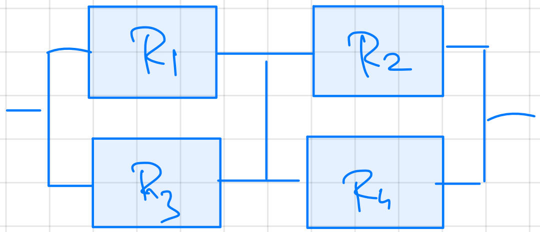
$R_{c_1}(S \mid R_5) = \dots$

Cazul 2: R_5 defect ($R_5 = 0$)

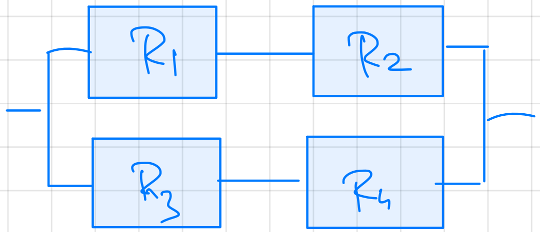
$R_{c_2}(S \mid \overline{R_5}) = \dots$

$$R_{TOTAL} = R_{c_1}(S \mid R_5) \cdot R_5 + R_{c_2}(S \mid \overline{R_5}) \cdot (1 - R_5)$$

$$R_{c_1}(S \mid R_5) = (R_1 \,\|\, R_3) \cdot (R_2 \,\|\, R_4) = (R_1 + R_3 - R_1 R_3)(R_2 + R_4 - R_4 R_2)$$



$$R_{c_2}(S \mid \overline{R_5}) = (R_1 R_2) \,\|\, R_3 R_4 = R_1 R_2 + R_3 R_4 - R_1 R_2 R_3 R_4$$



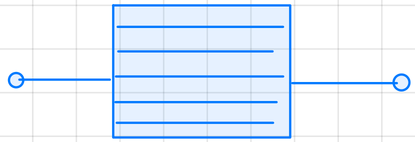
$R_{TOTAL} = \dots$

Fiabilitatea structurilor serie-paralel și paralel-serie

Să considerăm un modul format dintr-o celulă fotovoltaică.

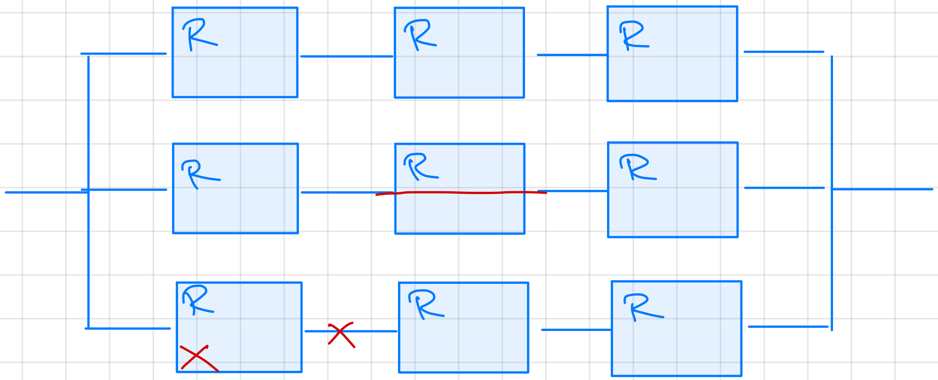
Moduri de defectare:

- întrerupere (prevalent) 
- scurtcircuit (relativ rar) 



Cum construiesc un panou fotovoltaic cu 9 celule care să fie fiabil?

Serie-Paralel

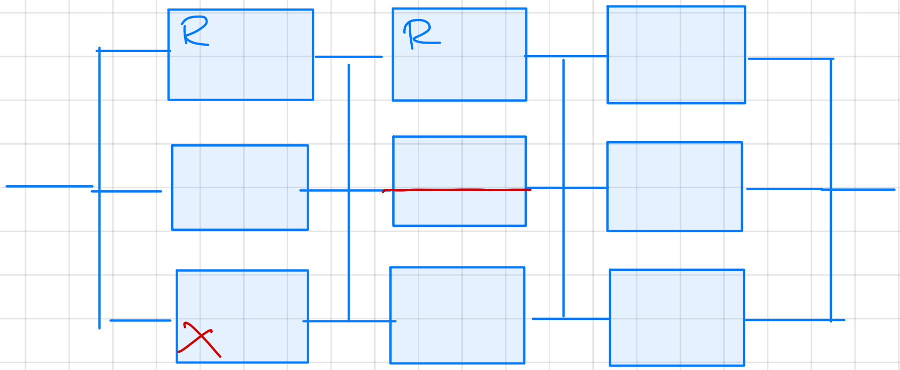


Scurtcircuit: pierdem 1 celulă din 9

întrerupere: pierdem 3 celule din 9

$$R_{SP} = 1-(1-R^3)^3$$

Paralel - Serie



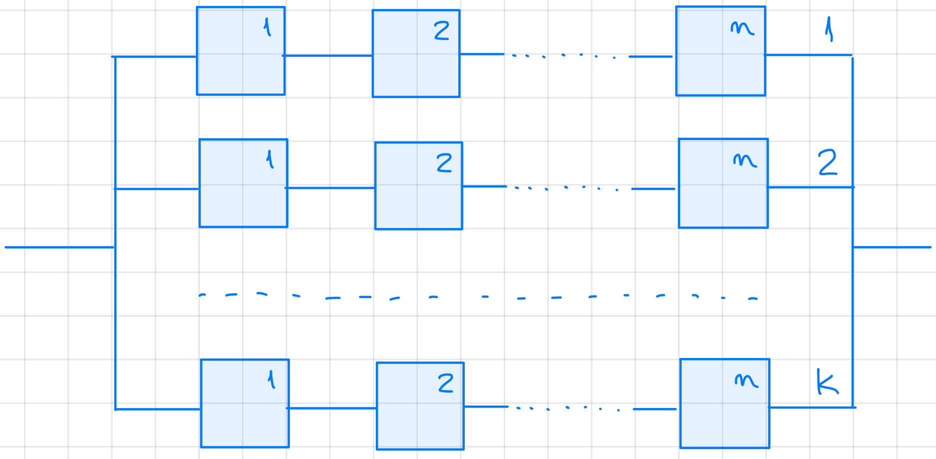
Scurtcircuit: pierdem 3 celule din 9

întrerupere: pierdem 1 celulă din 9

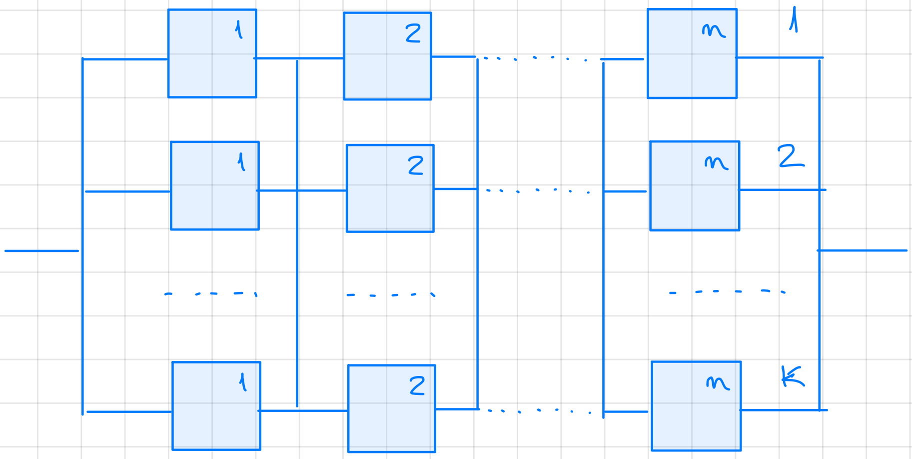
$$R_{PS} = (1-(1-R)^3)^3$$

Cazul general:

Serie-Paralel



Paralel – Serie

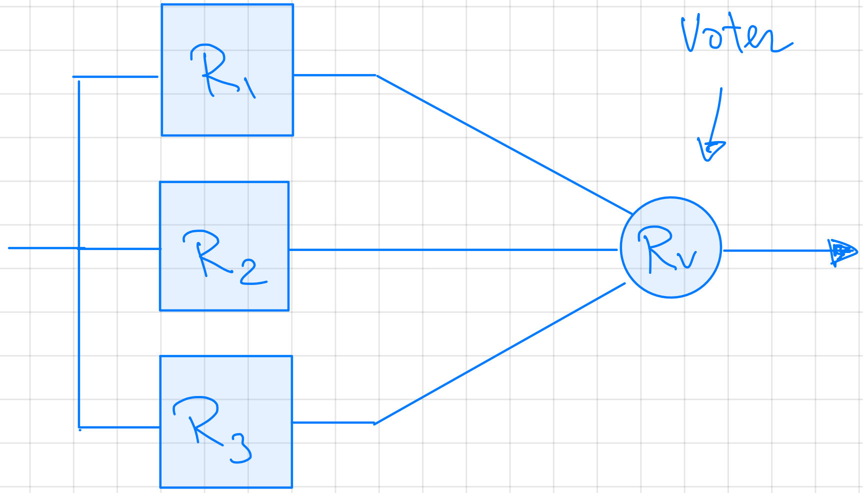


Presupunem că toate modulele sunt identice și au fiabilitatea R(t)

$$R_{SP} = 1 - (1 - R^n)^k$$

$$R_{PS} = \left[1 - (1 - R)^k\right]^n$$

Structuri cu votare majoritară



$$R_{2/3} = R_V \cdot \big(R_1R_2R_3 + R_1R_2(1-R_3) + R_1R_3(1-R_2) + (1-R_1)R_2R_3\big)$$

de obicei: $R_V >> R_1, R_2$ sau R_3

$$R_V \simeq 1$$

$$R_1 = R_2 = R_3 = R$$

$$R_{2/3} = R^3 + 3R^2(1-R) = R^3 + 3R^2 - 3R^3 = 3R^2 - 2R^3$$

dacă $R = 99\% \rightarrow R_{2/3} = 3 \cdot 0{,}99^2 - 2 \cdot 0{,}99^3 = 3 \cdot 0{,}98 - 2 \cdot 0{,}97 = 0{,}9997...$

$$R_{2/3} = 99{,}97\% > R$$

dacă $R = 10\% \rightarrow R_{2/3} = 3 \cdot 0{,}1^2 - 2 \cdot 0{,}1^3 = 3 \cdot 0{,}01 - 2 \cdot 0{,}001 = 0{,}028 = 2{,}8\%$

$$R_{2/3} = 2{,}8\% < R$$

$R_{2/3} > R$? $\Rightarrow 3R^2 - 2R^3 > R \Rightarrow 2R^3 - 3R^2 + R < 0$

$R(2R^2 - 3R + 1) < 0$, dar $R \in [0,1] \Rightarrow 2R^2 - 3R + 1 < 0 \Rightarrow$

$(R-1)(R-\frac{1}{2}) < 0 \Rightarrow$

$R \in (1/2, 1)$, deci:

$R_{2/3} > R$ doar dacă $R > 50\%$

R		$\frac{1}{2}$		1	
$R-1$	− − −	−	−	0	+ + +
$R-\frac{1}{2}$	− − −	0	+ +	+	+ +
$(R-1)(R-\frac{1}{2})$	+ + +	0	− −	0	+ + +

$$MTBF_{2/3} = \int_0^\infty R_{2/3}(t)\,dt = \int_0^\infty \left(3R^2(t) - 2R^3(t)\right)dt = 3\int_0^\infty e^{-2\lambda t}dt - 2\int_0^\infty e^{-3\lambda t}dt$$

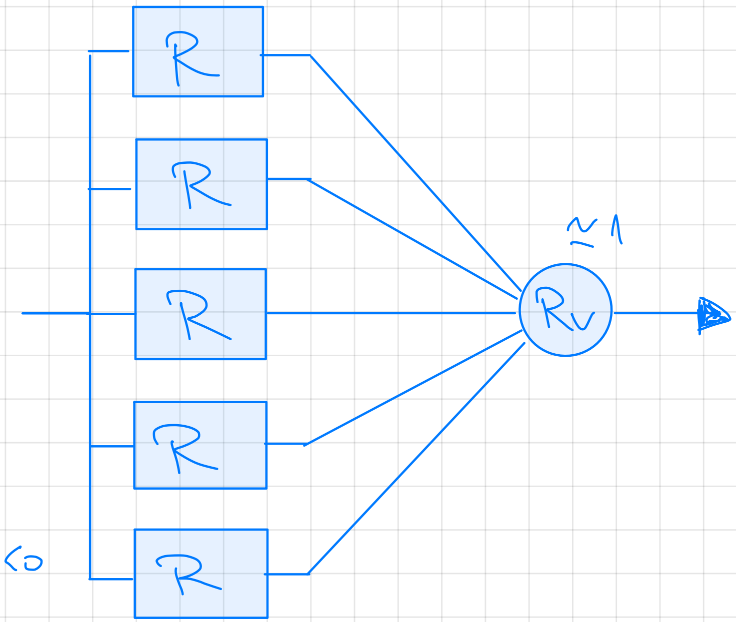
$R(t) = e^{-\lambda t}$

$$= \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{9-4}{6\lambda} = \frac{5}{6}\cdot\frac{1}{\lambda} = \frac{5}{6}\cdot MTBF$$

$$MTBF_{2/3} = \frac{5}{6} MTBF$$

$$MTBF_{2/3} < MTBF$$

$$R_{3/5} = R^5 + 5(1-R)R^4 + 10(1-R)^2R^3$$



$$R_{3/5} = R^5 + 5R^4 - 5R^5 + 10R^3 - 20R^4 + 10R^5 =$$
$$= 6R^5 - 15R^4 + 10R^3$$

$$R_{3/5} > R \ ? \Rightarrow 6R^5 - 15R^4 + 10R^3 > R \Rightarrow$$

$$\Rightarrow -6R^5 + 15R^4 - 10R^3 + R < 0 \Rightarrow R(1 - 10R^2 + 15R^3 - 6R^4) < 0$$

dar $R \geq 0 \Rightarrow 1 - 10R^2 + 15R^3 - 6R^4 < 0 \Rightarrow$

$$(R-1)\left(R-\frac{1}{2}\right)\left(R^2 - R - \frac{1}{3}\right) > 0 \Rightarrow \begin{cases} R \in \left(-\infty, \frac{1}{6}(3-\sqrt{21})\right) - \text{negativ} \ \times \\ R \in \left(\frac{1}{6}(3+\sqrt{21}), \infty\right) \rightarrow \text{pozitiv} > 1 \ \times \\ R \in \left(\frac{1}{2}, 1\right) \ \checkmark \end{cases}$$

Deci $R_{3/5} > R$ doar dacă $R > 50\%$

$$MTBF_{3/5} = \int_0^\infty R_{3/5}(t)\,dt = \int_0^\infty \left(6R^5(t) - 15R^4(t) + 10R^3(t)\right)dt = 6\int_0^\infty e^{-5\lambda t}dt - 15\int_0^\infty e^{-4\lambda t}dt +$$
$$+ 10\int_0^\infty e^{-3\lambda t}dt = {}^{12/}\frac{6}{5\lambda} - {}^{15/}\frac{15}{4\lambda} + {}^{20/}\frac{10}{3\lambda} = \frac{72-225+200}{60\lambda} = \frac{47}{60}\cdot\frac{1}{\lambda} = 0{,}783\ MTBF < MTBF$$

Caz general: votare majoritară n din 2n-1

$$R_{n|2n-1} = \sum_{i=0}^{n-1} C_{2n-1}^{i} \, R^{2n-1-i} (1-R)^{i}$$