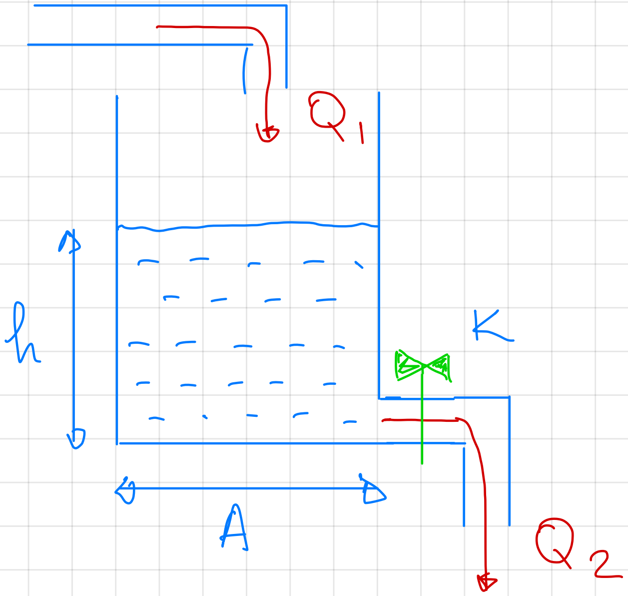


$K = 0 \rightarrow$ închis

$K > 0 \rightarrow$ deschis

$V = A \cdot h$

$Q_2 = K \cdot h$

$$\frac{dV}{dt} = Q_1 - Q_2 \Rightarrow$$

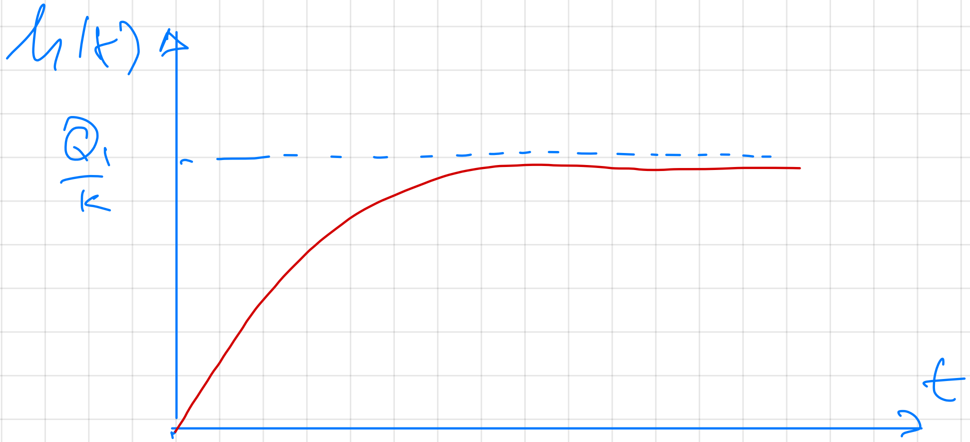
$$\left.\begin{aligned} &\frac{d(A \cdot h)}{dt} = Q_1 - K \cdot h \\ &\text{alegem } A = 1 \end{aligned}\right\} \Rightarrow \begin{aligned} &\frac{dh}{dt} = Q_1 - K h \\ &h(0) = 0 \end{aligned}$$

1. Cum variază înălțimea apei în raport cu timpul?

$$\left.\begin{aligned} &\frac{dh}{dt} = Q_1 - K h \\ &h(0) = 0 \end{aligned}\right\} \Rightarrow h(t) = \frac{Q_1}{K}\left(1 - e^{-Kt}\right)$$

$$\lim_{t \to \infty} h(t) = \frac{Q_1}{K}$$

$\frac{Q_i}{k} \rightarrow$ nivel la steady-state

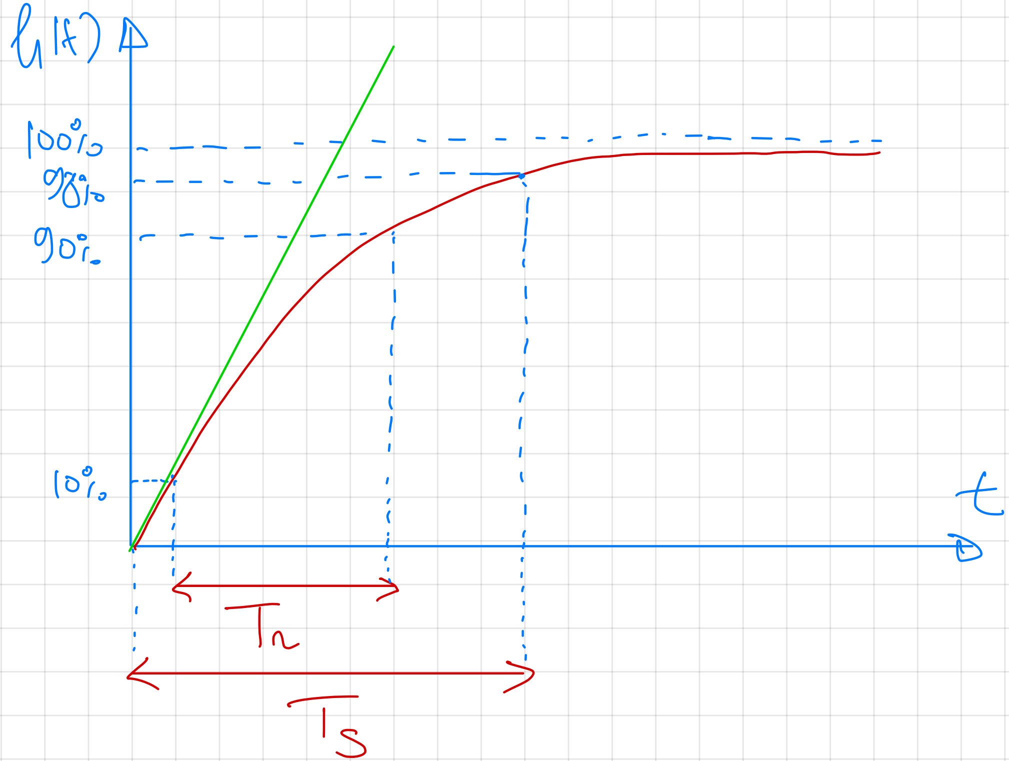


2. Cât de repede se umple rezervorul?

T_r - rise time

T_s - settling time

τ - constanta de timp



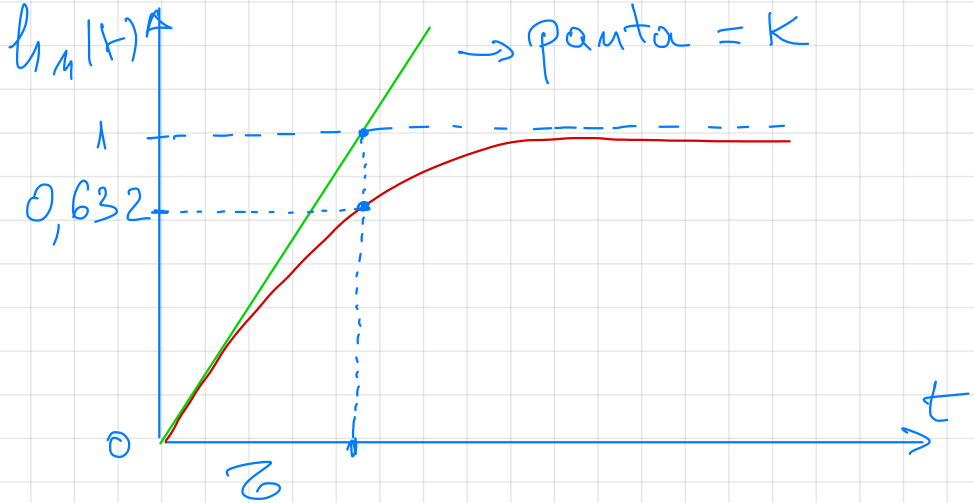
$$h(t) = \frac{Q_i}{k}\left(1-e^{-kt}\right) \Big| \cdot \frac{k}{Q_i} \Rightarrow$$

$$\frac{h(t)}{\frac{Q_i}{k}} = 1-e^{-kt} \Rightarrow \boxed{h_n(t) = 1-e^{-kt}}$$

$h_n(t)$ - înălțimea normalizată (valori între 0 și 1)

Care este panta la $t=0$?

$$\frac{d\,h_m(t)}{dt}\Big|_{t=0} = \frac{d(1-e^{-kt})}{dt}\Big|_{t=0} = K$$



$$\tau \overset{def.}{=} \frac{1}{K}$$

$$h_m(\tau) = 1-e^{-k\cdot\tau} = 1-e^{-k\cdot\frac{1}{k}} = 1-e^{-1} = 0,632..$$

De ex.: $K = 0,2/sec \Rightarrow \tau = 5\ sec$ → timpul ca h să ajungă la 63,2% din referință

τ este mare ⇒ sistemul e <u>lent</u>

τ este mic ⇒ sistemul e <u>rapid</u>

Rise Time (T_r)

$$0,9 = 1-e^{-k\cdot t_{90}} \Rightarrow t_{90} = -\frac{1}{k}\ln 0,1 = -\tau \ln 0,1$$

$$0,1 = 1-e^{-k\cdot t_{10}} \Rightarrow t_{10} = -\frac{1}{k}\ln 0,9 = -\tau \ln 0,9$$

$T_r = t_{90} - t_{10} = -\tau \ln 0{,}1 + \tau \ln 0{,}9 = \tau \cdot 2{,}19\ldots$

$T_r \simeq 2{,}2\tau$

Settling time (T_s)

$h_n(T_s) = 1 - e^{-K \cdot T_s} = 0{,}98 \Rightarrow T_s = -\tau \ln 0{,}02 = 3{,}912\tau$

$T_s \simeq 4\cdot\tau$

3. Cum influențează K nivelul apei?

$h(t) = \frac{Q_1}{K}\left(1 - e^{-Kt}\right)$ $\qquad h_{ss} = \frac{Q_1}{K}$

deci $K \uparrow \Rightarrow h_{ss} \downarrow$

$K \downarrow \Rightarrow h_{ss} \uparrow$

$$\frac{d h_{ss}}{dK} = -\frac{Q_1}{K^2}$$

Câștig logaritmic:

$$\frac{d\log h_{ss}}{d\log K} = \frac{\frac{1}{h_{ss}}}{\frac{1}{K}} \cdot \frac{dh_{ss}}{dK} = \frac{K}{h_{ss}} \cdot \frac{dh_{ss}}{dK} \longrightarrow \text{adimensional}$$

$$\frac{d\log h_{ss}}{d\log K} = \frac{K}{h_{ss}} \cdot \frac{dh_{ss}}{dK} \cdot \Big| \cdot \frac{h_{ss}}{K} \simeq \frac{h_{ss}\,\%}{K\,\%} \rightarrow \text{sunt schimbări procentuale}$$

$$L_K^h \simeq \frac{h_{ss}\,\%}{K\,\%}$$

De ex.: $L_K^h = 2 \Rightarrow$ 1% schimbare a K ne dă 2% schimbare a h_{ss}

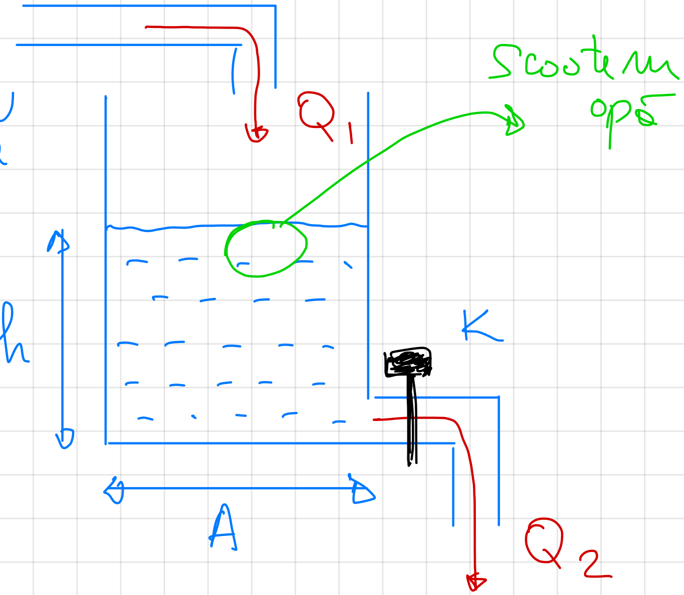
$$h_{ss} = \frac{Q_1}{K}, \quad \frac{dh_{ss}}{dK} = -\frac{Q_1}{K^2} \Big| \cdot \frac{K}{h_{ss}} \Leftrightarrow \frac{dh_{ss}}{dK} \cdot \frac{K}{h_{ss}} = -\frac{Q_1}{K \cdot h_{ss}} \Rightarrow$$

$$\Rightarrow \frac{dh_{ss}}{h_{ss}} \Big/ \frac{dK}{K} = -\frac{\cancel{Q_1}}{\cancel{K}} \cdot \frac{\cancel{K}}{\cancel{Q_1}} \Rightarrow \boxed{L_K^{h_{ss}} = -1}$$

4. Stabilitate (este sistemul stabil la perturbații?)

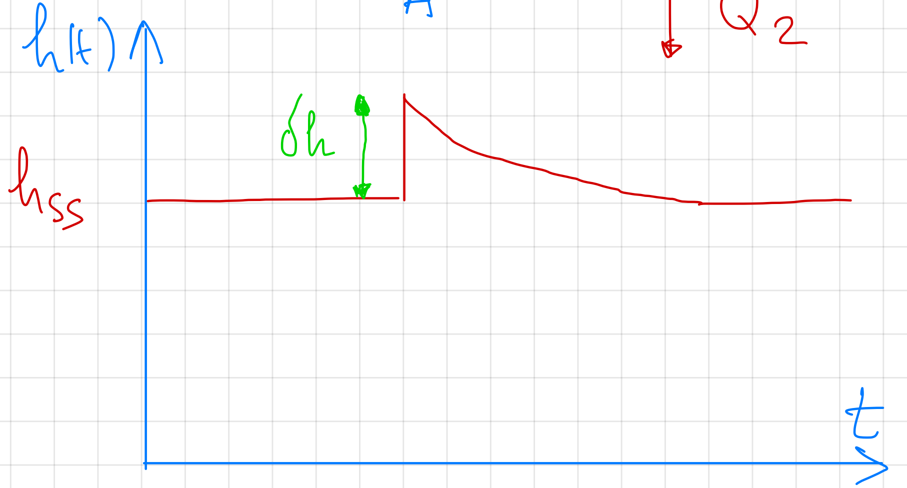
- Presupunem că sistemul a ajuns la steady-state
- Scoatem (sau adăugăm) o găleată de apă din rezervor
- Ce se întâmplă cu sistemul nostru?

δh → înălțimea cu care apa din rezervor scade (sau crește) dacă scoatem (sau adăugăm) apă



$$\frac{d(h_{ss} \pm \delta h)}{dt} = Q_1 - K(h_{ss} \pm \delta h) \Rightarrow$$

$$\cancel{\frac{d h_{ss}}{dt}} \pm \frac{d\,\delta h}{dt} = \underbrace{\cancel{Q_1 - K h_{ss}}}_{=\frac{d h_{ss}}{dt}} \mp K\delta h \Rightarrow$$



$$\boxed{\frac{d\,\delta h}{dt} = \pm K\delta h} \Rightarrow \frac{d\delta h}{\delta h} = \pm K dt \Rightarrow$$

$$\Rightarrow \int \frac{d\delta h}{\delta h} = \pm K\int dt \Rightarrow \ln(\delta h) = \pm Kt + c \Rightarrow$$

$$\Rightarrow \delta h = e^{-Kt+c},$$ dacă $t \to \infty$, $\delta h \to 0$ ⇒ **Sistemul este stabil la perturbații!**